CONSTRAINTS ON DISTRIBUTIVITY

By

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Simon Charlow and Veneeta Dayal
And approved by

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ABSTRACT OF THE DISSERTATION

Constraints on distributivity

By HOI KI LAW

Dissertation Director:
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Distributivity can be marked with lexical items like binominal each in English:

(1) The girls read three books each.

It has long been noted that some distributivity markers need to be licensed by the morphosyntactic makeup and/or interpretive properties of the predicate being distributed over (e.g. read three books in (1)). In this dissertation I investigate three distributivity markers that exhibit this type of licensing requirement:

1. binominal each in English (Safir and Stowell 1988, Zimmermann 2002, Champollion 2015, Kuhn 2017);

2. the verbal distributivity suffix saai in Cantonese (Tang 1996, Lee 2012);


The investigation leads to two findings. The first finding is that these licensing requirements should be understood as constraints on the dependencies arising from distributive quantification, which echo similar constraints proposed for various types of indefinites (Farkas 1997, 2002b, Brasoveanu and Farkas 2011, Henderson 2014, Kuhn 2017). A consequence of this finding is a more general
conception of constraints on dependencies: they are not only associated with indefinites (as conceived of in Farkas (2002b)), as they may be borne by distributivity markers.

The second finding is that constraints on dependencies may differ along a few parameters. One parameter determines whether a constraint makes reference to the internal mereological structure of dependencies, which arise from evaluating distributivity. Using the interactions of distributivity markers with extensive and intensive measure phrases (Zhang 2013), I conclude that the constraints under investigation make reference to the mereological nature of distributive dependencies. These constraints stand in contrast with constraints previously formulated for dependent indefinites (Farkas 1997, 2002b, Henderson 2014, Champollion 2015, Kuhn 2017), which do not need to access the mereological structure of dependencies. Another parameter determines whether a constraint requires dependence or independence. Using the contrast between binominal *each* and Mandarin *ge* on the one hand, and Cantonese *saai* on the other hand, I show that both parameters are used in natural language. This conclusion adds further support to the parallelism between constraints contributed by distributivity markers and those contributed by indefinites, as the dependence-independence parameter has also been used to characterize dependent indefinites and specific indefinites (e.g., Farkas 2002b, von Heusinger 2002).

To make constraints on dependencies formally explicit, I devise a version of dynamic plural logic with features from van den Berg (1996) and Brasoveanu (2008, 2013) to semantically represent dependencies arising from evaluating distributive quantification. The use of a dynamic logic, coupled with a delayed evaluation mechanism in terms of higher order meaning (Cresti 1995, de Swart 2000, Charlow (to appear)), allows the constraints to act as output constraints on distributive quantification, which mirror the use of output constraints in studies like Farkas (1997, 2002b), Henderson 2014, and Kuhn (2017).
ACKNOWLEDGEMENTS

This dissertation can be roughly broken down into dynamic semantics, distributivity, plurality, mereology, measurement, higher order meaning, cross-linguistic research, persistence (in a non-technical sense) and passion (in a non-technical sense). The people thanked below have helped me gain insight into one or more of these aspects. As always, there are more people to thank than there is space.

Simon Charlow (co-chair of my dissertation committee) joined Rutgers in 2014 and started teaching beautiful courses on dynamic semantics and monadic compositional semantics. The word ‘inspirational’ is an understatement of the effect of his teaching. My husband Haoze Li and I were in the same courses and we often spent hours admiring the way he packaged a lesson to make complex ideas flow naturally from simple ones. Thanks to him, I have learned that complexity is a set of simplicities in disguise, and there is nothing more satisfying than being able to use semantic tools as lego blocks. The dynamic semantics and higher order meaning part of the dissertation are credited to his teaching and research.

Veneeta Dayal (co-chair of my dissertation committee) taught me kinds, plurals, quantifiers, and questions, all of which (except for questions) play an important part in this dissertation. In addition, I have learned not just to appreciate the value of cross-linguistic research, but also how to put together threads of phenomena to form a holistic, theoretically interesting picture. Had it not been for Veneeta, this dissertation would have been a lot dryer for the writer and the reader. I also would like to thank Veneeta for reminding me to be open-minded and not be a fan of any framework. I have found this advice extremely liberating!

Ken Safir and Robert Henderson are both wonderful committee members to have. They have worked on topics that are directly related to the present work. Many of the key observations in the binominal each chapter go back to Ken’s joint work with Tim Stowell. The semantic architecture
in the whole dissertation is inspired by Robert’s work on dependent indefinites. I’m grateful to them for the beautiful research they did and the comments and suggestions they have given to me throughout the writing of this dissertation. A fun fact: Ken is a semanticist in disguise, and Robert has even more guises, as a morphologist, syntactician, and phonologist. A learning point: maybe don’t be a fan of any subfield.

I started working on Mandarin ge (the last chapter of this dissertation) after taking an extremely well-taught seminar on distributivity and event semantics with Lucas Champollion. His seminar touched upon not just distributivity, but also plurality and measurement, which are key players in this dissertation. Although I have not pursued an event-based framework for analyzing distributivity, there was a time when the core ideas of this dissertation lived in plural events rather than plural info-states. If not for the seminar with Lucas, I would not have gotten interested in distributivity at all, as I used to think that this topic was well explored enough.

I met with Anna Szabolcsi a few times to discuss research in this dissertation and those meetings have proven to be very important. An earlier version of the monotonicity constraint was too strong, as persistently pointed out by Anna (and less persistently by others). The weaker version proposed in the dissertation was developed following a meeting with Anna.

Kristen Syrett has to be praised for not shouting to me ‘hello, I work on distributivity!’ I only found out quite late in the game that I could have recruited more of her help. That said, through working with Kristen on other projects I have discovered that my heart is big enough for both theoretical linguistics and experimental linguistics. I cannot thank her enough for demonstrating to me how to fruitfully combine these two pursuits. Kristen’s time management and persistence are inspirational. Every time I find it too hard to balance my role as a researcher and a mother, I think of Kristen, her children, her dogs, her yoga, her teaching, her research, and her passions. Peace always comes to me afterwards.

I’m also indebted to Yimei Xiang for her friendship and support, to Jane Grimshaw for her inspiring teaching, to Maria Bittner for her courses in dynamic semantics, as well as to Akin Akinlabi and Paul de Lacy for the phonology courses I have taken with them.

Many fellow graduate students at Rutgers Linguistics have helped me adapt to the life in New Jersey. Sarah Hansen, Diti Bhadra, and Yağmur Sağ have provided me with the much needed friendship and company. Ümit Atlamaz and his lovely family always inspire me to become a better
parent. Events with Kunio Kinjo, Shu-hao Shih, Mingming Liu, Sha Zhu and Haoze Li formed some of my fondest memories about Rutgers. Above all, I admire my friends for their passions for linguistic research. There is no problem too hard for them. I’m blessed to have been in this highly stimulating linguistic community.

The rich experience outlined above is only possible because the linguistics department at Chinese University of Hong Kong had cultivated my interest in linguistics and supported my early research in syntax (with Candice Cheung) prior to my study at Rutgers. I thank the courageous group (Gladys Tang, Virginia Yip, Thomas Lee, Yang Gu, Ping Jiang) for their efforts in establishing the first linguistics department in Hong Kong, which directly benefited students like me.

Lastly, I’m very grateful to my family for their constant support and patience. Without their unconditional support, I would not have been able to pursue a research career, let alone writing a dissertation on theoretical linguistics.
DEDICATION

To my parents.
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LIST OF ABBREVIATIONS

The following glossing conventions are used for Cantonese and Mandarin examples:

<table>
<thead>
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<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ASP</td>
<td>Aspect marker</td>
</tr>
<tr>
<td>CL</td>
<td>Classifier</td>
</tr>
<tr>
<td>MOD</td>
<td>Modifier marker</td>
</tr>
<tr>
<td>PL</td>
<td>Plural marker</td>
</tr>
<tr>
<td>PROG</td>
<td>Progressive aspect marker</td>
</tr>
<tr>
<td>RES</td>
<td>Resultative verb marker</td>
</tr>
<tr>
<td>SFP</td>
<td>Sentence-final particle</td>
</tr>
<tr>
<td>SG</td>
<td>Singular marker</td>
</tr>
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</table>
1

INTRODUCTION

1.1 Overview

Research on distributivity has established that there are morphological markers across languages that signal distributive quantification (Roberts 1987, Link 1987, among many others). A well-known example is English each, as shown in (1).

(1) The girls each made a kite.

Many generalizations about distributive markers have been drawn that shed light on the process of distributive quantification. For instance, one requirement is that the subject being targeted by each for distributive quantification (henceforth, the (Distributive) Key,) has to contribute a plural entity. This requirement distinguishes the girls in (1) from the girl in (2), as only the former contributes a plurality for quantification. This requirement informs us that distributive quantification must not be vacuous—should there be only one girl, ordinary predication without distributivity already suffices.

(2) *The girl each made a kite.

1For the purpose of this study, ‘signaling’ distributivity does not equal contributing a distributive operator. While some distributive markers may indeed contribute a distributive operator as part of their semantics (as maybe the case with binominal each), others may merely require the presence of a distributive operator (as will be argued for in the cases of Cantonese saai and Mandarin ge). The main criterion for determining whether a distributive marker contributes a distributive operator is its compatibility with a distributive quantifier or another distributive marker. If it is not compatible with a distributive quantifier or another distributive marker, then it is taken to encode a distributive operator in its semantics.
As another example, some distributive markers, like *each*, cannot quantify over the denotation of a mass noun, as shown in (3), suggesting that these distributive markers need access to a set of neatly defined atoms (Roberts 1987, Link 1987, Zimmermann 2002, Champollion 2010).2,3

(3) *The water each leaked to the floor.

There are many other observations of this kind, which give rise to rich generalizations ranging from cover distributivity (when distributive quantification is over a cover induced on the Key; see Scha 1981, Schwarzchild 1996) to long-distance distributivity (when the distributive marker is not adjacent to the the Key; see Zimmermann 2002, Dotlačil 2012, Champollion 2017). However, it is fair to say that the majority of these generalizations pertain to the relationship between a distributive marker and a distributivity key.

Not till recently have linguists started to pay more attention to the relationship between distributive markers and expressions in their *(Distributed) Share*, i.e., the predicate being distributively predicated of the Key. A survey of the literature reveals that many distributive markers impose selectional requirements on what expressions may occur in the Share and the range of interpretations these expressions may assume. However, these selectional requirements on the Share are either poorly understood or do not form a coherent picture.

The aim of this dissertation is to gain a better understanding of the selectional requirements on the Share in general by investigating a few distributive markers that have been reported to bear these requirements. The overarching finding, which I report in the next few chapters, is that requirements on the Share are constraints on functional dependencies.4 More precisely, they are constraints

---

2There are many distributive markers that do not require a distributivity key with accessible atomic parts. A famous example is Mandarin *dou* (Lin 1998a). However, *dou*’s status as a distributive marker is disputed (Chen 2008, Xiang 2008, Liu 2016, Xiang 2016). English *all* is another often cited example, but its status has also been challenged by Brisson (1998, 2003) (cf. Champollion 2017).

3Two qualifications about mass nouns are relevant here. First, some mass nouns, sometimes known as fake mass nouns (e.g., *furniture, footwear*), can intuitively have atoms in their reference. However, they are still incompatible with *each* without plural morphology on the noun. Second, how to capture the count-mass distinction is of on-going interest in linguistics and philosophy of language. Link (1983) suggests that count nouns and mass nouns draw their references from distinct domains: the former are from atomic semilattices and the latter are from semilattices that are atomless. Chierchia (1998a) agrees with Link (1983) with respect to the reference of count nouns but argues that a mass noun draws its reference from a semilattice that includes both atoms and their sums. Rothstein (2004, 2010) suggests coding this version of count-mass distinction as a type difference. Chierchia (2010) later develops a single domain proposal, arguing that count nouns and mass nouns both draw their references from domains with atoms and their sums. However, with mass nouns the semilattices have atoms that are too vague to be counted.

4I use the term ‘functional dependency’ to refer to two sets A and B that stand in a certain relationship R. If R does not map the same individual in A to different values in B (in other words, if R is a function), then we can naturally say
on the functional dependencies arising from distributive quantification. This conclusion provides corroborations for two lines of research in the linguistic literature.

First, it provides additional evidence that distributive quantification brings about a set of functional dependencies and these dependencies should be made accessible to compositional semantics. Roberts (1987), Schein (1993), Kamp and Reyle (1993), Lasersohn (1995), Elworthy (1995), Krifka (1996a), van den Berg (1996), and Jackendoff (1996) are representative the studies that argue for the representation of the functional dependencies arising from distributivity. Since then, various attempts have been made to make available these dependencies, mainly using resources from Event Semantics (Schein 1993, Lasersohn 1995, Champollion 2017) and various versions of Dynamic Semantics (Elworthy 1995, Krifka 1996a, van den Berg 1996, Nouwen 2003, Brasoveanu 2008, 2011, Henderson 2014, Kuhn 2017). The fact that the very markers that signal distributivity also impose constraints on functional dependencies arising from distributive quantification provides an important and direct argument that distributivity makes available dependencies that must be compositionally available.

Second, it narrows the gap between garden-variety distributive markers and markers of dependent indefinites. Dependent indefinite markers have long been noted to signal narrow scope and/or co-variation (Choe 1987a, Farkas 1997, Balusu 2005, Henderson 2014, Cable 2007, and Kuhn 2017). An example of a dependent indefinite from Kaqchikel is given below as an illustration.

(4) K-onojel x-ø-ki-kano-j ju-jun wuj.
E3p-all CP-A3s-E3p-search-SS one-RED book
‘They looked for one book each.’ Kaqchikel

that B is functionally dependent on A. If R is a relation that maps a single value in A to more than one value in B, then a functional dependency can still be obtained by defining \( f : A \rightarrow P(B) \). For concreteness, suppose \( A = \{a, b\} \), \( B = \{c, d, e\} \) and they stand in a relation \( R = \{(a, c), (a, d), (b, e)\} \). We can define an \( f : A \rightarrow P(B) \), such that \( f(a) = \{c, d\} \) and \( f(b) = \{e\} \). This general definition of functional dependencies can be used for a functional structure or a relational structure.


\(^{6}\) Dependent indefinites are sometimes used interchangeably with distributive (or dependent) numerals to refer to the same or a closely related class of indefinites (e.g., Cable 2014, Champollion 2015). However, distributive/dependent numerals may also refer to a smaller set of expressions that consists of numeral expressions to the exclusion of non-numeral indefinites (Farkas 2015). When dependent indefinites are marked by reduplication, they are also called ‘reduplicated numerals’ (Gil 1988, Balusu 2005). Dependent indefinites do not form a homogeneous class, as pointed out in Henderson (2014) and Farkas (2015) (see also Yanovich 2005). The heterogeneity of dependent indefinites is not taken up in this dissertation.

b. Variation inference: More than one book was looked for.

The indefinite containing the distributive numeral, marked by partial reduplication, has to be interpreted distributively (giving rise to the distributive inference) and must co-vary with the Key (giving rise to the variation inference). According to the findings in this dissertation, there is no fundamental difference between distributive markers and markers of distributive numerals: they both signal distributive quantification and have the capacity to impose constraints on its outcome. When the constraint capacity is not used, we obtain ordinary distributivity. When the constraint capacity is used, we get distributive numerals and distributivity markers that bear selectional requirements on the Share. Building on the co-variation requirement of distributive numerals, this dissertation takes up a variety of novel requirements distributive markers in various languages impose on the functional dependencies of distributivity.

The distributive markers taken up in this research are English *each* (in three positions, binominal in (5-a), adverbial in (5-b), and determiner in (5-c)), Cantonese *saai* (6), and Mandarin *ge* (7).\(^7\)

(5)  
(a) The girls made one kite *each*.
(b) The girls *each* made a kite.
(c) Each girl made a kite.

(6)  
Di-neoizai zing-*saai* fungzeng.  
CL-girl make-SAAI kite  
‘The girls each made one or more kites.’  
\textit{Cantonese}

(7)  
Nühai-men *ge* zuo-le yi-zhi fungzeng.  
girl-PL GE make-ASP one-CL kite  
‘The girls each made a kite.’  
\textit{Mandarin}

What makes these distributive markers special is that they have all been noted as imposing additional morphosyntactic and/or interpretive requirements on their Share:

- Binominal *each* requires the support of a so-called counting quantifier in the distributed share

\(^7\)Cantonese *saai* does not have a correlate in Mandarin and Mandarin *ge* does not have a correlate in Cantonese. In some Chinese dialects (such as the Henan dialect), *ge* is used to mean ‘different’ or ‘odd’ (p.c. Mingming Liu). It is important to note that although the distributive marker *ge* is homophonous with the classifier *ge*, they are in fact distinct lexical items with different written forms.
(Safir and Stowell 1988, Sutton 1993, Szabolcsi 2010), and further requires that the counting quantifier obligatorily co-vary with the distributivity key (Choe 1987a). Determiner *each* has been reported to signal that the events participating in distributive predication are disjoint events, or differentiated events, in the sense of Tunstall (1998) (see also Beghelli and Stowell 1997, Brasoveanu and Dotlačil 2015). Although not reported in previous studies, adverbial *each* essentially patterns like determiner *each* in this respect.

- Cantonese *saai* generally resists counting quantifiers in the distributed share (Lee 1994, Tang 1996), unless they fail to co-vary with the distributivity key.

- Mandarin *ge* needs to be supported by a counting quantifier, an expression with a pronominal element bound by the distributivity key, or a special type of distributivity involving a ‘respectively’ reading (Kung 1993, Lin 1998b, Soh 2005, Tsai 2009, Lee et al. 2009a).

Despite having been documented and studied on a case-by-case basis, the selectional requirements these distributive markers imposes on the Share have not been studied together as a natural class of phenomena pertaining to distributivity. It is useful to take them up as a natural class for two reasons. For one thing, we will get a more holistic picture of the selectional requirements, which will help us understand their role in natural language. For another, since they are all found on markers that signal distributivity, we should expect them to be intimately related to distributive quantification. The following heuristic suggested in Szabolcsi (1997) is useful here.

What range of expressions actually participates in a given process is suggestive of exactly what that process consists in.

Specifically, the range of selectional requirements that may show up with distributive quantification is suggestive of what distributive quantification consists in. The contribution of this dissertation lies in recognizing a family of measurement-sensitive constraints and taking them to suggest that distributive quantification results in dependencies with a mereological structure.

Despite the apparent heterogeneity of the selectional requirements studied in this dissertation, I argue that they have a common core—they are all constraints on the functional dependencies arising from distributive quantification. The differences lie in the type of constraints, which we can understand in terms of a few parameters.
It is useful to discuss, at an informal level, how functional dependencies can be constructed with the help of a sentence like (8). Intuitively, this sentence can be verified in a number of ways. Two examples are shown in Figure 1.1.

(8) The girls each read a book.

In both scenarios, a girl stands in the reading relation with a book. In the one on the left, every girl read a different book. In the one on the right, two girls read the same book. However, both scenarios verify (8). Let us call these informal representations of how the girls stand in relation to the books they read functional dependencies. A sentence with a distributive marker may give rise to a set of functional dependencies, expressing relationships between the Key and various expressions in the Share. With this much background, we are ready to discuss different types of constraints that can be imposed on these functional dependencies.

First, they may differ on whether the functional dependency is required to exhibit dependence, as shown in Figure 1.2 (left) or independence, as shown in Figure 1.2 (right). The former requires that part of the distributed share co-varies with the distributivity key, as has been argued for distributive numerals and distributivity with binominal each in English, and will be argued for other uses of each as well as distributivity with Mandarin ge. The latter leads to the lack of co-variation of part of the distributed share relative to the distributivity key, as will be argued for distributivity with Cantonese saai.

Second, they may differ on whether (in)dependence is required at the value level or at the structural level. Informally speaking, a functional dependency is said to exhibit value (in)dependence when (in)dependence is determined without making reference to the internal mereological structure

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8 See also footnote 4.
9 Co-variation here does not pick out just injective mappings. It pick out mappings that are non-constant.
Figure 1.2: Functional dependencies that exhibit dependence (left) and independence (right)

of the dependency. Otherwise, it exhibits structural (in)dependence. More precise definitions of
these two terms are provided in Chapter 2, Definitions 8, 14 and 15.

A concrete example will help illustrate the difference. Suppose the interpretation of (9) estab-
lishes a functional dependency between a set of angles and their corresponding angle degrees, as
shown in Figure 1.3 (left) and the interpretation of (10) does so for a set of drinks and their corre-
spanding temperatures, as shown in Figure 1.3 (right). The two types of functional dependencies
look exactly the same and lack value dependence.

(9) The angles are 60 degrees each.

(10) *The drinks are 60 degrees (Fahrenheit) each.

Figure 1.3: Both functional dependencies lack value dependence

However, the functional dependency on the left differs crucially from the one on the right in an
important way. The former is built on an extensive measure function, which is additive, while the
latter is built on an intensive measure function, which is not additive. The definition of additivity
given in (11) is taken from Krifka (1998) (with his concatenation operator replaced by a summation
operator).

(11) A measure function $\mu$ (from a set of entities in $D$ to a set of positive real numbers) is additive
iff
∀x, y ∈ D : \mu(x \oplus y) = \mu(x) + \mu(y)

The functional dependency built with an extensive measure function licenses inferences about the measurement of the mereological sum of all the individuals in the functional dependency. In addition, there is a guarantee that bigger individuals are mapped to bigger measurements (e.g., in terms of numerical values), as shown in Figure 1.4 (left). However, the functional dependency built with an intensive measure function cannot license the same type of inference. We need much more information to find out the measurement of the sums (when the measurements of the parts are not the same), and there is no guarantee that bigger individuals will always be mapped to bigger degrees, as shown in 1.4 (right).

<table>
<thead>
<tr>
<th>Angle1+2+3</th>
<th>180°</th>
<th>Drink1+2+3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle1+2/1+3/2+3</td>
<td>120°</td>
<td>Drink1+2/1+3/2+3</td>
</tr>
<tr>
<td>Angle1/2/3</td>
<td>60°</td>
<td>Drink1/2/3</td>
</tr>
</tbody>
</table>

**Figure 1.4:** Functional dependencies with structural dependence (left) and without (left)

Constraints requiring value dependence have been explored in previous studies on distributive numerals (as well as dependent indefinites). In this study, I show that structural dependence is also relevant for formulating constraints on distributivity. In particular, I argue that English *each* and Mandarin *ge* exhibit structural dependence, while Cantonese *saai* exhibits structural independence. These claims are summarized in Table 1.1

<table>
<thead>
<tr>
<th>Value Structure</th>
<th>Dependence</th>
<th>Distributive numerals</th>
<th>Binominal <em>each</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Adverbial and determiner <em>each</em></td>
<td>Independence</td>
<td>N/A</td>
<td>Mandarin <em>ge</em></td>
</tr>
</tbody>
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<td>N/A</td>
<td>Mandarin <em>ge</em></td>
</tr>
</tbody>
</table>

Table 1.1: Major empirical claims in the dissertation

In summary, this dissertation advocates a treatment of garden-variety distributive markers along the lines of distributive numeral markers: distributive markers make a two-part contribution—signaling distributivity and imposing constraints on the functional dependencies arising from distributive
quantification. By doing so, it demonstrates that in addition to value dependence, a relatively well-studied constraint, there is a wider range of constraints that target functional dependencies contributed by distributivity.

Since having access to the functional dependencies of distributivity is of critical importance for modeling constraints on distributivity, I discuss how to build these functional dependencies in the next section.

1.2 Building functional dependencies

A functional dependency is a relation between two sets, such that determining a value in the first set uniquely determines a value in the second set. Many studies have observed that sentences with a distributive quantifier (every NP or each NP) or a distributive marker give rise to functional dependencies. For example, consider the following data:

(12) Every man\(x^x\) loves a woman\(y\). The old men\(x\) bring them\(y\) flowers to prove this. (van den Berg 1996:126)

(13) The students\(x\) each wrote an article\(y\). They\(x\) each sent it\(y\) to L&P. (Krifka 1996a:557)

The plural pronoun in (12) has to refer to the corresponding women loved by the old men, as introduced in the preceding sentence. Likewise, the singular pronoun in (13) picks out a different article written by each student, a dependency introduced in the preceding sentence.

A number of studies have motivated an extension of DRT or other versions of dynamic semantics to model anaphora to dependency (e.g., Elworthy 1995, Krifka 1996a, van den Berg 1996, Nouwen 2003, Brasoveanu 2007, 2008). This study follows van den Berg (1996), Brasoveanu (2007, 2008), Henderson (2014) and Kuhn (2017) in using Dynamic Plural Logic (DPlL) to model the functional dependencies established as a by-product of evaluating distributive quantification. I have chosen this framework for the following reasons:

- It is a plural logic capable of modeling and compositionally constructing functional dependencies, which are important for stating constraints on these dependencies.
• It is a dynamic logic capable of transmitting functional dependencies in the course of interpretation. These functional dependencies can be retrieved with the use of standard anaphoric devices in dynamic semantics, allowing a streamlined compositional analysis.

• Other recent studies have used a version of DPIL for studying distributive numerals (e.g., Henderson 2014, Champollion 2015, Kuhn 2017). Having the present study couched in a similar framework facilitates the comparison between the distributivity markers taken up in this study and distributive numerals discussed in previous studies.

I introduce the framework of DPIL used in this dissertation in Chapter 2. The backbone of the logic comes from van den Berg (1996). On top of that, I borrow domain pluralities and compositionality from Brasoveanu (2007, 2008) (but not dependency-introducing variable introduction (aka. random assignment) and distributively-evaluated lexical relations). A detailed comparison among the logic developed in this dissertation and its predecessors in van den Berg (1996) and Brasoveanu (2008) is given in Section 2.3 of Chapter 2.

1.3 Distributive markers studied in this dissertation

1.3.1 Binominal each

Binominal each is taken up in Chapter 3. The key observation is that noun phrases marked by binominal each pattern like dependent indefinites in requiring obligatory co-variation with the distributivity key, as shown in (14) (Safir and Stowell 1988, Champollion 2015, Kuhn 2017). In addition, these noun phrases must be either counting quantifiers (Sutton 1993, Szabolcsi 2010) or measure phrases with an extensive measure function (Zhang 2013), as shown in (15) – (16-b).

(14) The girls read two books each, namely, *Brave New World and Animal Farm.

(15) The girls read *some/two books each. Counting quantifier

(16) a. The angles are 60 degrees each. Extensive measurement
    b. *The drinks are 60 degrees each. Intensive measurement

These properties are used to motivate a constraint requiring structural dependence in the functional
dependencies established by distributivity with binominal each. The constraint is called a monotonic measurement constraint to indicate the importance of measurement in the formulation of this constraint.

1.3.2 Cantonese saai

Saai is a verbal suffixal serving to mark distributivity in Cantonese (Lee 1994, Tang 1996, Lee 2012, Lei 2017). Saai has been noted to resist indefinites in the Share, as shown in (17), but not definities, as shown in (18). Although previous studies have speculated that saai has a preference for specificity or definiteness (Lee 1994, Tang 1996), it has not been made clear how such a preference is related to saai’s role as a distributive marker. In addition, saai also displays a sensitivity to the type of measurement that is distinct from binominal each’s sensitivity: while intensive measurement can occur comfortably following saai, as shown in (19), extensive measurement cannot, as shown in (20).

(17) *Di-neoizai zing-saai jat-zek fungzeng.
     CL.PL-girl make-SAAI one-CL kite
     Intended: ‘The girls each made a kite.’ Indefinite

(18) Di-neoizai gin-saai go lousi.
     CL.PL-girl make-SAAI CL kite
     ‘The girls each saw the teacher.’ Definite

     CL.PL-girl buy-SAAI three-liter-pack-MOD water
     ‘The girls each bought water in three liter packs.’ Intensive

(20) ??Di-neoizai maai-saai saam-sing seoi.
     CL.PL-girl buy-SAAI three-liter-MOD water
     ‘The girls each bought three liters of water.’ Extensive

Saai is incompatible with most measure predicates so minimal pairs like (16-a) and (16-b) cannot be easily constructed.

In English, extensive measurement is typically expressed in the so-called ‘pseudo-partitive’ form (e.g., three liters of water) while an intensive measurement is expressed in the so-called ‘attributive’ form (e.g., three-liter water) (Schwarzschild 2006). Cantonese encodes the extensive-intensive distinction with the help of a combination of the modification marker ge and the nominal suffix zong ‘pack’. Generally speaking, when both zong and ge are present, a numeral-classifier complex is used intensively and when they are absent, the numeral-classifier complex is used extensively. However, when the classifier in a numeral-classifier complex is a container noun (e.g., seong ‘box’) or a measurement unit (e.g., sing ‘liter’), ge can be present in both the extensive and the intensive use, although the additional presence of zong ‘pack’ rules out the extensive use.
In Chapter 4, I show that there is a principled reason for the resistance to indefinites—*saai* requires a specific form of functional dependencies, in which a post-*saai* expression exhibits *independence* relative to the distributivity key. Sentences like (17) are ruled out because when a kite is chosen independently of the girls, there could only be one kite, and the girls could not distributively make the same kite. I explore two analyses to account for *saai*’s resistance to dependence, and ultimately argue in favor of an *independence constraint*. The constraint is shown to stand in opposition to the structural dependence requirement (i.e., the monotonic measurement constraint) of binominal *each*. Once understood as a constraint on the mereological structure of a distributive dependency, the sensitivity to the extensive-intensive distinction follows. In particular, the intensive measurement in (19) measures water volume per container (or per package), which does not change with more girls buying more water. However, the extensive measurement in (20) measures the water volume, which does change with more girls buying more water.

1.3.3 Generalized monotonicity: Mandarin *ge* and other uses of *each*

This chapter takes up Mandarin *ge* and the non-binominal uses of English *each*. *Ge* is an adverbial distributive marker in Mandarin. Like binominal *each* and Cantonese *saai*, *ge* imposes specific requirements on the Share (Lin 1998b, Soh 2005, Lee et al. 2009a, Tsai 2009). On one hand, it patterns like binominal *each* in favoring counting quantifiers and measure phrases with an extensive measure function in its distributed share.

(21) Zhe-xie haizi ge kan-le *(liang-chu) dianyin.
these-CL child GE see-ASP two-CL movie
‘The children each saw two movies.’ Counting quantifier

(22) Zhe-xie haizi ge zhong 100-bang.
these-CL child GE weigh 100-pounds
‘The children are 100 pounds each.’ Extensive measurement

(23) ??Zhe-xie yinliao ge re 50-du.
these-CL child GE heat 50-degrees
‘The drinks are 50 degrees each.’ Intensive measurement

On the other hand, it differs from binominal *each* in that it can be licensed by expressions interpreted as dependent on the Key despite their lack of a measurement component. These expressions include
I propose that the monotonic measurement constraint, formulated to account for the licensing requirements of binominal each, can be generalized to account for the licensing requirements of Mandarin ge. More specifically, while binominal each’s constraint can only be satisfied by structural dependence between individuals and measurements, ge’s constraint can be satisfied by structural dependence between individuals and measurements, as well as structural dependence between two sets of individuals.

A further generalization of the monotonicity constraint to allow events and their thematic dimensions to participate in the relevant mappings allows us to subsume the ‘event differentiation condition’ of non-binominal uses of each (Tunstall 1998, Brasoveanu and Dotlačil 2015). The event differentiation condition has been argued to be responsible for the additional inferences (indicated in italics) in sentences with each:

(26) a. The girls each walked to the park.
    ≈ The girls walked separately to the park.

    b. Each girl walked to the park.
    ≈ Each girl walked to the park by herself.

1.4 Conclusion

In short, I propose to treat distributive markers that impose selectional requirements on the Share as a natural class. Their requirements can be uniformly understood as constraints on the functional dependencies arising from distributive quantification. The fact that a glance at three languages reveals a set of distributive markers bearing such constraints is suggestive of two things. First, distributive
markers often have dual functions, signaling the presence of distributive quantification and imposing constraints on the outcome of distributive quantification (see also Balusu 2005, Henderson 2014, Kuhn 2017). Second, for the constraints to be satisfiable, distributivity must contribute dependencies that are available to compositional semantics, as argued in Krifka (1996a), van den Berg (1996), Nouwen (2003), Brasoveanu (2008), Henderson (2014), and Kuhn (2017).
2.1 Overview

In this dissertation, functional dependencies are formalized using sets of assignment functions, as in Dynamic Plural Logic (van den Berg 1996; see also Nouwen 2003 for an assignment-free implementation) and Plural Compositional DRT (Brasoveanu 2007, 2008, 2010, 2013; see also Dotlačil 2010, Henderson 2014 and Kuhn 2017). The core semantic framework is based primarily on van den Berg (1996), with the following enrichments and modifications: (i) subsentential compositionality, as borrowed from PCDRT, (ii) Linkean referential pluralities in the range of assignment functions, also borrowed from PCDRT, and (iii) degrees in the range of assignment functions.

These modifications are empirically motivated. Subsentential compositionality is needed for modeling how chunks of meaning are pieced together. The DPIL in van den Berg (1996) does not have any referential plurality as referential pluralities are completely replaced plurals generated by a set of assignments. However, referential pluralities are needed in DPILM for modeling expressions whose references do not have a well-defined or accessible atomic tier, such as *a lot of milk* and *a large amount of sewage* (Quine 1960, Link 1983, Chierchia 2010; cf. Chierchia 1998a, Rothstein 2004, 2010).\(^1\) Generalizing to the ‘worst case’, i.e., to mass nouns, all noun phrases can introduce referential pluralities in DPILM. Since functional dependencies involving these expressions are an important subject of investigation in this dissertation, it is necessary to allow them to be generated

\(^1\)See also footnote (2) in Chapter 1 for more discussion on mass noun reference.
in the first place. Degrees are needed to model functional dependencies involving individuals and their measurements, which is also an important subject of investigation in this dissertation. To make it easier to distinguish between the original DPIL and the modified version here, I call the current framework Dynamic Plural Logic with Measurement (DPILM).

This chapter has two parts. In section 2.2, I lay out the core semantics of DPILM. Then, in Section 2.3 I review the major differences between DPILM, DPIL, and PCDRT.

2.2 Dynamic Plural Logic with Measurement

2.2.1 Types and model

Like PCDRT, DPILM is a typed logic. Table 2.1 lists all the primitive types and the objects associated with them:

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Variables</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individuals</td>
<td>$e$</td>
<td>$x, y, z$</td>
<td>$a, b, a+b$</td>
</tr>
<tr>
<td>Events</td>
<td>$\nu$</td>
<td>$\epsilon, \epsilon'$</td>
<td>eat1, eat2</td>
</tr>
<tr>
<td>Degrees</td>
<td>$\sigma$</td>
<td>$d, d'$</td>
<td>$\langle 80kg, \text{weight}, \text{john} \rangle$</td>
</tr>
<tr>
<td>Variable assignments</td>
<td>$s$</td>
<td>$g, h, ...$</td>
<td></td>
</tr>
<tr>
<td>Truth value</td>
<td>$t$</td>
<td>–</td>
<td>1, 0</td>
</tr>
</tbody>
</table>

Table 2.1: Basic types in DPILM

In addition to the individual variables available in most versions of DPIL and PCDRT, DPILM has also events of type $\nu$ (see Henderson 2014), and degrees of type $\sigma$. Events variables will be used when we talk about the event differentiation requirement of adverbial and determiner each in Chapter 3. Degree variables will be used to talk about functional dependencies that hold between individuals and their measurements, as in the investigation of binominal each, Cantonese saai, and Mandarin ge.

Previous studies have argued that degrees have more structure than a simple numerical value.
In this dissertation, they are modeled as a \textit{triple}: the first coordinate of this triple stores a degree name (i.e., a point on a scale, such as 80kg), the second coordinate stores a measurement dimension (e.g., \textit{length}, \textit{weight}), and the third coordinate stores the individual being measured. The representation of the measurement dimension and the individual being measured is motivated by the need to model the functional dependencies between a degree variable and an individual variable, i.e., the individual variables storing the values the measurement of which yields the information stored in the degree variable.

Function types are recursively built out of primitive types. Table 2.2 lists some frequently used function types in the dissertation:

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Abbreviation</th>
<th>Variables</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Info-state</td>
<td>$s \rightarrow t$</td>
<td></td>
<td>$G, H, ...$</td>
<td></td>
</tr>
<tr>
<td>Dynamic proposition</td>
<td>$(s \rightarrow t) \rightarrow (s \rightarrow t) \rightarrow t$</td>
<td>$t$</td>
<td>$p, q$</td>
<td>Ann left.</td>
</tr>
<tr>
<td>Dynamic property</td>
<td>$e \rightarrow (s \rightarrow t) \rightarrow (s \rightarrow t) \rightarrow t$</td>
<td>$e \rightarrow t$</td>
<td>$P, P'$</td>
<td>pretty, smile</td>
</tr>
<tr>
<td>Dynamic relation</td>
<td>$e \rightarrow e \rightarrow (s \rightarrow t) \rightarrow (s \rightarrow t) \rightarrow t$</td>
<td>$e \rightarrow e \rightarrow t$</td>
<td>$R, R'$</td>
<td>kiss, see</td>
</tr>
<tr>
<td>Measure function</td>
<td>$e \rightarrow d$</td>
<td>$m$</td>
<td>$\mu_{\text{temp}}, \mu_{\text{vol}}$</td>
<td></td>
</tr>
</tbody>
</table>

\textbf{Table 2.2:} Function types in DPLM

I work with standard models $M = \langle D_e, D_v, D_d, D_s, I \rangle$, where $D_e$ is the domain of individuals, $D_v$ is the domain of events, $D_d$ is the domain of degrees, $D_s$ is the domain of variable assignments and $I$ is the basic interpretation function where $I(R) \subseteq D^n$ for any $n$-ary relation $R$. I assume that $D_e$ and $D_v$ are each subject to the axioms of classical extensional mereology; that is, they are equipped with partial orders $\leq_e$ and $\leq_v$ and sum operations $\oplus_e$ and $\oplus_v$ such that each $\oplus_i$ is the least upper bound of its $\leq_i$ (for additional details, see Krifka 1998; Champollion 2017). $D_d$ is assumed to be a set of triples storing a degree name, a measurement dimension, and an individual being measured. Degree names are totally ordered points on the relevant scales.
2.2.2 Information states and value projections

Like DPIL and PCDRT, a DPILM information state is a set of assignments. Interpreting a formula or dynamic proposition yields a relation between information states (info-states). This differs minimally from the more well known DPL (Groenendijk and Stokhof 1991), which interprets a formula as a relation between assignments, rather than sets of assignments.

**Definition 1 (Information state)**

An information state is a set of assignments.

**Definition 2 (Assignments)**

An assignment \( g \) takes a variable and when defined, returns a (possibly plural) individual.\(^2\)

Following a common practice in the literature, an information state is represented as a matrix. The first row has the variables, introduced into the info-state so far. The first column lists the assignments in it. All other cells store values obtained by applying an assignment to a variable.

\[
\begin{array}{cccc}
G & \ldots & x & y & \ldots \\
g_1 & \ldots & a & e & \ldots \\
g_2 & \ldots & b & c \oplus d & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\end{array}
\]

**Figure 2.1**: A sample info-state

Introducing referential pluralities comes with a price tag, but it is necessary to deal with domains that lack well-defined atomicity, like the domains of mass nouns, atelic events, and spatial and temporal intervals (see, for example Bach 1986). The system thus developed has two types of pluralities. Discourse plurality, also known as evaluation plurality, comes from considering a plurality\(^2\)Note that d-refs are modeled as variables in the present work, as in the traditions of Discourse Representation Theory (DRT, Kamp 1981; Kamp and Reyle 1993), File Change Semantics (FCS, Heim 1982, 1983a), Dynamic Predicate Logic (DPL, Groenendijk and Stokhof 1991), and Dynamic Plural Logic (DPIL, van den Berg 1996). In PCDRT, d-refs are modeled as individual concepts that take an assignment and return an individual (i.e., type \( s \rightarrow e \) for individual d-refs; see Brasoveanu 2008:137–138 for the benefits of modeling d-refs as individual concepts; see also the 'register'-type in Muskens 1996). The difference in the two types of treatment does not pertain to the main points of this dissertation. I opt for the former for its simplicity.

\(^2\)
of assignments. Referential plurality, also known as domain plurality, comes from considering a single assignment function that assigns a plurality to a variable. When we talk about cardinality and measurement, we need to be careful and keep these two types of plurality apart. Fortunately, thanks to the collective evaluation of lexical relations, to be defined in Section 2.2.5, we only need to make reference to evaluation-level cardinality in this framework.

In addition, following van den Berg (1996) and Brasoveanu (2010), I incorporate the dummy individual $\star$ in the range of assignment functions. The dummy individual $\star$ is the universal falsifier, i.e., any lexical relation with $\star$ as its argument, like $\text{read}(a, \star)$, is false (Brasoveanu 2010). The dummy individual is useful in a few ways. First, it can be used to model info-states that don’t contain any information, as visualized in Figure 2.2. Such an info-state is sometimes referred to as $\{\emptyset\}$. Relatedly, as pointed out in van den Berg (1996:Ch 2.4), information growth can be modeled by replacing dummy individuals with real individuals.

Second, the dummy individual is useful for defining generalized quantifiers in dynamic plural logic, which we will turn to in Section 2.2.9. Assuming the dummy individual also enables us to keep the logic simple, as we can work with total assignments rather than partial assignments (Brasoveanu 2010).

Given an info-state, we can define ways to project values in this info-state and in its sub-states. When a variable stores a set of nominal values, the corresponding projection function yields a set of values, as shown in (1). We can also project values stored in a variable by taking into consideration values stored in another variable, using the parameterized projection functions in (2) and (3).

**Definition 3 (Value projections)**

1. $G(u) := \{g(u) \mid g \in G \& g(u) \neq \star\}$
2. $G\|_{u=\alpha}(u') := \{g(u') \mid g \in G \& g(u) = \alpha\}$
3. $G\|_{u \in U}(u') := \{g(u') \mid g \in G \& g(u) \in U\}$
Parameterized projection functions enable us to compute functional dependencies between two (or more) variables, and hence will be frequently used throughout this dissertation.

When a variable stores a plural degree in an info-state, applying the projection function to it yields the concatenation of the plurality of degrees, i.e., a *single* degree, which is a triple, as shown in (4). This triple is obtained by applying the measurement function stored in the second coordinates of the plural degree $\bigoplus G_{i=2}^i(d)$ to the plural individual stored in the third coordinates of the plural degree $\bigoplus G_{i=3}^i(d)$. A projection function can not only be parameterized based on a variable, as in $G|_{x=a}(y)$, but it can also be parameterized based on a particular coordinate of a variable, as in $G_{i=1}^i(d)$.3

A rich degree ontology is assumed to model the fact that measurement tracks the mereological structure of individuals, which are in turned tracked by sets of assignments in an info-state. Because a degree contains all the necessary ingredients for building a new degree, we can effectively model the dependency among degrees, individuals and assignments.4 The reader should take the triple structure of a degree to mean that all the information necessary for computing a degree is recorded in discourse, including the individuals, the measurement function and the outcome of the measurement. This assumption is partially shared by studies like Schwarzschild (2006) and Wellwood (2015), who assume that at least the dimension of a measure function is contextually provided.

We can choose to only project the numerical coordinate of a degree variable, using the notation in (5). Like the parameterized projection function for nominal variables, the projection function for degree variables can also be parameterized, as shown in (6).

**Definition 4 (Degree projections)**

(4) \[ G(d) := \bigoplus \text{measure function} \bigoplus \text{plural individual} (\bigoplus G_{i=2}^i(d) (\bigoplus G_{i=3}^i(d))), \] where $G_{i=2}, G_{i=3}$ are the measure function and the (possibly plural) individual stored in $G(d)$.

---

3I thank Robert Henderson (p.c.) for suggesting that the coordinates in a degree triple can be treated as part of the parameterization of the projection function.

4An alternative, but not simpler, approach is to let individuals, degrees (as numerical values), and measure functions live as different variables, perhaps introduced by different parts of a degree expression. Although this approach does not require us to posit a degree as a triple, every time a plural degree name is computed it is still necessary to retrieve the measure function and apply it to the individuals that are being mapped to the plural degree name. In other words, a projection function still needs to take two variables to compute a degree (as numerical values). An assignment function taking two coordinates in a variable to construct a degree name is isomorphic to an assignment function taking two variables to construct a degree name.
(5) \( G^i=1(d) := \text{the first coordinate of } \bigoplus G^i=2(d) \bigoplus G^i=3(d) \)

(6) \( G^i=1(x=a) := \text{the first coordinate of } \bigoplus G^i=2(x=a) \bigoplus G^i=3(x=a) \)

For concreteness, let us consider the info-state in Figure 2.3.

<table>
<thead>
<tr>
<th>( G )</th>
<th>( x )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 )</td>
<td>( a )</td>
<td>( \langle 100\text{kg}, \mu\text{weight}, a \rangle )</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>( b )</td>
<td>( \langle 80\text{kg}, \mu\text{weight}, b \rangle )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Figure 2.3: Projecting degrees

\( G(d) \) yields a triple of the form in (7), and \( G^i=1(d) \) yields the degree name in (8). The parameterized version \( G^i=1(x=a) \) returns (9).

(7) \( G(d) := \bigoplus G^i=2(d) \bigoplus G^i=3(d) \)

\( = \langle 180\text{kg}, \mu\text{weight}, a\oplus b \rangle \)

(8) \( G^i=1(d) := 180\text{kg} \)

(9) \( G^i=1(x=a) := 80\text{kg} \)

2.2.3 Truth and connectives

A sentence is modeled as a dynamic proposition, a device for changing context. Feeding an input info-state to a dynamic proposition returns a set of updated info-states. The truth of a dynamic proposition is defined in the expected form, i.e, existential quantification over output info-states, following DPL/DPIL/PCDRT.

Definition 5 (Truth)

A dynamic proposition \( \phi \) is true (= \( \top \)) with respect to an input info-state \( G \) iff there is an output state \( H \) such that \( \langle G, H \rangle \in \llbracket \phi \rrbracket \).
The truth condition of a dynamic proposition is represented with the help of the interpretation function \( G\vec{\cdot}H \), which is a more iconic notation for the relation of info-states. The definitions of the propositional connectives are provided below. They basically follow the definitions in DPL, with assignments substituted for sets of assignments.

**Definition 6 (Connectives)**

1. \( G\vec{\phi \land \psi}H = T \) iff \( \exists K. G\vec{\phi}K = T \land K\vec{\psi}H = T \)
2. \( G\vec{\phi \lor \psi}H = T \) iff \( G\vec{\phi}H = T \) or \( G\vec{\psi}H = T \)
3. \( G\vec{\phi \rightarrow \psi}H = T \) iff \( G = H \land \forall K.H\vec{\phi}K = T \implies \exists J.K\vec{\psi}J = T \)
4. \( G\vec{\neg \phi}H = T \) iff \( G = H \) and \( \neg \exists K.G\vec{\phi}K = T \)

Conjunction is both internally and externally dynamic. Since it is internally dynamic, if variables are introduced in the first conjunct, their values and dependencies are available for the interpretation of the second conjunct (though not vice versa). Since it is externally dynamic, values and dependencies associated with variables introduced in both conjuncts are available outside the scope of conjunction. Disjunction is externally dynamic but internally static, which is taken from a later part of Groenendijk and Stokhof (1991) (Definition 53, page 88; see also Stone (1992) and Charlow (2014: Chapter 4.6) for discussions of dynamic disjunction, and Rooth and Partee (1982) for discussions of the wide scope behavior of disjunction). An externally dynamic disjunction is useful for analyzing the interaction between Cantonese *saai* and disjunction (to be discussed in Chapter 4). Implication is internally dynamic but externally static. Likewise, negation is internally dynamic but externally static (see Krahmer and Muskens 1995 and van den Berg 1996 for modified versions of the negation operator that tracks variables introduced in its scope).

### 2.2.4 Variable introduction and the lack of dependence

In DPLM, existential quantifiers are responsible for introducing variables. Variable introduction, also known as random assignment, is defined as in 7. It introduces all values in a set \( D \) as values of a variable \( u \).
Definition 7 (Variable introduction)

\[ G \models \exists u \models H = \top \iff \text{there is a set } D \text{ such that } H = \{ g^{u \mapsto \alpha} \mid g \in G \land \alpha \in D \} \]

where \( D \) is a subset of \( D_e, D_v \) or \( D_d \) in \( M \)

This version of variable introduction differs from both the DPlL version and the PCDRT version. Unlike the PCDRT version but like the DPlL version, it does not introduce dependence between the introduced variable and any existing variable. Unlike the DPlL version but like the PCDRT version, a single assignment is free to assign any value, atomic or not, to a variable. The result is a variable introduction that may introduce plurality but is still dependence-free.\(^5\)

An example should help us see how this variable introduction works. Assume a model with a domain \( D \) containing the individuals \( c, d \) and \( c \circ d \). Introducing a variable \( x \) with \( \exists x \) into an info-state \( G \) involves the following steps, which are also illustrated in Figure 2.4.

(10) Steps for introducing a new variable \( x \) to an info-state \( G \)

a. Pick a non-empty subset \( D \) of values from \( D_e \)

b. For each \( g \) in \( G \) and each value \( d \) in \( D \), extend \( g \) to include \( x \) in the domain of \( g \) and \( d \) in the range of \( g \), and collect the results in a set of assignments \( H \) (i.e., an info-state). The number of assignments in \( H \) is the cardinality of the cross product of \( G \) and \( D \) (|\( G \times D |) \). So, if \( G \) has two assignments and \( D \) is a singleton, there are two assignments in \( H \). If \( G \) has two assignments and \( D \) has two members, then \( H \) has four assignments, so on and so on.

c. Repeat the above steps for each non-empty subset \( D \) of \( D_e \). The total number of output info-states generable from introducing \( x \) into \( G \) is the cardinality of the power set of \( D_e \) minus the empty set (i.e., \( 2^{|D_e|} - 1 \)). So, if \( D_e \) has three members, then the total number of output info-states are seven.

Since we’re bringing referential plurality into the range of assignment functions, it is necessary for us to represent pluralities as well as singletons. I have chosen to represent them as atomic individuals

---

\(^5\)Whether or not assignment functions may range over pluralities and whether or not new variable introduction is dependence-free are two independent design choices of a dynamic plural logic.
Figure 2.4: Variable introduction in DPILM

and sum-individuals, following the tradition of Link (1983).⁶,⁷

Due to the dependency-free nature of variation introduction in DPILM, in none of these output info-states does the value of the variable \( y \) depend on the value of the variable \( x \) or vice versa. To better see this, we need the formal notion of value dependence:

**Definition 8 (Value dependence)**

\( y \) is value-dependent on \( x \) in an information state \( G \) iff there are \( a, b \in G \) \( x \) \( G \rvert_{x=a} y \neq G \rvert_{x=b} y \)

For value dependence between two variables \( y \) and \( x \) to hold, a variable (say \( x \)) should not always be assigned the same value for different values assigned to \( y \). In other words, the values stored in \( x \)

---

⁶An alternative is to represent pluralities as sets following the tradition of Scha (1981), Schwarzschild (1996) and van den Berg (1996). My decision is primarily based on readability—the plurality resulting from sets of assignments are represented as sets already, and having referential pluralities modeled as sets inside these sets is not very aesthetically appealing.

⁷As suggested in Brasoveanu (2011), a bonus for having referential pluralities in the range of assignment functions is that cover-based distributivity as proposed in Schwarzschild (1996) can be modeled without using covers.
is not constant relative to the values in \( y \). Obviously, \( x \) is not dependent on \( y \) in any of the info-states shown in Figure 2.4. Take \( H_3 \) and \( H_5 \) as examples:

\[
\begin{align*}
(11) \quad H_3|_{x=a}(y) &= \{c, d\} \\
(12) \quad H_3|_{x=b}(y) &= \{c, d\} \\
(13) \quad H_5|_{x=a}(y) &= \{c, c\oplus d\} \\
(14) \quad H_5|_{x=b}(y) &= \{c, c\oplus d\}
\end{align*}
\]

In both info-states \( H(y) \) yields the same value when \( x \) is restricted to a different value. Hence, \( y \) is not value-dependent on \( x \) in \( H_3 \) or \( H_5 \).

### 2.2.5 Lexical relations

All lexical predicates are cumulatively closed by default, following the assumptions in Landman (2000), Kratzer (2007), Brasoveanu (2013) and Champollion (2017). For example, if \( x \in \text{boy} \) and \( y \in \text{boy} \), then \( x \oplus y \in \text{boy} \); and if \( \langle x, y \rangle \in \text{saw} \) and \( \langle x', y' \rangle \in \text{saw} \), then \( \langle x \oplus y, x' \oplus y' \rangle \in \text{saw} \). For this reason, I do not mark a predicate with “*” to indicate cumulative closure. Lexical relations are interpreted according to Definition 9 and the relevant operation of discourse-level summation is given in Definition 10.

**Definition 9 (Lexical relations)**

(i) \( G[R(x_1, \ldots, x_n)]_H = \top \) is true iff \( G = H \land \left( \bigoplus G(x_1), \ldots, \bigoplus G(x_n) \right) \in I(R) \)

(ii) \( G[x = a]_H = \top \) iff \( G = H \land \bigoplus G(x) = I(a) \), where \( x \) is a variable and \( a \) a constant.

**Definition 10 (Discourse-level summation)**

\( \bigoplus G(u) = \bigoplus \{g(u) : g \in G\} \)

As suggested in clause (i) of Definition 9, lexical relations are satisfied collectively, like DPIL defined in van den Berg (1996). I use a summation operator to bring a set of individuals to a sum individual when checking for lexical relations. Clause (ii) of Definition 9 is a special lexical relation that holds between a variable and a constant when the collective value assigned to variable by a set of assignment functions is the same as the value the constant denotes in the model. This piece of notation is used when a variable is set to a fixed value, such as a proper name. Note that in Brasoveanu’s (2008) PCDRT, lexical relations are distributively checked, as a result of his definition of variable introduction. I’ll return to this issue in Section 2.3.
2.2.6 Cardinality and measurement

Although there are two types of pluralities in DPlLM, measurement of these pluralities is done in a uniform manner. As suspected, it is done by collapsing all evaluation-level pluralities into a domain-level plurality and taking its measurement. This is true of both cardinality measurement (Definition 12) and other types of measurement (Definition 13). Specifically, the cardinality test is on the atomic parts of the individual obtained from discourse-level summation (see also Henderson 2014:53 for a similar way to find the atomic parts of a discourse plurality).

Definition 11 (Cardinality)

\[
\mathcal{G}[|u| = d]\mathcal{H} = \mathcal{T} \iff \mathcal{G} = \mathcal{H} \land |\{u' \mid u' \leq \mathcal{E}H(u) \land u' \text{ is an atom}\}| = d
\]

Measurement other than cardinality is defined in a similar way in Definition 12 (i), only this time it is not necessary to access the atomic parts of a referential plurality (i.e., plurality in the range of a single assignment). We just need to apply a measure function (with a parameterized dimension, such as height or volume) to the referential plurality and obtain a degree, following Krifka (1998).\(^8\) Cardinality measurement (Definition 11) can receive a notational variant closer to non-cardinality measurement, as shown in Definition 12 (ii). This notation is used when an emphasis on the representation of a measure function is warranted.

Definition 12 (Measurement)

(i) \[
\mathcal{G}[\mu u = d]\mathcal{H} = \mathcal{T} \iff \mathcal{G} = \mathcal{H} \land \mu(\bigoplus H(u)) = d
\]

(ii) \[
\mathcal{G}[\mu_{\text{card}} u = d]\mathcal{H} = \mathcal{T} \iff \mathcal{G} = \mathcal{H} \land |\{u' \mid u' \leq \bigoplus H(u) \land u' \text{ is an atom}\}| = d
\]

2.2.7 Distributivity and dependence

Like DPIL, functional dependencies are generated via distributivity, which is modeled as a distributive operator \(\delta\), in DPlLM.

---

\(^8\)In fact, the measure function can be decomposed into two parts: a function that relates entities to degrees and a function that relates degrees to numbers (Lønning 1987; Champollion 2017). Suppose John weights 68 kilograms. This fact is represented as follows. The function weight maps John to a degree, and then the function kilogram maps the degree to the number 68.
**Definition 13 (Distributivity)**

\[ G\left[\delta_u(\phi)\right]H = \top \iff G(u) = H(u) \land \forall \alpha \in G(x).G_{|_{u=\alpha}}\left[\phi\right]H_{|_{u=\alpha}} = \top \]

The distributive operator splits up the input info-state into substates based on the values stored in the subscripted variable. It then checks that the formula in its scope, i.e., \(\phi\), holds for each sub-state. Hence, for each sub-state, a distributivity update generates a set of output sub-states. These sets of sub-states are then pointwisely put back together to form the output info-state. If \(\phi\) carries with it an existential quantifier, the new variable gets passed to the output. This way, DPILM opens up a door for introducing dependency into info-states.

Let’s take a concrete example to see how \(\delta_u\) works. Suppose we have a formula with an existential expression in the scope of a distributive operator, i.e., \(\delta_y(\exists x)\). Regarding the info-state \(G\), \(\delta_x(\exists y)\) first splits up the input info-state along the \(y\) dimension, resulting in two atomic sub-states, as shown in Figure 3.11. Then intermediate sub-states are created by updating \(y\) to each of the two atomic sub-states and assigning random values to \(y\). Note that within each leg of the distributive update, there is no dependence relation between \(x\) and \(y\). However, after collecting the intermediate sub-states to form the set of output info-state, some of the output info-states actually exhibit value dependence between \(x\) and \(y\), for example \(H_2\) and \(H_3\) in this case.\(^9\)

For concreteness, as shown in (15) and (16), \(y\) stores different values in \(H_2\) when \(x\) is restricted to different values. So, in \(H_2\), \(y\) is dependent on \(x\) (and vice versa).

\[(15) \quad H_2|_{x=a}(y) = \{c\} \]
\[(16) \quad H_2|_{x=b}(y) = \{d\} \]

**2.2.8 Types of functional dependencies in DPILM**

Now that there is a way to build a nontrivial functional dependency, we can discuss different types of functional dependencies depending on how variables stand in relation to each other. As will be shown in the subsequent chapters, there are distributive markers that impose constraints on the type of dependencies a distributive quantification brings about.

\(^9\)The possible output info-states after evaluating distributivity in Figure 3.11 is the cross product of the two sets of outputs obtained from splitting the evaluation. Since there are three info-states in each leg of the evaluation, the total number of info-states in the final output is nine. Only three sample info-states are shown for space reasons.
We have already seen the first type of dependency. Two variables in a dependency are said to exhibit value dependence (Definition 8, repeated below) when one co-varies with the other.

**Definition 8 (Value dependence)**

\( y \) is value-dependent on \( x \) in an information state \( G \) iff there are \( a, b \in G \times x : G|_{x=a} y \neq G|_{x=b} y \)

In addition to value dependence, there are dependencies that exhibit structural dependence. The formal definition of structural dependence for individual variables is given below:
Definition 14 (Structural dependence (for individual variables))

\( y \) is structurally dependent on \( x \) in an information state \( G \) iff (i) and (ii) holds.

i. there are distinct nonempty sets \( A, B \subseteq G(x) \): \( G|_{x \in A}(y) \neq G|_{x \in B}(y) \)

ii. for all distinct nonempty sets \( A, B \subseteq G(x) \): if \( A \subseteq B \), then \( G|_{x \in A} y \subseteq G|_{x \in B} y \).

(i) is equivalent to value dependence (see Definition 8). (ii) states that when \( x \) stores more individuals, \( y \) should not store fewer individuals. This essentially requires that the functional dependency between \( x \) and \( y \) be correlated for their cardinalities. Since variable introduction in DPILM is dependence-free, a newly introduced variable also does not stand in any structural dependence relation with another variable. Again, take \( G_3 \) and \( G_5 \) as examples:

\[
\begin{align*}
(17) & \quad G_3|_{x \in \{a\}}(y) = \{c, d\} \\
(18) & \quad G_3|_{x \in \{b\}}(y) = \{c, d\} \\
(19) & \quad G_3|_{x \in \{a, b\}}(y) = \{c, d\} \\
(20) & \quad G_5|_{x \in \{a\}}(y) = \{c, c \oplus d\} \\
(21) & \quad G_5|_{x \in \{b\}}(y) = \{c, c \oplus d\} \\
(22) & \quad G_5|_{x \in \{a, b\}}(y) = \{c, c \oplus d\}
\end{align*}
\]

It is easy to see that clause (i) is violated for the functional dependency between \( x \) and \( y \) in these info-states. In other words, there is no structural dependence between \( x \) and \( y \).

As pointed out earlier, when a dependent variable stores individual values, value dependence entails structural dependence. For this reason, \( y \) is also structurally dependent on \( x \) in the info-states \( H_2 \) and \( H_3 \). The following calculation confirms this: at least two different subsets stored in \( x \) are linked to different values in \( y \), and more values in \( x \) are never linked to fewer values in \( y \).

\[
\begin{align*}
(23) & \quad H_2|_{x \in \{a\}}(y) = \{c\} \\
(24) & \quad H_2|_{x \in \{b\}}(y) = \{d\} \\
(25) & \quad H_2|_{x \in \{a, b\}}(y) = \{c, d\}
\end{align*}
\]
When a degree variable is introduced inside the scope of a distributive operator, the relevance of structural dependence comes in. To see this, let us consider the following info-state, with \( d \) and \( d' \) introduced as a result of distributive quantification over values stored in \( x \).

<table>
<thead>
<tr>
<th>( G )</th>
<th>( x )</th>
<th>( d )</th>
<th>( d' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_1 )</td>
<td>( a )</td>
<td>(80kg, weight, a)</td>
<td>(36.5°C, temp, a)</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>( b )</td>
<td>(80kg, weight, b)</td>
<td>(36.5°C, temp, b)</td>
</tr>
</tbody>
</table>

Figure 2.6: Extensive measurement

Recall that degrees are modeled as triples. The first coordinate of such a triple stores a degree name, modeled as a point on a scale, the second coordinate stores a measure function, and the third coordinate stores the (possibly plural) individual being measured. When the measure function stored in the second coordinate is extensive, as in the case of \( d \), it tracks the size of the third coordinate, resulting in a bigger value (i.e., a higher point on a scale) in the first coordinate when there is a bigger plurality in the third coordinate.

\[
(26) \quad G_{x \in \{a\}}^{i=1}(d) = 80\text{kg}
\]

\[
(27) \quad G_{x \in \{b\}}^{i=1}(d) = 80\text{kg}
\]

\[
(28) \quad G_{x \in \{a,b\}}^{i=1}(d) = 160\text{kg}
\]

However, when the measure function stored in the second coordinate is intensive, as in the case of the temperature measure function stored in \( d' \), it fails to track the size of the third coordinate. As a result, even when there are more values in the third coordinate, the first coordinate does not necessarily have a bigger value, as demonstrated below:

\[
(29) \quad G_{x \in \{a\}}^{i=1}(d') = 36.5°C
\]

\[
(30) \quad G_{x \in \{b\}}^{i=1}(d') = 36.5°C
\]

\[
(31) \quad G_{x \in \{a,b\}}^{i=1}(d') = 36.5°C
\]

In many cases, measuring the plurality in the third coordinate with an intensive measure function
may not be straightforward. For example, the temperature of two non-overlapping drinks cannot be easily determined. Nor can the speed of two cars. However, it does not mean that there is no way of measuring a collective temperature (or speed). One way to do so is to calculate an average temperature (or speed).\textsuperscript{10} Since an average always falls somewhere between two (or more) measurements, an intensive measure function still fails to be additive.

The following definition of structural dependence for degrees captures the above intuition:

**Definition 15 (Structural dependence (for degree variables))**

d is structurally dependent on \( x \) in an information state \( G \) iff (i) and (ii) holds.

i. there are distinct nonempty sets \( A, B \subseteq G(x) \): \( G|_{x \in A}^{i=1}(d) \neq G|_{x \in B}^{i=1}(d) \)

ii. for all distinct nonempty sets \( A, B \subseteq G(x) \): if \( A \subseteq B \), then \( G|_{x \in A}^{i=1}(d) \leq G|_{x \in B}^{i=1}(d) \).

### 2.2.9 Generalized quantifiers

Most of the work in this dissertation does not use a full-blown semantics of dynamic generalized quantifiers but a simplified version commonly used in studies like Brasoveanu (2008), Henderson (2014), Kuhn (2017). The simplified version introduces one less d-ref (i.e., the one corresponding to the restriction property) and maximization (i.e., the corresponding maximization over the restriction d-ref) than the full-blown one. I include a discussion of dynamic generalized quantifiers because additional adjustments need to be introduced on top of the original formulation of dynamic generalized quantifiers in van den Berg (1996), due to the incorporation of referential pluralities.

Like DPIL and PCDRT, the definition of generalized quantifiers in DPILM is built on the classical theory of generalized quantifiers (Barwise and Cooper 1981; Keenan and Stavi 1986). In the classical theory, a generalized quantificational determiner (\( \text{Det} \)) denotes a relation of two sets of individuals (type \( e \rightarrow t \)). In DPILM, a generalized quantificational determiner (\( \text{Det} \)) also denotes a relation, of two dynamic propositions (type \( s \rightarrow t \rightarrow s \rightarrow t \rightarrow t \)). Recall that a dynamic proposition is a relation of two info-states, and a d-ref in an info-state stores a set of values, assigned to

\textsuperscript{10}It is an empirical question how readily natural language allows such a use of intensive measure functions. I do not have a good answer to this question at this point. However, I suspect that if two objects can be construed as a group, then it is easier to use the average of the intensive measurements of the parts as a measurement of the group. An alternative way to evaluate \( G|_{x \in \{a,b\}}^{i=1}(d') \) in (31) is to evaluate it to ‘undefined’, as suggested to me by Simon Charlow (p.c.), Robert Henderson (p.c.), and Philippe Schlenker (p.c.). This is a possible option and holds the potential to simplify the monotonicity constraints developed in the later chapters. I reserve the exploration of this option for future research.
it by a set of assignments. Therefore, it is possible to construct a set of individuals from a dynamic proposition with the help of d-refs (see also Dekker (1993) on the related operation Existential Disclosure).

A schema for relating static distributive generalized quantifier to their dynamic correlates is given in Definition 16. The notations used come from Brasoveanu (2010) (the translation schema offered in van den Berg 1996 Section 4 of Chapter 4 is less compact but essentially the same, except for the complexity involved in $\text{Det}_{\oplus}$, to be made precise in Definition 20).

**Definition 16 (The translation schema of Det)**

$\text{Det}^{u,u'} (\phi, \psi) := \max^u(\delta_u(\phi)) \land \max^{u' \sqsubseteq u}(\delta_{u'}(\psi)) \land \text{Det}_{\oplus}(u,u')$

In simple words, a dynamic determiner introduces two d-refs $u$ and $u'$, and requires that they stand in the relationship specified by the static correlate of the dynamic quantificational determiner $\text{Det}_{\oplus}$. The two d-refs are subject to two additional conditions. First, both $u$ and $u'$ have to be maximal, relative to a restriction proposition $\phi$ and a scope proposition $\psi$, respectively. Definition 17 spells out how maximization works. In addition, the subset relation between $u'$ and $u$ is structure-preserving, so that the values in $u'$ stand in the same relationship that the corresponding values in $u$ stand with values in other variables. The structure-preserving subset relation is spelled out in detail in Definition 18. Combining these two gives rise to the full definition of $\max^{u' \sqsubseteq u}$ in Definition 19.

The maximization operator, defined in Definition 17, introduces a set of entities and the sum of these entities is the maximal one that satisfies $\phi$. Restrictor maximization and scope maximization make sure that the values satisfying the restriction formula and the scope formula are maximal values. They are important as we don’t want to accidentally make true every girl left in a scenario of five girls by only introducing a proper subset of these girls via variable introduction and checking that every single one of these values left.

**Definition 17 (Maximization)**

$G[\max^u(\phi)]H = \top$ iff $G[\exists u \land \phi]H = \top$ and $\neg \exists K. \bigoplus H(u) < \bigoplus K(u) \land G[\phi]K = \top$

---

11It is possible to define non-distributive quantifiers. I refer readers who are interested in non-distributive quantifiers to van den Berg (1996:Ch. 3, Ch. 4.4).
The structure-preserving subset relation is defined in Definition 18, following Brasoveanu (2010) (see also subset assignment in van den Berg 1996, with the symbol \( \subseteq \) in place of \( \sqsubseteq \)). It requires that a d-ref \( u' \) inherits all the dependencies established between the corresponding values in \( u \) and values in other variables. This can be achieved by forcing all the assignments that assign a value to \( u' \) to also assign the same value to \( u \). If there is any value in \( u \) that is not assigned to \( u' \), the relevant assignments assign a dummy individual \( \star \) to \( u' \). A sample of values stored in \( u \) and \( u' \) (\( u' \sqsubseteq u \)) is given in Figure 2.7. The values stored in \( u' \) form a subset of the values stored in \( u \). In addition, except for the assignment that assigns a dummy individual, all the assignments assign the same values to \( u \) and \( u' \). The first matrix in Figure 2.7 fulfills the requirements of \( u' \sqsubseteq u \), but the other two violate condition (a) and condition (b) in Definition 18.

**Definition 18 (Structure-preserving subset relation)**

\[
G \sqsubseteq u \parallel H = \top \text{ iff } G = H \text{ and }
\]

\[\begin{align*}
a. & \quad \forall h \in H. h(u') = h(u) \lor h(u') = \star \quad \text{and} \\
b. & \quad \forall h \in H. h(u) \in H(u') \rightarrow h(u) = h(u')
\end{align*}\]

**Figure 2.7:** An illustration of the structure preserving subset relation

Putting together maximization and structure-preserving subset relation, we obtain the definition for a maximal structure preserving subset:

**Definition 19 (maximal structure preserving subset)**

\[
G \sqsubseteq u \parallel \max u' \parallel H = \top \text{ iff } G \sqsubseteq \max u' \parallel \phi \parallel H = \top \text{ and } \parallel u' \subseteq u \parallel = \top
\]

\( \det \parallel \) represents the standard GQ relation of sets of elements. Since these elements may be in the form of pluralities in the range of assignment functions, it is necessary to extract all the atomic
parts from these sets of elements.

**Definition 20 (The GQ relation of sum entities)**

\[ G[\text{Det}_0(u, u')]H \iff G = H \text{ and } \text{Det} \{ a \mid a \leq \bigoplus H(u) \& \text{atom}(a) \}, \{ a' \mid a' \leq \bigoplus H(u') \& \text{atom}(a') \} \]

Note that every dynamic determiner contains a static \( \text{Det}_0 \). As a result, any static GQ can be translated into a dynamic GQ. For example, the sentence in (32) can be translated as (33), with the first sentence contributing the first three conjuncts and the second sentence the last conjunct.

(32) Every student came in. They sat down.

(33) \[ \max^x \left( \delta_x(\text{stdt } x) \right) \land \max^{x' \subseteq x} \left( \delta_{x'}(\text{come.in } x') \right) \land \text{every}_0(x, x') \land \text{sat.down}(x') \]

In the first conjunct, a d-ref \( x \) is introduced that stores all the students. In the second conjunct, a d-ref \( x' \) is introduced that stores a structured subset of \( x \) and is a maximal set of students that came in. In the third conjunct, every\(_0\) stands for a static subset relation, i.e., \( \subseteq \). It says that the set of the atomic students is a subset of the set of all the atomic students who came in. Since the first sentence in (32) introduces two d-refs \( x \) and \( x' \) storing all the students and all the students that came in, respectively, it is predicted that both d-refs can be picked up by subsequent pronouns. For example, the plural pronoun they in (32) refers to every student via one of these d-refs.\(^{12}\)

### 2.3 Comparisons with DPIL and PCDRT

As discussed in the last section, DPILM is a hybrid of DPIL and PCDRT. It borrows an essential assumption from each of DPIL and PCDRT, as shown in Figure 2.8. On one hand, DPILM follows PCDRT and assumes that there are two types of pluralities, i.e., domain-level pluralities and evaluation-level pluralities. On the other hand, DPILM patterns like DPIL in that introduction of variables doesn’t generate dependency. As a result, DPILM not only can incorporate measurement into dynamic semantics for plurals, but also maintains that dependence between variables can only be generated via distributivity.

\(^{12}\) In theory, both d-refs are available for anaphora. However, Nouwen (2003) shows that anaphora to the scope d-refs is more readily available than anaphora to the restriction d-ref. In the case of every, both d-refs store the same individuals, so it does not matter which d-ref is used for resolving the anaphora with they.
2.3.1 Comparison with DPlL

van den Berg (1996) presents and elaborates the formal properties of DPlL. The basic idea of DPlL is that, instead of enriching our ontology with plural entities, as suggested by Link (1983), we can enrich our logic, by assuming that contexts of evaluation consist of sets of assignments. So, Linkean plural individuals are replaced by evaluation-level plurality. The domain of individuals contain only atomic individuals, no sum-entities. As a consequence, measurement only needs to be defined at the evaluation level. For example, the numeral phrase *three boys*, which is translated as (34), requires the values stored in the variable $x$ to be three atomic boys.

\[(34) \exists x \land \text{boy } x \land x = 3\]

In DPlL, variable introduction is defined in the same way as DPlLM (i.e., Definition 7), with the exception that the assignment functions do not range over plurals (as the domain of individuals only contain atoms), as shown in 21.

**Definition 21 (Variable introduction (DPlL))**

\[G[\exists_{\text{DPlL}} x]H = T \text{ iff there is a set } D \text{ s.t. } H = \{g^a \rightarrow x \mid g \in G \land a \in D\}\]

where $D$ is a subset of $D_e$, which contains atomic entities

Cardinality of a variable is determined by the set of individuals in the range of a plurality of assignment functions, as indicated in Definition 22. Since DPlL does not have referential pluralities, there
is no need to break down the individuals in the range of each assignment function to determine the cardinality of a variable. This contrasts with cardinality in DPILM (Definition 11, repeated below), which does require breaking down the referential pluralities into atoms in the range of each assignment and counting all the derived atoms.

**Definition 22 (Cardinality (DPIL))**

\[ G[x = n]H = \top \text{ iff } G = H \land |H(x)| = n \]

**Definition 11 (Cardinality (DPILM))**

\[ G[u = d]H = \top \text{ iff } G = H \land \{|u' | u' \leq \bigoplus H(u) \land u' \text{ is an atom}\} = d \]

Without extra assumptions, DPIL has difficulty modeling mass nouns, like *water* and *coffee*. This is because mass nouns do not have a well-defined atomic tier (e.g., Quine 1960, Link 1983, Chierchia 2010; cf. Chierchia 1998a, Rothstein 2004, 2010). Hence, \( \exists_{DPIL}x \) cannot assign appropriate values to a variable with the *water* property. This problem could be resolved by assuming that the domain of individuals contain atomic units of a mass noun. However, the defect of this solution is also obvious. It predicts that mass nouns can be counted without requiring the occurrence of any explicit measure units. This prediction does not sit well with facts from natural language about mass nouns and measurement.

If we would like to handle mass nouns in a dynamic semantics for plurals, the easiest way is to assume that the domain of individuals in the model is closed under sum formation. That is, sum entities are available and they can have the property characterized by a mass noun. Adding sum entities to DPIL directly leads to DPILM, which inherits most definitions from DPIL.

### 2.3.2 Comparison with PCDRT

DPILM is similar to PCDRT in the assumption that a single assignment function may range over pluralities. In other words, referential pluralities are allowed in both PCDRT and DPIL. However, DPILM differs from PCDRT in two important respects: (i) variable introduction and evaluation of lexical relations. Since the design feature of variable introduction determines the design feature of evaluating lexical relations, I first take up variable introduction and then turn to lexical relations.
Variable introduction

An essential difference between DPILM and PCDRT lies in whether variable introduction is allowed to introduce dependence among variables. In a series of studies (Brasoveanu 2007, 2008, 2010), Brasoveanu defines variable introduction as in Definition 23. A variable introduction $\xi x$ is successful as long as each input assignment $g$ has a successor output assignment $h$ that differs from $g$ at most on the value of $x$ and vice-versa, each output assignment $h$ has a predecessor input assignment $g$ that differs at most from $h$ on the value assigned to $x$. In other words, as long as all the assignments in an input info-state are preserved in the corresponding output info-state, there is no restriction on what value each assignment associates to $x$. They may associate the same value(s) with it (in which case there is no dependence) or they may associate different values with it (in which case there is dependence). This is crucially different from the DPILM variable introduction, which forces all assignments to assign the same value(s) to a variable.

**Definition 23 (Variable introduction (PCDRT))**

$G[\xi x]H = T$ iff for all $g \in G$, there is a $h \in H$ s.t. $h = g^{x \rightarrow \alpha}$ and for all $h \in H$, there is a $g \in G$ s.t. $h = g^{x \rightarrow \alpha}$, where $\alpha$ is an element in $D_e$

Assume a model with an individual domain including $c$, $d$ and $c \oplus d$. Introducing a variable $x$ with $\xi x$ results in many more outputs than with $\exists x$ in DPILM, as visualized in Figure 2.9.\(^{13}\) Crucially, the output of $\xi x$ can give rise to dependencies when $\alpha$ belongs to a non-singleton domain. As demonstrated by the output info-states $G_4 - G_7$ in Figure 2.9, the values stored in $y$ can be associated with different values stored in $x$.

Evaluation of lexical relations

Not only does this version of variable introduction bring in dependencies, it also requires a distributive evaluation of lexical relations, as indicated in Definition 24.

**Definition 24 (Lexical relations (PCDRT))**

$G[R(x_1, ..., x_n)]H = T$ iff $G = H \land \forall h \in H, \langle h(x_1), ..., h(x_n) \rangle \in I(R)$

\(^{13}\) $\xi x$ generates all the outputs $\exists x$ generates plus (potentially infinitely) many more.
A lexical relation holds between two variables just in case for every assignment, the pair of values associated with the two variables stand in the said relation. This contrasts with how lexical relations are evaluated in DPILM (as well as DPIL). According to Definition 9 (repeated below), lexical relations are evaluated collectively in DPILM. What this means is that a lexical relation holds between two variables just in case the collective pair of values assigned to the two variables by all the assignments stand in the said relation.

**Definition 9 (Lexical relations (DPILM))**

\[ G \models R(x_1, ..., x_n) \models H = \top \text{ is true iff } G = H \ & \left( \bigoplus G(x_1), ..., \bigoplus G(x_n) \right) \in I(R) \]

The contrast in how lexical relations are evaluated stems from the distinct design features in variable introduction in the two logical systems. In DPILM, a newly introduced variable does not depend on any other extant variable by default. So, every value in a newly introduced variable is ‘related’ to every value in another variable (for all info-states). To evaluate a lexical relation distributively will result in a very strong claim—every value in a variable stands in the said relation to every value in
another variable. To avoid making such a strong claim, it is necessary to check a lexical relation collectively.

The state of affairs is very different in PCDRT, in which a newly introduced variable may depend on an extant one. Since a value in a newly introduced variable does not stand in relation to every value in another variable for all info-states, checking lexical relations distributively has a much milder effect. Some info-states will be ruled out if the two variables do not stand in the right relation, but others will survive. Moreover, since dependencies between variables can be generated, checking lexical relations collectively will ‘waste’ the dependencies. After all, collective evaluation of a lexical relation ignores all the dependencies among the relevant variables.

**Which variable introduction to prefer?**

The reader may notice that there is a principled way to map a DPlLM-type of logic to a PCDRT-type of logic, by elaborate use of the distributive operator $\delta_x$. Specifically, by *always* evaluating variable introduction and lexical relations in DPlLM in the scope of a distributive operator, we will end up with the same variable introduction and lexical relation evaluation in PCDRT.\(^\text{14}\)

The essential difference between PCDRT and DPlL/DPlLM, then, lies in whether distributivity and the dependencies arising from it are taken to come from a default distributive interpretation procedure of the logic or from a lexical source (i.e., an overt or covert distributive operator that accompanies the distributive interpretation). The former presumably generates dependencies in more contexts than the latter. There is no *a priori* reason to favor one over the other. However, two empirical consideration have been discussed that shed light on the choice, one supporting the PCDRT variable introduction and the other the DPlL/DPlLM variable introduction. I discuss them in turn below.

The empirical fact that supports PCDRT variable introduction comes from mixed readings of donkey sentences (*Brasoveanu 2007, 2008*). An example involving a mixed reading is given in (35).

(35) Every person who buys a\(^x\) book on amazon.com and has a\(^x\)' credit card uses it\(_{x'}\) to pay for

\(^{14}\)This is not strictly correct, as variable introduction in PCDRT places no limit on the number of assignments on an output info-state after variable introduction nor is there a limit on how many output info-states a variable introduction can generate. Both numbers are fixed in DPlLM (as well as in DPlL), as illustrated earlier in (10).
Brasoveanu (2008) notes that this sentence is compatible with a situation in which (i) every person buys more than one book from amazon.com, (ii) every person has more than one credit card, and (iii) uses different credit cards to pay for different books. His account has two assumptions. First, a book introduces a maximal set of books, for each person. This gives rise to the strong reading of a book. Second, a credit card introduces a non-maximal credit card for each book introduced earlier. This models the weak interpretation of a credit card. In addition, due to the fact that a credit card is introduced in the distributive scope of a book, there can be co-variation between the books and the credit cards. Based on this type of dependence between two donkey indefinites, Brasoveanu (2007, 2008) motivates the PCDRT variable introduction, which allows a newly introduced variable to be dependent on an extant one without using a distributive operator.

However, Champollion et al. (to appear) argue that mixed readings of donkey sentences should be analyzed as involving truth value gaps, which are adjusted pragmatically. If their proposal is on the right track, then there is no need to use dependency-introducing variable introduction to model mixed readings of donkey sentences.

The empirical consideration that favors DPILM variable introduction concerns expressions that require dependencies to be licensed. Examples of such expressions are dependent indefinites and the distributive markers investigated in this dissertation. As many studies have observed, dependent indefinites piggyback on a distributive interpretation to be licensed (Farkas 1997, 2002b, Yanovich 2005, Brasoveanu and Farkas 2011, Henderson 2014, Kuhn 2017). In this dissertation, I show that the hallmark inferences associated with dependent indefinites can also be borne by distributive markers. Both classes of phenomena suggest that there is an intimate relationship between distributive interpretation and dependencies. However, this relationship is understood quite differently in DPILM and PCDRT.

In DPILM (as well as DPIL), dependencies arise only when variable introduction is interpreted in the scope of a distributive operator. When a distributive operator is missing, variable introduction does not introduce dependency. For this reason, expressions that require dependencies naturally require a distributive operator. Since a distributive operator is responsible for the distributive interpretation, it is naturally predicted that expressions requiring dependencies are parasitic on the
distributive interpretation.

For concreteness, consider sentences such as (36-a) and (37-a) under a non-distributive interpretation, as indicated in (36-b) and (36-b), respectively.\(^{15}\)

\[(36)\]
a. Three boys\(^x\) made five kites\(^y\).
   \[∃x \land \text{boy } x \land |x| = 3 \land ∃y \land \text{kite } y \land |y| = 5 \land \text{make } y x\]

b. The girls\(^x\) solved the problems\(^y\).
   \[\max^x(\text{girl } x) \land \max^y(\text{problem } y) \land \text{solve } y x\]

Since the two sentences pattern similarly in DPILM (as well as in PCDRT), I only offer a discussion of (36-a) here. Interpreting three boys generates a set of output info-states with a variable \(x\) that stores a set of boy pluralities whose collective cardinality is three. Then, interpreting five kites against each info-state in this set of output yields another set of output info-states, each of which has an additional variable \(y\) that stores a set of kite pluralities whose collective cardinality is five. Note that because of the lack of a distributive operator, there is no dependency between \(x\) and \(y\). Lastly, the verb contributes a test making sure that the collective values in \(x\) and the collective values in \(y\) stand in a making relation. An illustration of such an update is given in Figure 2.10.

The non-distributive interpretation is compatible with a collective interpretation as well as a cumulative interpretation. In fact, without extra machinery, DPILM does not distinguish between a collective interpretation and a cumulative interpretation (see also Roberts 1987, Landman 1989, Link 1998, who also do not distinguish between the two readings; cf. Scha 1981, Landman 2000). Since the non-distributive interpretation lacks dependencies, it is not surprising that it fails to license dependent indefinites (Kuhn 2017). Since only distributive interpretations may generate dependencies, it is also not surprising that lexical items that need access to dependencies also function as distributive markers, a phenomenon taken up in this dissertation.

In PCDRT, by contrast, a non-distributive interpretation may still exhibit dependencies among variables. Consider the PCDRT-theoretic translation of (36-a) in (38). I have used small capitals to translate lexical relations here to remind us that lexical relations are evaluated distributively in

\(^{15}(36-a)\) does not have maximization over the two variables, so there may be more than three boys and five kites that stand in a making relation in the model. Maximization may be introduced but is not relevant to the discussion here.
PCDRT, except for the cardinality tests, which are evaluated collectively (i.e., globally) (Brasoveanu 2013).

(38)  \( \exists x \land \text{BOY} \land |x| = 3 \land \exists y \land \text{KITE} \land |y| = 5 \land \text{MAKE} \land y \land x \)

In (38), variable introduction makes available two variables, one storing a set of boys and the other storing a set of kites. We can conveniently use the collective cardinality measurement defined for DPILM in Definition 11 to interpret the cardinality tests. The definition tells us that the cardinality of the two variables are, collectively, three and five, respectively. Lastly, the two variables are required to stand in a making relation.

**Definition 11 (Cardinality (DPILM))**

\[
G \models |u| = d \models H = T \iff G = H \land |\{u' \mid u' \leq \bigoplus H(u) \land u' \text{ is an atom}\}| = d
\]

A sample flow of update is given in Figure 2.11. What is important is that since variable introduction may encode dependency in PCDRT, many info-states in the output set may encode dependence.
between the variable storing the boys and the variable storing the kites. The outputs in the sample update are two examples.

\[
\begin{array}{c|c}
G & x \\
g_1 & b1 \\
g_2 & b2 \oplus b3 \\
\end{array}
\] 
\[
\begin{array}{c|c|c|c}
G & x & y \\
g_1 & b1 & k1 \oplus k2 \\
g_2 & b2 \oplus b3 & k3 \ldots k5 \\
\end{array}
\]

\[
\begin{array}{c|c}
G & x \\
g_1 & b4 \oplus b5 \\
g_2 & b6 \\
\end{array}
\] 
\[
\begin{array}{c|c|c|c}
G & x & y \\
g_1 & b4 \oplus b5 & k6 \oplus k7 \\
g_2 & b6 & k8 \ldots k10 \\
\end{array}
\]

Figure 2.11: Dependency in a cumulative reading (PCDRT)

If a dependent indefinite requires variable dependence and a cumulative interpretation provides variable dependence, it is natural to expect that a dependent indefinite should be licensed by a cumulative interpretation.\(^{16}\) However, as pointed out by Kuhn (2017), dependent indefinites are, in fact, not acceptable without a distributive interpretation. In other words, dependent indefinites and distributivity seem to go hand in hand.

To model the fact that dependent indefinites are ruled out without a distributive interpretation, Kuhn (2017) and Henderson (2014) introduce different assumptions. Kuhn (2017) suggests building distributivity directly into the semantics of dependent indefinites. In particular, a dependent indefinite is assumed to carry two important components. One component expresses the need for variable dependence (known as evaluation plurality, to be discussed in the next paragraph), another expresses the need for distributively evaluating the numeral contribution.\(^{17}\) While bundling distributivity and

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\(^{16}\)One may suspect that such a dependent indefinite is independently ruled out for its odd interpretation. I disagree. Consider a hypothetical example below.

(i) Three boys looked for five-five books between them.

If the cumulative reading supported the dependent indefinite, the above sentence would have the following meaning: three boys looked for a total of five books, and not all the boys looked for the same books. This is compatible with the cumulative interpretation.

\(^{17}\)Since the scope of such a distributive operator is limited to the numeral contribution, Kuhn makes a desirable prediction that a dependent indefinite can be conjoined with a collectively interpreted noun phrase.
a requirement for variable dependence at the lexical level models the intimate relationship between
distributivity and dependency-looking expressions, it is not explanatory. For example, one may still
wonder why bundling cumulativity and a requirement for variable dependence does not exist.

As an attempt to model the limited distribution of dependent indefinites, Henderson (2014) sug-
gests a way to remove unwanted dependencies generated from variable introduction in PCDRT.\(^\text{18}\)
In particular, he assumes that expressions that introduce variables into an info-state come in two
types: those that allow variable dependence and those that disallow it (see also a development of
this assumption in Champollion et al. 2017). In other words, Henderson (2014)’s PCDRT has two
modes of variable introduction: the dependency-free one proposed in DPILM (and DPIL, barring
referential pluralities), as well as the dependency-introducing one proposed in Brasoveanu (2008).
This fact may not be immediately obvious since the locus of the distinction does not lie in variable
introduction but is formulated as a pair of cardinality tests, which are given in Definition 25.\(^\text{19}\)

**Definition 25 (Evaluation cardinality (Henderson 2014))**

\[
\begin{align*}
G \models x = 1 & \Rightarrow H = T \text{ iff } G = H \land |H(x)| = 1 \\
G \models x > 1 & \Rightarrow H = T \text{ iff } G = H \land |H(x)| > 1
\end{align*}
\]

\(x = 1\) says that there is only a single, unique value in the range of a collection of assignment functions
in an info-state. Recall that \(H(x)\) is a set. The set may contain atoms or pluralities, but \(x = 1\)
requires that there be only one element in the set. So, either a single atom, or a single plurality.
By contrast, \(x > 1\) says that there must be more than one value in the range of a collection of
assignment functions. In other words, there must be at least two assignment functions that assign

---

\(^{18}\) Henderson (2014) does not explicitly discuss cumulative interpretations. However, if his device for removing
dependencies is extended to sentences with a cumulative interpretation, the prediction is that the sentences will not encode
variable dependence. Kuhn (2015:71–72) also has a discussion of this point.

\(^{19}\) The reader may notice that evaluation cardinality in Henderson (2014) is defined in the same way as the cardinality
test in van den Berg (1996). This is not surprising. What evaluation cardinality does is to count the number of values in
the range of a set of assignments without trying to break down the referential pluralities. In other words, if an assignment
ranges over a referential plurality, the plurality still only counts as one, as counting does not go into the subparts of the
referential plurality. Since DPIL only has atoms in the range of assignment functions, its cardinality test is devised not
to look into the subparts of the individuals in the range of a single assignment function. For this reason, an evaluation
cardinality test in Henderson (2014) is the same as the cardinality test in van den Berg (1996).
different values to \( x \). Importantly, Henderson (2014) shows that a noun phrase set to be evaluation singular may still give rise to dependency if there is a distributive operator scoping over it.

An evaluation cardinality test is to be distinguished from a referential cardinality test (translated as \( \text{ONE} \ x \), \( \text{TWO} \ x \), etc). The latter is just an ordinary predicate in PCDRT and hence is evaluated distributively. So, \( \text{TREE} \ x \) is true when there are three atoms in the range of each assignment function applying to \( x \). A concrete definition of referential cardinality is given in Definition 26.

**Definition 26 (Referential cardinality)**

\[
G \models \text{ONE}(x) \iff G = H & \forall h \in H.\{x' \mid x' \leq h(x) & \text{atom}(x)\} = 1
\]

According to Henderson (2014), plain indefinites are evaluation singular. So, (36-a) is translated as (39) in Henderson’s PCDRT.

\[
(39) \quad \xi x \land \text{BOY} x \land x = 1 \land \text{TREE} x \land \xi y \land y = 1 \land \text{FIVE} y \land \text{KITE} y \land y = 5 \land \text{MAKE} y x
\]

Since both \( x \) and \( y \) are required to be evaluation singular, interpreting (39) does not lead to a set of info-states with dependencies between \( x \) and \( y \) anymore. Rather, the info-states in the output will be akin to the ones in Figure 2.12. Although represented differently, the cumulative reading as represented in Figure 2.12 (Henderson-style PCDRT) and the cumulative reading as represented in Figure 2.10 (DPILM) encode the same, dependency-free, information.

\[
\begin{array}{c|c|c}
G & x & y \\
\hline
G & x & y \\
\hline
G & x & y \\
\hline
G & x & y
\end{array}
\]

**Figure 2.12:** No dependency in a cumulative reading (PCDRT as in Henderson 2014)

Although using evaluation singularity as proposed in Henderson (2014) provides a way to explain why non-distributive readings do not generally license dependent indefinites, the nature of evaluation singularity does not sit well with PCDRT’s spirit, namely, dependencies among variables
are freely available. The use of evaluation singularity is precisely to get rid of these dependencies. If we are to assume that evaluation singularity is associated with the majority of expressions that trigger variable introduction, we lose the only essential difference between PCDRT and DPILM: freely available vs. restricted dependencies. For this reason, I take Henderson (2014)’s revisions of PCDRT as motivation for a logic more line with DPILM for analyzing expressions that require access to distributive dependencies.\footnote{Since the cumulative reading does not encode dependency in DPILM, the present framework predicts that there can not be markers of dependence residing in a cumulative construction, unlike a distributive construction.}

### 2.4 Summary

We have introduced the framework of DPILM and discussed its relations to van den Berg (1996)’s DPIL, Brasoveanu (2008)’s PCDRT and Henderson (2014)’s PCDRT. In the next three chapters, I use this framework to analyze three distributive markers: binominal *each*, Cantonese *saai*, and Mandarin *ge*. I show that although these distributive markers bear distinct constraints on distributive dependencies, the framework outlined in this chapter can handle them in a unified manner.
3.1 Introduction

Each is a distributive marker in English. It may appear in the determiner position, as shown in (1), the pre-verbal, adverbial position, as shown in (2), or the post-nominal position, as shown in (3). When appearing in the last position, it is often referred to as binominal each, following the terminology of Safir and Stowell (1988).\(^1\)

(1) Each girl saw two movies.

(2) The girls each saw two movies.

(3) The girls saw two movies each.

This chapter is devoted to the binominal use of each for it has many interesting properties not shared by determiner and adverbial each.\(^2\) In particular, binominal each is known to impose morphosyntactic and interpretive requirements on its host, i.e., the noun phrase immediately preceding it. These requirements include: the counting quantifier requirement (Safir and Stowell 1988, Sutton

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\(^1\)Other names that have been given to binominal each includes ‘anti-quantifier (Choe 1987a), ‘shifted each’ (Postal 1974), and ‘adnominal each’ (Champollion 2016).

\(^2\)In Chapter 5, I show that determiner and adverbial each do share important properties with binominal each.
1993, Szabolcsi 2010), the variation requirement (Cable 2014, Champollion 2015, Kuhn 2017), and the extensive measurement requirement (Zhang 2013).

I argue that these requirements can be accounted for in a unified manner. Specifically, I argue that binominal each not only signals distributivity, but also contributes a monotonic measurement constraint on the dependency arising from distributive quantification. Specifically, the constraint requires a monotonic mapping from the size of the plurality contributed by the Key to the measurements contributed by the host of binominal each.

Although the semantics proposed for binominal each aligns it with markers of dependent indefinites in broad terms, there is an important difference between the analyses developed in this study and the analyses pursued by previous studies on dependent indefinites (e.g., Henderson 2014, Champollion 2015, Kuhn 2017). The difference lies in whether or not a constraint on a functional dependency makes use of the mereological structure of the dependency. Previous accounts of dependent indefinites do not access the mereological structure of functional dependencies. However, with help from binominal each, I show that it is crucial to treat functional dependencies as having a mereological structure.

To build functional dependencies with a mereological structure, I use the version of dynamic plural logic developed in the previous chapter, which allows distributive quantification to give rise to functional dependencies in the form of a set of assignments. Binominal each is able to compositionally access these dependencies after taking split scope (together with its host) over distributivity with the help of a higher order meaning (Cresti 1995, de Swart 2000, Charlow, to appear). The use of higher order meaning for accessing dependencies compositionally is another feature that distinguishes the present study from previous studies on distributive numerals (e.g., Henderson 2014, Kuhn 2017).

This chapter proceeds as follows. Section 3.2 takes up the three selectional requirements binominal each imposes on its host. Section 3.3 offers the informal generalization that binominal each imposes a monotonic measurement constraint on the internal mereological structure of the functional dependencies arising from distributivity. Section 3.4 provides a formal, compositional implementation of the monotonicity constraint in the framework of DPlLM. Section 3.5.2 takes up the interaction between binominal each and negation. Section 3.6 provides a comparison of my analysis of binominal each with the analyses put forward in previous studies. Section 3.7 concludes.
3.2 The selectional requirements of binominal each on its host

In this section, I discuss three properties of binominal each that does not directly follow from it being a distributive marker. These three properties all have to do with the noun phrase that immediately precedes binominal each (underlined in (4)).

(4) The girls saw two movies each.

To facilitate the discussion, let me introduce some terminology for referring to different parts of this sentence. The noun phrase that immediately precedes binominal each is called the host of binominal each. Following the terminology established in Chapter 1, the noun phrase being distributively quantified, typically a plural expression occupying the subject position, is called the Key. The whole predicate following the Key is called the Share.

3.2.1 Variation requirement

Safir and Stowell (1988) is the earliest study, as far as I know, to notice the variation requirement of binominal each. They observed that in a sentence like (5), there is a strong preference that the girls did not all see the same two movies. In fact, if one tries to add a continuation clause to identify two particular movies, as in done (6), the result is unacceptable.

(5) The girls saw two movies each.

(6) *The girls saw two movies each, namely Avatar and Ice Age.

Safir and Stowell (1988) treat binominal each as a polyadic distributive operator that quantifies over sets provided by two nominals at the same time (hence the name ‘binominal’.) According to them, the quantification in (5) results in a one-to-one correspondence between girls and movies, such that each girl saw a different set of two movies.3

Moltmann (1991) points out that the one-to-one correspondence condition is too strong. This is because a sentence like (5) is judged true not only when there is a one-to-one correspondence between the girls and the movies (or a total variation of the movies), such as the scenario shown in

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3Safir and Stowell (1988) credits Jim Higginbotham p.c.) for suggesting the one-to-one correspondence condition.
Figure 3.1 (left), but also when there is only partial variation of the movies, such as the scenario shown in Figure 3.1 (right).

![Figure 3.1: Two scenarios in which (5) is judged true.](image)

Molatmann suggests weakening the variation requirement to a condition of distinct d-refs, noting that distinct d-refs do not necessarily have distinct values. Recent studies that recognize the variation requirement, such as Cable (2014), Champollion (2015), and Kuhn (2017), borrow insights from distributive numerals and model the variation requirement of binominal each along the same lines as the variation requirement of distributive numerals.

Generally speaking, a distributive numeral is a numeral phrase with a morphological marker that induces a distributive interpretation of the sentence. In addition, the morphological marker bears an additional component requiring the numeral phrase to contribute a witness that co-varies with the Key. The following sentence from Kaqchikel (from Henderson 2014) illustrates a distributed numeral marked by numeral reduplication:


‘They looked for one book each.’ Kaqchikel


b. Variation inference: More than one book was looked for.

Couched in various frameworks, Farkas (1997, 2002a,b), Balusu (2005) and Henderson (2014) have proposed a plurality condition for capturing the variation requirement of distributive numerals. The plurality condition requires that a distributive numeral must be associated with at least two distinct

---

4Distributive numerals are known by various names. They are called ‘dependent indefinites’ in Farkas (1997, 2002a,b), Henderson (2014), Kuhn (2017). In languages that use reduplication to mark distributive numerals, they are commonly referred to as ‘reduplicated numerals’ (Gil 1988, Balusu 2005). Since the distributive numerals signal distributivity without being close to the Key, they have also been characterized as exhibiting ‘distance distributivity’ (Zimmermann 2002, Cable 2014).
values after distributivity is evaluated.\(^5\) I review how the variation requirement of distributive numerals is treated in Henderson (2014) in Section 3.6.1. It suffices at this point to know that attempts to extend his treatment of distributive numerals to binominal each, such as Champollion (2015) and Kuhn (2017), have been successful in modeling the variation requirement of binominal each.

A few other strategies have been explored to model the variation requirement. In Choe (1987a), the variation requirement is used to signal the obligatory narrow scope of the host of binominal each. A binominal each is called an ‘anti-quantifier’ because Choe (1987a) takes the variation requirement to indicate that the host of binominal each necessarily takes narrow scope, contrary to the scope flexibility of ordinary quantifiers. To see the contrast between a ‘quantifier’ and an ‘anti-quantifier’, consider (8) and (9). (8) is ambiguous between a wide-scope interpretation of every girl (8-a) and a narrow-scope interpretation of the quantifier (8-b), relative to the indefinite. Conversely, we can say that the quantifier contributed by the indefinite is ambiguous between a narrow-scope interpretation and a wide-scope interpretation.

(8) Every girl saw a monkey.
   a. For every girl, there is a monkey that she saw.
   b. There is a monkey such that every girl saw it.

However, the ‘quantifier’ one monkey each in (9) lacks the wide scope interpretation. It is only compatible with a narrow-scope interpretation, due to its variation requirement. For this reason, it is called an ‘anti-quantifier’.

(9) The girls saw one monkey each.
   a. For every girl, there is a monkey that she saw.
   b. #There is a monkey such that every girl saw it.

However, it must be made clear that while narrow scope may give rise to co-variation, it does not guarantee it. To see this, note that (9-a) is compatible with a scenario in which all the girls saw the

---

\(^5\)The plurality requirement is weaker than a one-to-one correspondence. However, Henderson (2014:fn.15) notes that although the plurality requirement seems to be truth-conditionally adequate, native speakers of Kaqchikel have a preference for full covariation, i.e., a one-to-one correspondence. A similar preference seems to also hold for binominal each (Simon Charlow, p.c.).
same monkey. For this reason, the variation requirement cannot be simply restated as a narrow-scope requirement.

Another possibility that has been considered, in Kuhn (2017), is to generate the variation requirement as an implicature. For example, the wide scope indefinite interpretation (9-b) entails the narrow-scope indefinite interpretation (9-a). By using binominal *each* to explicitly signal the narrow-scope indefinite interpretation, one indicates that the wide scope indefinite interpretation is false, hence triggering a covariation implicature. This possibility is briefly considered in Henderson (2014) and discussed in more detail in Kuhn (2015: Ch.3.6).

Both Henderson (2014) and Kuhn (2017) reject a scalar implicature account for the variation requirement. Henderson’s main objection is that scalar implicatures should be cancellable when the context fails to license it. However, distributive numerals, when failed to be licensed, are ungrammatical.\footnote{Kuhn cautions against using cancellability to diagnose scalar implicatures, as more recent studies have identified a host of grammaticalized, obligatory scalar implicatures (see Chierchia (2006), Chierchia et al. 2011, Fox 2007, a.o.).} Using data from American Sign Language, Kuhn argues that it is desirable to analyze distributive numerals and quantifier-internal adjectives like *same* and *different* as a unified class of phenomena. His concern with a scalar implicature approach is that it lacks generality: while it may be a reasonable account for distributive numerals and binominal *each*, it cannot be extended to quantifier-internal adjectives.

In short, the variation requirement of binominal *each* has received a few theoretical treatments. While the narrow scope approach (Choe 1987a) and the implicature approach face some difficulties, the plurality approach defended in many extant studies are empirically adequate for treating the variation requirement.

The beyond-distributivity properties of binominal *each*, however, are not limited to the variation requirement. In fact, there are two other requirements of binominal *each* that cannot be accounted for by studies that only target the variation requirement. I discuss these requirements in the next two subsections.

### 3.2.2 Counting Quantifier Requirement

It is generally agreed that binominal *each* forms a constituent with its host (Burzio 1986, Safir and Stowell 1988). In addition, studies have documented that binominal *each* seem to select some forms...
of indefinites as its host (e.g., Safir and Stowell 1988, Zimmermann 2002, Stowell 2013). The most precise description, I believe, comes from Sutton (1993). In particular, Sutton (1993) concludes that only counting quantifiers, i.e., noun phrases with (modified) numerals or vague quantity words like many, a few or several, can host binominal each (see also Szabolcsi 2010). All other noun phrases are rejected. The contrast is illustrated in (10) and (11).

\[
(10) \quad \text{The boys saw} \begin{cases} 
\text{two} \\
\text{at least two} \\
\text{more than two} \\
\text{a few} \\
\text{several} \\
\text{many} \\
\text{a lot of} 
\end{cases} \text{movies each.}
\]

\[
(11) \quad *\text{The boys saw} \begin{cases} 
\emptyset \\
\text{some} \\
\text{a certain} \\
\text{the} \\
\text{those} \\
\text{few} \\
\text{most} \\
\text{all} 
\end{cases} \text{movie(s) each.}
\]

Most previous studies that handle the counting quantifier requirement take it to be syntactic in nature. For example, Zimmermann (2002) takes binominal each to only compose with an indefinite. However, this analysis over-generates, as many indefinites in (11) cannot host binominal each, such as some movies, a certain movie. In fact, many speakers even dislike regular indefinites with the

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7 The term counting quantifier does not have an agreed-upon definition in linguistics. For example, while few is taken to be a counting quantifier in Beghelli and Stowell (1997), it is not treated as one in Sutton (1993).

8 To some speakers, some and few are better than the rest in (11) (Simon Charlow, p.c.). Safir and Stowell (1988) suggest that some of the NPs is slightly more acceptable than some NPs when it serves as the host. They judge the latter as unacceptable.
determiner *a* for hosting binominal *each*, according to Safir and Stowell (1988:(7a)):

(12) %The men saw a jewel each.

Alternatively, Cable (2014) proposes that binominal *each* takes a number term as one of its arguments. This has the effect of ruling out the hostile hosts in (11). The study may even correctly rule in the friendly hosts in (10) should it treat non-numerical counting quantifier determiners (such as *a few* and *many*) as generalized quantifiers over degrees that must undergo quantifier raising: the movement makes available a degree variable, which may have the same type as number terms (Kennedy 2015). Cable’s account represents a step forward in understanding the counting quantifier constraint: counting quantifiers are special because they have number terms as part of their semantics. However, in the next subsection, I show that the number component in a counting quantifier does not reliably distinguish hostile hosts from friendly hosts. What matters is the measurement component embedded in a counting quantifier.

### 3.2.3 Extensive Measurement Requirement and Monotonicity

Zhang (2013) notices that the counting quantifier constraint is insufficient even as a description. Concretely, Zhang (2013) observes that the type of measurement also plays a crucial role in constraining what counting quantifiers may host binominal *each*: extensive measurements give rise to friendly hosts but intensive measurements give rise to hostile hosts.

It is widely assumed that numeral expressions such as *two students* and *seven feet* have more structure than meets the eye. In addition to the number word and the common noun, they also contain measure functions like *cardinality*, *height*, *weight*, *speed*, and *temperature*. According to Lønning (1987), a measure function denotes a mapping between a class of physical objects and a degree scale that preserves a certain empirically given ordering relation, such as “be lighter than” or “be cooler than.” Degrees are further mapped to numbers by unit functions like *pound* or *kilogram*. Krifka (1989, 1998) classifies measure functions into two types—extensive and intensive measure functions. Crucially, these two types of measure functions differ with respect to the property *additivity*. More concretely, *weight* is extensive since for any object, its weight is equal to the weight of all its parts added together; whereas *temperature* is intensive since the temperature of an object is
not always equal to adding up the temperature of its parts.

The examples in (13) and (14) demonstrate Zhang’s observation that binominal each can only be hosted by a noun phrase with an extensive measure function. To rule out the concern that some of the intensive measure functions may give rise to a more complex structure, as in the case of speed, or a less natural noun phrase, as in the case of purity, a minimal pair using the measure phrase 60 degrees is offered. In (13-d), 60 degrees is a measurement of the angles, and in (14-a), the same form is a measurement of the temperature of drinks.

(13) a. The boys read two books each.  
    b. The girls walked three miles each.  
    c. The windows are four feet (tall) each.  
    d. The angles are 60 degrees each.

(14) a. *The drinks are 60 degrees (Fahrenheit) each.  
    b. *The girls walked at three miles-per-hour each.  
    c. *The gold rings are 24 Karat each.

Zhang (2013) proposes to understand the extensive measurement requirement as follows: the measurement of the Key should be positively correlated with the measurement of the host. I think Zhang’s generalization is essentially correct. Other than Zhang (2013) and my attempt in this chapter, I am not aware of any previous study on binominal each that has an account for the extensive measurement requirement.

In fact, binominal each is not the only natural language item that cares about the distinction between extensive and intensive measurement. Schwarzschild (2002a, 2006) points out a similar contrast in pseudo-partitives: pseudo-partitives admit extensive measurement, as in (15), but reject intensive measurement, as in (16).

(15) a. two pounds of cherries  
    b. thirty liters of water

(16) a. *five degrees Celsius of the water in this bottle  
    b. *five miles an hour of running
In addition, Wellwood (2015) observes similar contrasts in comparatives. Both sentences in (17) can express comparisons involving extensive measurement, but neither can express a comparison involving intensive measurement. For example, in (17-a) the amount of the soup that Al bought is larger than the amount of the soup that Bill bought. The amount may be understood in terms of volume or weight, but not temperature.

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(17) a. Al bought as much soup as Bill did. volume, weight, *temperature
b. Al ran as much as Bill did. time, distance, *speed

Schwarzschild (2002a, 2006) accounts for the sensitivity of measurement constructions to types of measure function by invoking the notion of monotonicity. Wellwood (2015) provides a formal definition of this monotonicity condition on measurement, as shown in (18). This condition requires that the part-whole structure of the domain of a measure function be preserved in the domain of degrees.

(18) **Monotonic Measurement** \( (\text{Wellwood 2015}) \)

A measure function \( \mu \) is monotonic iff

a. there exists \( x, y \in D_{\text{part}} \), such that \( x \neq y \), and

b. for all \( x, y \in D_{\text{part}} \), if \( x \sqsubset_{\text{part}} y \), then \( \mu(x) <_{\text{deg}} \mu(y) \)

The monotonicity condition of Schwarzschild and Wellwood says the following: only monotonic measure functions can be used in measurement constructions like pseudo-partitives or comparatives. Consequently, extensive measure functions, but not intensive ones, pass the condition. Consider a portion of coffee, \( c \), and two of its proper parts, \( c_1 \) and \( c_2 \). \( c \) necessarily measures a greater degree by volume or weight than that of the parts \( c_1 \) and \( c_2 \), but \( c, c_1 \) and \( c_2 \) typically have the same temperature. If they don’t, the temperature of \( c \) falls somewhere between the temperature of \( c_1 \) and the temperatures of \( c_2 \), making temperature non-monotonic.

It is reasonable to assume that constructions with binominal *each* also obey some version of the monotonicity condition. This will have the following effect: to qualify as a host for binominal *each*, a noun phrase must have a measure function, and the measure function must be an extensive measure function to satisfy the monotonicity condition. It is clear that the monotonicity condition
straightforwardly accounts for the sensitivity of binominal *each* towards extensive and intensive measure functions. In addition, it illuminates the counting quantifier requirement. Counting quantifiers are essentially measure phrases, typically involving the extensive measure function *cardinality*. By contrast, bare nouns and indefinites do not contribute any measure function, making them unsuitable hosts. Additionally, Schwarzschild (2006) shows that NPs with Q-adjectives like *many, a few, a little* and *a lot of* have a syntax similar to measurement phrases and must be associated with a monotonic measure function. In this respect, it is not surprising that NPs formed with Q-adjectives pattern like counting quantifiers and extensive measure phrases.

### 3.3 A monotonicity constraint for binominal *each*, informally

In this work, I propose that a sentence with binominal *each* has a two-part contribution: distributivity and monotonicity. The former may be contributed by binominal *each* itself, as argued in Kuhn (2017) and many other studies that simply treat *each* as a distributive operator (e.g., Zimmermann 2002, Dotlačil 2012, Champollion 2017), or by a separate distributive operator, as suggested in Champollion (2015) following Henderson’s (2014) semantics for distributive numerals. I leave both options as open possibilities since there is considerable inter-speaker variation regarding how acceptable binominal *each* is when a distributive quantifier is present (Champollion 2015, Kuhn 2017):

(19) %Every boy saw two movies each.

The monotonicity inference is assembled with help from the distributivity inference as well as the ingredients provided by the host. Leaving a fully compositional implementation until Section 3.4, let me spell out the formation of the monotonicity inference in plain English below.

The distributivity inference provides a set of functional dependencies indicating the relationship between the individual parts of the Key and information provided by various expressions in the share. For example, (20) provides us with, at the very least, a set of functional dependencies encoding the relationships between the boys, the movie-watching events, and the movies being watched. I have singled out the dependency between the boys and the movies being watched in Figure 3.2.
(20) The boys saw two movies each.

\[ f : *\text{boy} \rightarrow *\text{movie} \]

Figure 3.2: Dependency established via distributivity

Here, boy1 saw movie1 and movie2, while boy2 saw movie3 and movie4. The movies seen between the two boys are movie1, movie2, movie3 and movie4. Let’s assume a function \( f \) that maps each boy to the movies he saw and also sums of boys to the sums of movies they saw. In other words, \( f \) encodes the functional dependency induced by distributivity and is cumulatively closed (marked by *, following Link 1983).

The host of binominal each provides two important ingredients: (i) a measurement function \( \mu \), and (ii) a label for the range of \( f \). The second is important because more than one functional dependency can be formed out of any distributive quantification. The host indicates which functional dependency is being considered.

With \( f \) and \( \mu \) in hand, we can define a monotonic measurement condition checked in association with the functional dependency of distributivity, as in (21) (I abbreviate this condition \( \text{dm} \), with ‘d’ a mnemonic for distributivity and ‘m’ a mnemonic for monotonicity and measurement).

(21) **Monotonic measurement in association with distributivity (\text{dm}, first approximation)**

A measure function \( \mu \) satisfies \( \text{dm} \) iff there is a function \( f \) such that\(^9\)

a. NON-DECREASING MAPPING

For all \( a, a' \in \text{Dom}(f) \). \( a \leq a' \rightarrow \mu(f(a)) \leq \mu(f(a')) \), and

b. NON-CONSTANT MAPPING

There are distinct \( b, b' \in \text{Dom}(f) \). \( \mu(f(b)) \neq \mu(f(b')) \)

---

\(^9\)Recall that degrees are modeled as triples in this study and their first coordinates (i.e., degree names) are fully ordered relative to a scale and a relation. So, \( \mu(f(a)) \leq \mu(f(a')) \) iff the first coordinate of the triple arising from \( \mu(f(a)) \) is at least as great as the first coordinate of the triple arising from \( \mu(f(a')) \), likewise for \( \mu(f(a)) = \mu(f(a')) \).
(21-a) requires that the part-whole relation found in the domain of $f$ (i.e., the boys) be mapped non-decreasingly to the measurement of the range of $f$ (i.e., the movies). Since $f$ encodes the functional dependency induced by distributivity, this amounts to making reference to the structure of distributivity. In other words, we are referring to parts of a distributivity dependency that stand in a part-whole relation. Modulo the association with $f$, (21-a) is a standard definition of non-decreasing monotone functions. It is weaker than the definition of monotonic measure functions found in Wellwood (2015), which picks out strictly increasing functions among the non-decreasing ones. I will return to this difference after demonstrating how the definition in (21) works as a whole.

(21-b) requires measurement variability in the range of $f$.

3.3.1 Capturing the extensive measuring requirement

Let me start by demonstrating how the monotonic measurement condition captures the extensive measurement requirement. Consider (20) with a function $f$ as illustrated in Figure 3.2. The measure function in this case is cardinality (or $\mu_{\text{card}}$). It is clear that (20) in this setup satisfies (21). First, suppose we take elements $b_1$, $b_2$ and $b_1 \oplus b_2$, the former two are proper subparts of the last one. $f$ maps $b_1$ to $m_1 \oplus m_2$, $b_2$ to $m_3 \oplus m_4$ and $b_1 \oplus b_2$ to $m_1 \oplus m_2 \oplus m_3 \oplus m_4$. The cardinality function $\mu$ maps $f(b_1)$ to 2, $f(b_2)$ also to 2, and $f(b_1 \oplus b_2)$ to 4, as shown in Figure 3.3. Since the measurement of the range of $f$ does not decrease (in fact, it increases) as we consider increasingly bigger elements in the domain of $f$, we can conclude that (21-a) is satisfied. In addition, there are at least two elements in the domain of $f$ that get mapped to elements in the range of $f$ that also yield different measurements. For example, $b_1$ and $b_1 \oplus b_2$ are such a pair, so are $b_2$ and $b_1 \oplus b_2$. We can conclude that (21-b) is also satisfied.

(21-a) alone is a rather weak condition. In fact, as long as the measure function involved in the
host of binominal each is extensive, it is always satisfied, regardless of how many elements there are in the domain and the range of $f$. One can verify this by constructing scenarios with only one element in the domain of $f$ and/or only one element in the range. In addition, if $\mu_{\text{dim}}$ is intensive, as long as the range of $f$ is a singleton, or the range of $\mu_{\text{dim}}$ is a singleton, (21-a) is satisfied. Therefore, to have the right strength, (21-a) has to be complemented by (21-b).

What (21-b) requires is that the values stored in the range of $f$ must yield different degrees after being measured by a measure function. This rules out the possibility of all values in the range of $f$ having the same measured degree. For example, (22) has an intensive measure function temperature (or $\mu_{\text{temp}}$) that typically yields a uniform degree for all the values in the range of $f$, as illustrated in Figure 3.4. It is predicted to fail non-constant mapping, i.e., (21-b), and hence violate the monotonic measurement condition. Note that it does not violate NON-DECREASING MAPPING in (21-a), as the measurement is indeed a non-decreasing mapping of the domain of $f$, albeit in a trivial way as there is only one degree in the range of $\mu_{\text{temp}}$.

(22) *The boys bought 60-degree coffee each.

![Figure 3.4: Intensive measurement does not track the internal structure of distributivity](image)

If measuring the range of $f$ indeed yields different degrees, as in the case of cardinality measurement as illustrated in Figure 3.3, non-constant mapping is satisfied.

Interestingly, $f$ and $\mu_{\text{dim}}$ may happen to be the same function, and the contrast between extensive and intensive measurement still holds, as shown in (23-a) and (23-b).

(23) a. *The coffees are 60 degrees each.

b. The angles are 60 degrees each.
In these two examples, a predicative measure phrase helps map individuals in the Key to the corresponding degrees of measurement as indicated by the measure phrase. For concreteness, the coffees (or angles) are distributively checked for their temperature (or degree). So, \( f \) encodes a functional dependency between coffees (or angles) and their temperatures (or degrees). \( \mu_{\text{temp}} \) (or \( \mu_{\text{ang}} \)) is identical to \( f \) in being a temperature (or angle) measure function. Both sentences satisfy NON-DECREASING MAPPING as stated in (21-a). However, (23-a) fails NON-CONSTANT MAPPING while (23-b) satisfies it. This is because there is only one temperature, i.e., 60 degrees, in association with \( f \) in (23-a), but two degrees, i.e., 60 degrees and 120 degrees, in association with \( f \) in (23-b). The contrast is illustrated in Figure 3.5.

One may suspect that (21-b) alone is sufficient to guarantee the variation inference and the privilege of extensive measure functions. It is not. It can be satisfied with an intensive measure function as long as the function yields different degrees for different values in the range of \( f \). For example, consider binominal each whose host is a measure phrase with a modified numeral, such as (24-a) and (24-b). Figure 3.6 illustrates how \( f \) and \( \mu_{\text{dim}} \) works in these two sentences.

(24)  

a. *The drinks are more than 60 degrees each.

b. The angles are more than 60 degrees each.

(24-a) satisfies NON-CONSTANT MAPPING (as well as evaluation-level plurality), as the range of \( f \) has different degrees. However, it is still not well-formed. This is because it violates NON-DECREASING MAPPING: there is a pair of elements in the domain of \( f \) that stand in a part-whole relation whose corresponding measurement fails to preserve the order of the pair, as indicated by the crossing lines in Figure 3.6a. By contrast, (24-b) satisfies both NON-CONSTANT MAPPING (as
3.3.2 Capturing the counting quantifier requirement

Lastly, we predict that noun phrases without an appropriate measure function component cannot host binominal each. A natural question that arises is how we can diagnose the presence of a measure function component. I do not have a comprehensive answer at this point. However, compatibility with unit functions like pound(s) and mile(s) seems to be a rather reliable test: if a determiner-like expression is compatible with measure units like pounds and miles, then it can form a noun phrase that can host binominal each. Some examples are given in Table 3.1.  

It has been pointed out that noun phrases with the indefinite article a are better than those with the determiner some in hosting binominal each, although not all speakers accept them equally well (Safir and Stowell 1988, Szabolcsi 2010, Milačić et al. 2015), as illustrated in (25).

(25) a. %The boys read a book each.
   b. *The boys read some book(s) each.

Interestingly, a is also compatible with unit functions in ways that some is not. Of course, this is not

---

10Some sometimes does occur with unit functions, as in gained some inches and lost some pounds. In these cases, the unit functions are interpreted as standing in for the entities they measure, i.e., height and weight, respectively. I have been informed that some + units are more friendly hosts than ordinary some NPs (Simon Charlow, p.c.):

(i) The boys lost some pounds each over the summer.

(ii) *The boys lost some marbles each over the summer.
### Table 3.1: Expressions that can (and cannot) form a host for binominal *each*

<table>
<thead>
<tr>
<th>Expressions</th>
<th>Compatibility with measure units</th>
<th>Host binominal <em>each</em></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>(modified) numerals</em></td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>two, at least/most two,</td>
<td>e.g., two pounds</td>
<td>e.g., two books each</td>
</tr>
<tr>
<td>more/less than two</td>
<td>more than five miles</td>
<td></td>
</tr>
<tr>
<td>[Hackl (2000), Kennedy (2015)]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>quantity expressions</strong></td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>a few, a couple, many</td>
<td>e.g., a few gallons</td>
<td>e.g., many movies each</td>
</tr>
<tr>
<td>[Rett (2014), Solt (2015)]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>quantity comparative</strong></td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>more, as many (much) as</td>
<td>e.g., as many pounds as</td>
<td>as many books each</td>
</tr>
<tr>
<td>[Wellwood (2015)]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>quantificational determiners</strong></td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>no, some, few, most, every, all</td>
<td>e.g., <em>most miles</em></td>
<td>e.g., *most books each</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
at all surprising given that many linguists have argued that *a* is derived from *one* synchronically or diachronically (e.g., Perlmutter 1970, Chierchia 2013, Kayne 2015). Given these considerations, it is conceivable that *a* is ambiguous between a numeral *one* and an existential determiner, while *some* is only an existential quantificational determiner without a measure function component.\footnote{It is perhaps too simple to think that *some* NPs have a simple existential quantifier status. For example, it has been observed that when the common noun restriction is a singular count noun, as in *some girl*, the quantifier carries an epistemic effect (Farkas 2002b, Alonso Ovalle and Menendez Benito 2003.)}

(26)  
  a. a mile, a pound, an inch  
  b. *some mile(s), *some pound(s), *some inch(es)

### 3.3.3 NON-DECREASING + NON-CONSTANT VS. STRICTLY INCREASING

The decision to use a weaker form of monotonicity, one in terms of a non-decreasing mapping, instead of a strong form requiring a strictly increasing mapping, as suggested in Wellwood (2015), is empirically motivated. Consider (27). It is judged true in a scenario like Figure 3.7a, in which both boy1 and boy3 saw movie1, while boy2 saw movie2. Since the range of the mapping function serves as the domain of the measure function, we can compose the two functions to form a composite function, $\mu_{\text{dim}} \circ f$ as illustrated in Figure 3.7b: the domain of the function is the values associated with the Key, i.e., the boys in this case, and its range is the measurement of the values introduced by the host, i.e., the cardinality of the movies.

(27) The boys watched one movie each.

In this situation, the cardinality of $f(b_1)$ is the same as that of $f(b_1 \oplus b_3)$. Similarly, the respective cardinality of $f(b_1 \oplus b_2)$ and $f(b_2 \oplus b_3)$ is the same as that of $f(b_1 \oplus b_2 \oplus b_3)$. In other words, the composite function is non-injective.

If the monotonicity constraint is formulated to require a strictly increasing mapping, like (28), then the situation in Figure 3.7 is predicted to be incompatible with (27). This is because a strictly increasing mapping does not allow two elements that stand in a proper subpart relation in the domain of $\mu_{\text{dim}} \circ f$ to be mapped to the same element in the range of $\mu_{\text{dim}} \circ f$. However, there are a few pairs of elements in the domain of $\mu_{\text{dim}} \circ f$ that stand in a proper subpart relation but are mapped to the same element in the range of $\mu_{\text{dim}} \circ f$. For example, the pair $b_1 \oplus b_3$ and $b_1$, as well as the pair...
\[ f : \text{*boy} \rightarrow \text{*movie} \]

\[ \mu_{\text{card}} \circ f : \text{*boy} \rightarrow D_d \]

(a) Dependency between boys and movies

(b) Dependency established by \( \mu_{\text{card}} \circ f \)

Figure 3.7: Non-decreasing mapping

\[ b_1 \oplus b_2 \oplus b_3 \text{ and } b_1 \oplus b_2. \]

(28) **Strictly increasing dm** (rejected)

For all \( a, a' \in \text{Dom}(f) \), \( a < a' \rightarrow \mu(f \ a) < \mu(f \ a') \)

However, formulating the monotonicity constraint as a non-decreasing and non-constant mapping, as in (21), does not run into this problem. The composite function in Figure 3.7b is non-decreasing and non-constant. Therefore, it is predicted that (27) is acceptable in the scenario depicted in Figure 3.7a. This is indeed a welcome result, as binominal *each* only requires partial variation, a property that also holds for dependent indefinites as noted in Henderson (2014:fn15) see also footnote 5 earlier in this chapter).

### 3.4 Formalizing the monotonic measurement condition

Now that the monotonic measurement condition has been established, we are ready to supplement it with a more compositional semantics. In fact, it is not difficult to imagine what kind of framework we need to implement this condition compositionally. The framework should satisfy the following criteria:

- Criterion 1: It should allow us to talk about measure functions of various sorts.
- Criterion 2: It should allow us to represent the functional dependencies arising from distributive quantification and refer back to them. In other words, it should make concrete how \( f \) is
assembled.

- Criterion 3: Since measurement kicks in after $f$ is established, we need a way to split up the contribution of a host of binominal *each*, evaluating one part (i.e., the basic semantics of the host) inside the scope of distributivity and the other part (i.e., the monotonic measurement condition) outside the scope of distributivity. The former provides the necessary ingredients for building the functional dependencies of distributivity and hence the function $f$. The latter can access $f$ after it is assembled.

Criterion 1 is very easy to satisfy. Any framework that can be enriched to include pluralities and measure functions can be used to model monotonicity. Therefore, a decisive choice depends on the remaining two criteria.

A well-known framework satisfying Criterion 2 is Dynamic Plural Logic of van den Berg (1996) and its close cousin Plural Compositional DRT, devised in Brasoveanu (2007, 2008, 2013). Both approaches have been used to model phenomena that need access to the functional dependencies of distributivity, such as quantificational subordination (van den Berg 1996, Nouwen 2003), quantifier-internal adjectives and reciprocals (Dotlačil 2010), as well as distributive numerals (Henderson 2014, Champollion 2015 and Kuhn 2017).

In Chapter 2, I have developed a hybrid approach, DPlLM, which is intermediate between DPlL and PCDRT, and enriched to include various sorts of measurement. The logic lets assignment functions range over not only atomic individuals, as in van den Berg (1996), but also plural individuals, as suggested in Brasoveanu (2008). However, it sides with van den Berg (1996) in inhibiting dependency introduced by random assignment. To introduce dependency into discourse, a distributive operator has to be used. This is crucially different from the PCDRT tradition, which allows any random assignment to introduce dependency into discourse. I show, in Section 3.6.1, that this choice explains why the monotonicity condition is only seen with distributive predication.\footnote{There are other frameworks that track distributivity dependency. For example, Schein (1993), Lasersohn (1995) and Champollion (2017) develop accounts for distributivity based on event semantics, in which the dependency is retrievable from events. There are also event-based analysis on distributive numerals, such as Balusu (2005) and Cable (2014). For another example, Huang (1996) develops a semantics for distributivity based on skolem functions, in which the distributivity dependency can be retrieved by using skolem functions. There are few studies that use the skolem function approach to analyze distributive numerals with the exception of Milačić et al. (2015). The merit of DPlL is that it not only tracks the dependency in context, but the built-in anaphoric device (i.e., discourse variables) allows us to access the dependency relatively easily. I thank Veneeta Dayal and Simon Charlow for discussing these alternative frameworks with me.}

\footnote{Besides van den Berg (1996), many studies have observed that distributive quantification has a much easier time...}
Criterion 3 essentially asks for a split-scope mechanism. Several alternatives have been explored in the literature. One option is by means of a post-supposition (Henderson 2014, Champollion 2015. Kuhn (2017) points out that post-suppositions, without further assumptions, predict the lack of locality in the licensing of distributive numerals. The prediction is not borne out, as binominal each and its Key cannot be separated by a scope island. Consider the following examples from English and Hungarian (‘<...>’ marks a scope island and judgments due to the credited sources):

(29) *The boys said <Mary captured two snakes each>. (Safir and Stowell 1988;(48))

(30) a. Jones proved the prisoners guilty with one accusation each.
    b. Bob made/let Sam and Tom leave on two occasions each. (Safir and Stowell 1988:(36a-b))

(31) ??The linguists thought <two theories each were refuted>. (Simon Charlow, p.c.)

(32) The linguists want two theories each to be refuted. (Simon Charlow, p.c.)

(33) Minden professzor két-két diákról mondta, hogy meglepné ha <diplomát every professor two-two students-of said that surprised if diploma szereznének>. receive ‘Every professor said of two students that he would be surprised if they graduated.’ (Hungarian, Kuhn 2017:(100))

(34) *Minden professzor azt mondta, hogy meglepné ha <két-két diák diplomát szerezne>. every professor DEM said that surprised if two-two student diploma receive ‘Every professor said of two students that he would be surprised if they graduated.’ (Hungarian, Kuhn 2017:(101))

In (29) and (31), the distributive numerals are inside tensed clauses, which have been independently identified as a scope island for quantifiers (e.g., May 1985, Beghelli 1995, Barker 2002, Charlow 2014). In (30) and (32), the distributive numerals are inside untensed (ECM) clauses, which have been observed not to be a scope island for quantifiers (e.g., May 1985). The fact that distributive introducing dependency than non-distributive quantification, such as cumulative and collective quantification. Some examples are Nouwen (2003) and Solomon (2011).
numerals introduced by binominal *each* are subject to the same locality conditions governing quantifier scope suggests that a locality-sensitive mechanism should be used for licensing distributive numerals.

To model the island sensitivity of distributive numerals, Kuhn (2017) suggests a scope-taking analysis, in which a distributive numeral like *two theories each* has to undergo quantifier-raising (QR) to take wide scope. A drawback of Kuhn’s QR analysis (discussed in Kuhn 2017 and credited to an anonymous reviewer), is that it fails to account for the grammaticality of distributive numerals with a bound pronoun inside them.

(35) Minden rendező benevezte két-két filmjét.

every director entered two-two film-POSS.-3SG-ACC

‘Every $x$ director entered two films of his $x$ (in the competition).’ (Hungarian, Kuhn 2017:(107))

In this Hungarian example, the noun phrase restriction of the distributive numeral has a (possessor) pronominal bound by the quantifier that licenses the distributive numeral. If the distributive numeral has to take wide scope over its licensor to be licensed, then the pronoun is left unbound.

Based on considerations of island sensitivity and pronominal binding, Charlow (to appear) suggests a scope-taking mechanism involving higher order meaning. While deferring a more detailed discussion until Section 3.4.3, it suffices to note that Charlow’s higher-order meaning approach has a very similar empirical coverage as the post-supposition approach, with the exception of island sensitivity, which favors the former. In this study, I adopt the higher-order meaning approach for it has better empirical coverage, although the choice is largely immaterial to the main claim that binominal *each* makes reference to the mereological structure of a distributivity dependency.

It should be clear by now what kind of framework is needed to account for the crucial properties of binominal *each* discussed earlier. In the next sections, the essential components of such a framework are provided. I begin by discussing the general framework in Section 3.4.1, followed by translating the monotonic measurement condition into this framework, and lastly in Section 3.4.3 the monotonicity condition is implemented in a compositional manner.
3.4.1 Formal background: DPILM

The background for the account is DPILM, as outlined in Chapter 2. Recall DPILM is a dynamic logic using info-states for encoding discourse information. An information state is a set of assignment functions, which is capable of encoding functional dependencies. In addition, by drawing subsets from a set of assignments, we can access the internal mereological structure of the functional dependencies contributed by distributivity.

3.4.2 Monotonic measurement condition in DPILM

Recall that in Section 3.3, I have sketched the main proposal of this chapter: binominal each introduces a constraint known as the monotonic measurement constraint, checking the monotonic property of measure functions relative to the internal mereological structure of the functional dependency established via distributivity.

(36) **Monotonic measurement in association with distributivity (dm, with f)**

A measure function \( \mu \) is \( \text{dm} \) iff there is a function \( f \) such that

a. **NON-DECREASING MAPPING**
   
   For all \( a, a' \in \text{Dom}(f) \). \( a \leq a' \rightarrow \mu(f \ a) \leq \mu(f \ a') \), and

b. **NON-CONSTANT MAPPING**

   There are distinct \( b, b' \in \text{Dom}(f) \). \( \mu(f \ b) \neq \mu(f \ b') \)

The checking of \( \text{dm} \) is facilitated by a function \( f \) that maps values stored in the Key to corresponding values stored in the host. The natural correlate of this \( f \) in DPILM is sets of assignment functions, i.e., info-states. To see this, recall that info-states encode not just values assigned to variables and dependencies among different variables, but also internal structures of these dependencies. In more concrete terms, with help of info-states, not only can we retrieve values associated with the Key and the host of binominal each, given that distributivity is externally dynamic in this logic, we can also make precise reference to the corresponding values in the host for all the atomic values and their combinations (i.e., pluralities) in the Key. Having access to this structured dependency allows us to conduct measurement on it to check \( \text{dm} \). Translating \( \text{dm} \) as a dynamic proposition into DPILM, we
obtain (37).  

**Definition 27 (Monotonic measurement in association with distributivity (dm, in DPILM))**

(37) \[ G[dm_{x,y}(\mu)]H = \top \text{ iff} \]

a. \[ H = G \]

b. For all \( A, A' \subseteq G(x) . A \subseteq A' \rightarrow \mu\left( \bigoplus_{x \in A} G(y) \right) \leq \mu\left( \bigoplus_{x \in A'} G(y) \right) \]

c. There are distinct \( B, B' \subseteq G(x) . \mu\left( \bigoplus_{x \in B} G(y) \right) \neq \mu\left( \bigoplus_{x \in B'} G(y) \right) \]

To begin with, \( dm \) bears two anaphoric indices. The first one corresponds to the variable introduced by interpreting the Key and the second one corresponds to the variable introduced by interpreting the host. There is a longstanding tradition in granting binominal *each* an anaphoric component, started in the early work of Burzio (1986) and Safir and Stowell (1988) and later adopted in Dotlačil (2012), Cable (2014), and Kuhn (2017). I provide independent justification for using anaphoric indices in Section 3.5.2, where I discuss how negation interrupts dynamic binding in binominal *each* constructions.

To check for \( dm \) of a measure function in DPILM, we need to access the values stored in the variable the measure function applies to. (37) says that the measure function \( \mu \) is monotonic on the dependency between \( x \) and \( y \) iff

- (37-a): Checking \( dm \) does not change the info-state in any way (i.e., it’s a test).
- (37-b): Measuring \( y \)'s values in an info-state storing less \( x \)'s values does not yield a bigger number (or degree) than measuring \( y \)'s values in an info-state storing more \( x \)'s values.
- (37-c): In the input info-state, there are at least two sub-parts storing different \( x \)'s values that also yield different measurement of \( y \)'s values.

In addition, I propose that the monotonicity condition in (37) is introduced as an ‘output context constraint’ in the sense of Farkas (2002b) and Lauer (2009, 2012). In particular, (37) is treated as a constraint that is checked after the at-issue content has been established. If the at-issue content

---

\(^{14}\)

The domain and the range of the function \( f \) in (36) are closed under sum formation. To model this, we consider in an info-state subsets of values assigned to the variable \( x \) corresponding to the Key (i.e., the domain of \( f \)) and to the variable \( y \) corresponding to the host of binominal *each* (i.e., the range of \( f \)). Since the host is subject to a measurement transformation, sets of values in \( y \) are mapped to mereological sums using \( \bigoplus \). See Definition 10 in Chapter 2.
cannot pass the test, then the truth condition denoted by the sentence is not defined. As a result, the
sentence is *undefined*, rather than *false*. This is to model the fact that sentences with binominal *each*
that fail \( dm \) (for various reasons) are judged unacceptable and not false, as illustrated below:

(38)  
   a. *The drinks are 60 degrees (Fahrenheit) each.
   b. *The boys read some books each.
   c. *The boys read one book each, namely *Emma*.

The constraint is formulated in (39). The connective \( \overline{\wedge} \) indicates that the constraint \( \psi \) applies after
evaluating the at-issue content \( \phi \).

**Definition** Output context constraint

(39)  
\[
G [\phi \overline{\wedge} \psi] H = G [\phi] H \text{ if } H [\psi] H = T; \text{ otherwise, undefined}.^{15}
\]

This definition says: the at-issue content given by \( \phi \) has a truth value only if the output context of \( \phi \)
admits \( \psi \). A constraint behaves in a similar way to a presupposition in being a definedness condition,
but it differs from a presupposition as the definedness condition is imposed on the *output* context,
instead of the *input* context. This way of understanding the monotonic measurement constraint
makes novel and supported predictions about how it interacts with negation, as discussed in 3.5.2.

3.4.3 Composition

Like *Nouwen* (2003) and *Brasoveanu* (2008), I assume that DPILM is a typed logic. It includes
basic types and derived types as in (40): \( e \) for entities, \( t \) for truth values, \( s \) for assignments, \( d \) for
degrees, and a derived type \( \tau \rightarrow \tau \) for functions.

(40)  
\[
\tau ::= e \mid t \mid s \mid d \mid n \mid \tau \rightarrow \tau
\]

To keep type description reader-friendly, the following type abbreviations are used:

---
^{15} It is nontrivial to have a logic with an undefined truth value. In this dissertation, I assume a sentence has a truth value
only when defined and undefinedness projects following a weak Kleene logic (Kleene 1952). However, this is likely
insufficient as weak Kleene logic is known to be insufficient for describing presupposition projection (Beaver and Geurts
2014).
I propose that a noun phrase hosting a binominal *each* is a measure phrase. Depending on whether the measure phrase occurs in an argument position, as in (41-a) and (41-b), or a predicate position, as in (41-c), it has slightly different types.

(41)  

a. John bought two apples.  
b. John bought three pounds of chicken.  
c. John is six feet (tall).

In an argument position, a measure phrase is a dynamic generalized quantifier (GQ), of type \((e \to t) \to t\). In a predicate position, a measure phrase is simply a predicate, of type \(e \to t\). However, unlike ordinary dynamic GQs and predicates, measure phrases have two additional components: a measure function and a measure head. The internal structures of different measure phrases are given in Figure 3.8.

Argumental measure phrases, analyzed as GQs, are shown in Figure 3.8a and Figure 3.8b. If the measure phrase is a cardinal GQ, the measure head is a silent determiner akin to the silent *many* in Hackl (2000). The measure head takes a number, a property and a measure function and returns a GQ. This measure head is defined in (42-a). If the measure phrase is a non-cardinal GQ, the measure head is assumed to be provided by a measure unit like *pounds*, which takes a number, a property, and a measure function and returns a GQ, as defined in (42-b).

(42)  

a. \[
\text{many}^\# := \lambda n \lambda P \lambda m \lambda P'. \exists y \; (P \land y \land P' \land y \land m \; y = \langle n, m, y \rangle)
\]
The cardinality measure head many selects (with help of agreement or some other means) a cardinality measure function $\mu_{\text{card}}$ (type $e \rightarrow n$), while a non-cardinality measure head like pound selects an non-cardinality measure function, like $\mu_{\text{weight}}$ (type $e \rightarrow d$). A measure function is assumed to be syntactically present and further away from a measure head, unlike that in Hackl (2000), which builds the measure function into the meaning of a measure head.

If a measure phrase is predicative and has a nominal predicate (as in this is two pounds of chicken), then the measure head takes the same ingredients, but returns a predicate rather than a GQ. The corresponding definitions of the predicative measure heads are given in (43-a) and (44-a). Lastly, sometimes a measure phrase may not contain a common head noun at all, as in this is two pounds.
I assume that a measure head may optionally not take a nominal predicate as one of its arguments, giving rise to a measure phrase. Sample definitions of the measure heads are given in (43-b) and (44-b).

\[(43)\]

a. \[\text{many}^\text{NP} := \lambda n \lambda P \lambda m \lambda u. P \ u \land m \ u = \langle n, m, u \rangle\]
b. \[\text{many}^\text{MP} := \lambda n \lambda m \lambda u. m \ u = \langle n, m, u \rangle\]

\[(44)\]

a. \[\text{pound}^\text{NP} := \lambda n \lambda P \lambda m \lambda u. P \ u \land m^y\text{alt} \ u = \langle n \ lbs, m, u \rangle\]
b. \[\text{pound}^\text{MP} := \lambda n \lambda m \lambda u. m^y\text{alt} \ u = \langle n \ lbs, m, u \rangle\]

With the assumptions about the internal structure of a measure phrase fleshed out, we are now ready to add binominal \textit{each} to the structure. I assume that binominal \textit{each} attaches to a measure function and turns the whole measure phrase into a higher-order meaning. Concretely, in a cardinal GQ, binominal \textit{each} maps the GQ into a higher-order GQ by turning the measure function from an argument status (it is sought by a \(m \to Q\) function) to a function status (it now seeks a \(m \to Q\) function), as shown in Figure 3.9a. Similarly, in a measure phrase predicate, \textit{each} attaches to the measure function and turns the whole measure phrase predicate into a higher order predicate, as shown in Figure 3.9b.

Since binominal \textit{each} can be hosted by both argumental and predicative measure phrases, and predicative measure phrases with or without a common noun component, we need to allow it to be type-polymorphic. I offer a schema for defining binominal \textit{each} in (45-a), where \(f\)'s range may be any type \(\alpha\). In addition, when a measure phrase does not introduce any discourse variables, as in the case of a predicative measure phrase, \textit{each} only needs to bear one anaphoric index, i.e., the anaphoric index for the variable storing the individuals measured by the measure function \(\mu_{\text{dim}}\). This is shown in (45-b).

\[(45)\]

a. \[\text{each}_{x, y} := \lambda m \lambda f \lambda c. c(f \ m) \bar{\alpha} \ dm_{x, y} \ m \ \ m \to (m \to \alpha) \to ((\alpha \to t) \to t)\]
b. \[\text{each}_{x} := \lambda m \lambda f \lambda c. c(f \ m) \bar{\alpha} \ dm_{x} \ m \ \ m \to (m \to \alpha) \to ((\alpha \to t) \to t)\]

As already can be seen in (45-a) and (45-b), after turning a GQ (or predicate) into a higher-order GQ (or a higher-order predicate), binominal \textit{each} is capable of introducing a monotonic measurement constraint in a place different from where the original GQ (or predicate) takes scope. For
example, in (45-a), the ‘lower-order’ GQ $f \; m$ takes scope inside $c$, but the monotonic measurement constraint is introduced outside $c$.

To see a concrete example, after composing with all the ingredients inside an argumental cardinal measure phrase, a host with binominal each essentially denotes a higher-order dynamic GQ, as shown in (46).

(46) **two many** movies $\mu_{\text{card}} \; \text{each}_{x,y} = \lambda c. c \left( \lambda P. \exists y \; \text{movie} \; y \land \mu_{\text{card}} \; y = (2, \mu_{\text{card}}, y) \land P \; y \right) \wedge \text{dm}_{x,y}(\mu_{\text{card}})$

This higher-order dynamic GQ looks for a function from GQ to truth values, puts the GQ (i.e., *two movies*) back in the scope of this function and introduces a monotonic measurement constraint
two^\| movies each_{x,y}$

\[Q \rightarrow t\]

\[\Lambda u\]

\[t\]

\[\text{the boys}^x\]

\[Q \rightarrow t\]

\[e \rightarrow t\]

\[\text{dist}\]

\[\Lambda u\]

\[t\]

\[\text{saw} t_u t_w\]

\[\beta\]

\[\alpha\]

a. $\beta = \lambda Q. \max^x \left( \text{boys}_x \right) \land \delta_x \left( \text{atom}_x \land Q \left( \lambda u. \text{saw}_u x \right) \right)$

---

**Figure 3.10:** Scope taking of a higher order dynamic GQ

outside the scope of this function. Figure 3.10 shows the Logical Form of a sentence with a higher-order dynamic GQ. A higher-order GQ essentially has a ‘split scope’ mechanism that allows *two movies* to scope both inside and outside of distributivity. Scoping it inside distributivity gives us the correct narrow scope reading of *two movies* and scoping it outside of distributivity allows the monotonicity constraint to ‘associate’ with the internal structure of distributivity dependency.

Assuming the lexical entries in Table 3.3 for the definite NP *the boys*, the verb *saw* and the covert distributive operator, we obtain the final meaning of the LF, as shown in (47).

(47) $\text{two}^\| \text{movies each}_{x,y} \left( \lambda Q. \text{the boys}^x \land \delta_x \left( \text{atom}_x \land Q \left( \lambda u. \text{saw}_u x \right) \right) \right)$

---

\[16\] A trace in the form of $t_u$ or $t_w'$ is treated as a special variable, which is subject to the interpretation of a special $\zeta$-assignment and does not interact with assignments in an info-state. Abstraction is done with an abstraction operator, in the manner of Heim and Kratzer (1998), as shown in (i):

(i) $G[\Lambda u \beta]H := \lambda x. [\beta]^{s_{x \leftarrow x}}$
<table>
<thead>
<tr>
<th>Expression</th>
<th>Denotation</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>the boys</td>
<td>$\lambda P. \max^x (\text{boys } x) \land P(x)$</td>
<td>Q</td>
</tr>
<tr>
<td>saw</td>
<td>$\lambda u \lambda u'. \text{saw } u u'$</td>
<td>$e \to e \to t$</td>
</tr>
<tr>
<td>dist</td>
<td>$\lambda P \lambda u. \delta u (\text{atom } u \land P u)$</td>
<td>$(e \to t) \to (e \to t)$</td>
</tr>
</tbody>
</table>

Table 3.3: Definite NPs, verbs and the distributive operator

As shown in (47), the split scope mechanism allows two movies to scope inside the distributive operator but dm to scope outside the distributive operator. The ‘association-with-distributivity’ effect is clearly seen in the dm test in (47-b). The test bears an index $x$, which is the same index borne by the distributive operator, i.e., the variable that stores values based on which an info-state is split up into sub-states to check for distributivity.

To test for $dm$, we first assemble the distributivity update. Assuming a scenario in which three boys each saw two different movies, the output of the distributivity update can be visualized in Figure 3.11.

The monotonic measurement constraint, spelled out in (48), is evaluated against the output of the distributivity update. It first requires that the info-state be split up into sub-states each storing one or more values in the variable $x$. With three values in $x$, 7 such sub-states can be found (excluding the empty sub-state, which stores no value in $x$). Then, it compares these sub-states, requiring that if a sub-state whose $x$-value is a proper subset of the $x$-value of another sub-state, then measuring $y$’s cardinality in the former sub-state does not yield a bigger number than measuring $y$ in the latter sub-state.

(48) $G[dm_{x,y}(\mu_{\text{card}})]H = T$ iff

a. $H = G$ and

b. $\forall A, A' \subseteq G \times x. A \subseteq A' \to \mu_{\text{card}} \left( \bigoplus_{x \in A} G \right) \leq \mu_{\text{card}} \left( \bigoplus_{x \in A'} G \right)$
Figure 3.11: Distributivity update

c. \( \exists B, B' \subseteq G \ x. \ \mu_{\text{card}} \left( \bigoplus_{x \in B} y \right) \neq \mu_{\text{card}} \left( \bigoplus_{x \in B'} y \right) \)

For concreteness, let’s consider two info-states, shown in Figure 3.12, that verify \( \text{dm} \). In info-state \( G \), three boys each watched a different set of two movies. The cardinality of \( y \) (i.e., the movies) in each \( x \) sub-state is provided under the matrix. Since the cardinality of \( y \) never decreases in a bigger sub-state containing more \( x \)-values, non-decreasing mapping is satisfied. In addition, the cardinality of \( y \) is not constant in all the \( x \) sub-states, non-constant mapping is satisfied. As a result, \( \text{dm} \) is satisfied by Info-State \( G \). Another info-state that also verifies \( \text{dm} \) is Info-State \( G' \), which has two boys seeing two identical movies but a third boy seeing two different movies. Again, this info-state satisfies both non-decreasing mapping and non-constant mapping, hence also \( \text{dm} \).

Of course, not all distributivity updates satisfy \( \text{dm} \). If the values stored in \( y \) does not vary across the distributivity dependency, as in Info-State \( G'' \), in Figure 3.13, \( \text{dm} \) is violated. Recall that since \( \text{dm} \) is modeled as a constraint, the predicted judgment for the corresponding sentence containing
Figure 3.12: Info-states that verify the boys saw two movies each

a binominal each is infelicitous, or unacceptable, rather than false. This is how \( \text{dm} \) captures the variation inference triggered by binominal each.

When the measure phrase is a predicate, as in (49-a) and (49-b), the measure phrase does not introduce a discourse variable. \( \text{dm} \) is checked by just using a single discourse variable, i.e., the variable storing the values for the key (the relevant angles for (49-a) and the relevant coffees for (49-b)).

\[
\text{(49)} \quad \begin{align*}
\text{a.} & \quad \text{The angles are 60 degrees each.} \\
\text{b.} & \quad \text{*The coffees are 60 degrees each.}
\end{align*}
\]

The corresponding monotonic measurement constraints have a similar form, as shown in (50), differing only with respect to whether the values stored in \( x \) are angles or coffees, and whether the measure function measures angle degree or temperature.

\[
\text{(50)} \quad G \models \text{dm}_x(\mu_{\text{angle/temp}}) \models H = T \iff \\
\begin{align*}
\text{a.} & \quad H = G \\
\text{b.} & \quad \forall A, A' \subseteq G \times x. A \subseteq A' \rightarrow \mu_{\text{angle/temp}} \left( \bigoplus_{x \in A} G \right) \leq \mu_{\text{angle/temp}} \left( \bigoplus_{x \in A'} G \right)
\end{align*}
\]
Figure 3.13: An info-state that fails to verify the boys saw two movies each

c. \( \exists B, B' \subseteq G \ x. \, \mu_{angle/temp} \left( \bigoplus_{x \in B} G \right) \neq \mu_{angle/temp} \left( \bigoplus_{x \in B'} G \right) \)

As shown in the info-states in Figure 3.14, it is possible to satisfy \( \text{dm} \) if the measure function is extensive, as in the case of \( \mu_{angle} \) (Info-State \( H \)), but not if the measure function is intensive, as in the case of \( \mu_{temp} \) (Info-State \( H' \)).

### 3.4.4 Interim summary

I have demonstrated how to translate \( \text{dm} \) as an output constraint in DPILM, a dynamic plural logic enriched with domain pluralities and measure functions but otherwise faithful to van den Berg (1996) (with the exception of negation, see Section 2.2.3) of Chapter 2. The use of plural logic enables us to model distributivity-induced dependency as a discourse plurality, and marrying plural logic with a dynamic logic allows us to record this dependency and its internal structure. The anaphoric component on binominal each retrieves this dependency, and the monotonic measurement

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17When the measure function is intensive, there is no way to satisfy \( \text{dm} \). However, when the measure function is extensive, whether or not \( \text{dm} \) is satisfied is context-dependent, as it matters what values are associated with the variable being measured.
constraint makes crucial use of the internal structure of this dependency.

In the next few sections, I discuss two extensions of the current study. The first extension takes up the interaction between binominal *each* and negation, with the goal of showing that their interaction follows from the dynamic framework we are using. The second extension generalizes the monotonicity constraint to cover the event differentiation condition of adverbial and determiner *each*, pointed out in studies such as Vendler (1962), Beghelli and Stowell (1997), Tunstall (1998) and Brasoveanu and Dotlačil (2015).

Following the two extensions, I offer a comparison of the proposal developed in this study and proposals developed in previous studies.

### 3.5 Output constraints and negation

#### 3.5.1 More on output constraints

I have suggested to model the additional inference of binominal *each* as an output constraint. The truth conditions of the constraint are given in Figure 3.4 (assuming $\phi$ stands in for the truth value of

\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_1\}}) = \langle 60^\circ, \mu_{\text{angle}}, a_1 \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_1\}}) = \langle 60^\circ F, \mu_{\text{temp}}, c_1 \rangle\]
\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_2\}} y) = \langle 60^\circ, \mu_{\text{angle}}, a_2 \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_2\}} x) = \langle 60^\circ F, \mu_{\text{temp}}, c_2 \rangle\]
\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_3\}} y) = \langle 60^\circ, \mu_{\text{angle}}, a_3 \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_3\}} x) = \langle 60^\circ F, \mu_{\text{temp}}, c_3 \rangle\]
\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_1, a_2\}} y) = \langle 120^\circ, \mu_{\text{angle}}, a_{12} \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_1, c_2\}} x) = \langle 60^\circ F, \mu_{\text{temp}}, c_{12} \rangle\]
\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_1, a_3\}} y) = \langle 120^\circ, \mu_{\text{angle}}, a_{13} \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_1, c_3\}} x) = \langle 60^\circ F, \mu_{\text{temp}}, c_{13} \rangle\]
\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_2, a_3\}} y) = \langle 120^\circ, \mu_{\text{angle}}, a_{13} \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_2, c_3\}} x) = \langle 60^\circ F, \mu_{\text{temp}}, c_{23} \rangle\]
\[\mu_{\text{angle}}(\bigoplus H|_{x \in \{a_1, a_2, a_3\}} y) = \langle 180^\circ, \mu_{\text{angle}}, a_{123} \rangle\]
\[\mu_{\text{temp}}(\bigoplus H'|_{x \in \{c_1, c_2, c_3\}} x) = \langle 60^\circ F, \mu_{\text{temp}}, c_{123} \rangle\]
distributivity, $\psi$ stands in for the truth value of the monotonicity constraint, and $\overline{\lambda}$ stands in for the truth value for evaluating the constraint relative to distributivity.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\psi$</th>
<th>$\overline{\lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>#</td>
</tr>
<tr>
<td>0</td>
<td>#</td>
<td>#</td>
</tr>
</tbody>
</table>

Table 3.4: The truth condition of evaluating the monotonicity constraint against the output of distributivity

When distributivity is true, a sentence with binominal *each* is either true (if the monotonicity constraint is also true, as shown in Figure 3.15 (a)–(b)) or undefined (if the monotonicity constraint is not true, as shown in Figure 3.15 (c) – (d)).

![Figure 3.15](image)

When distributivity is false, the distributive update leads to an empty set of output info-states. Since $dm_{x,y}$ bears a pair of anaphoric indices, these indices triggers presupposition failure as they cannot receive a proper interpretation. As a result, the constraint is predicted to be undefined. Relatedly, the result of evaluating the constraint against distributivity is undefined. What this means is that when distributivity is false, a sentence with binominal *each* is judged unacceptable, rather than false. This,
however, is incompatible with native speakers’ judgement. As far as I know, the two scenarios in Figure 3.16, which do not verify the distributivity inference due to Carol not watching any movie, are judged false rather than undefined.

The discrepancy seems to pose a challenge for analyzing the monotonicity inference of binominal each as an output constraint.\textsuperscript{18} However, the discrepancy is not unique to the output constraints in dynamic semantics, it is also shared by standard pronominal anaphora in examples like (51).

(51) A man\textsuperscript{x} came in. He\textsubscript{x} sat down.

Suppose we are in a world in which no man came in. Evaluating the first sentence in (51) returns an empty set of info-states. In other words, there is no info-state in the output that has a variable \textsuperscript{x}. Since the second sentence has a pronoun, which presupposes the presence of the d-ref \textsubscript{x}, (51) is predicted to trigger a presupposition failure. However, the sentence is judged to be false, instead of undefined.

The situation becomes clearer when our dynamic semantics is enriched with intensions.\textsuperscript{19} In such a dynamic semantics, a proposition is a relation between world-info-state pairs, as shown in Figure 3.17. As one can see, interpreting (51) against \langle w_1, \emptyset \rangle leads to a non-empty set of outputs while interpreting it against \langle w_2, \emptyset \rangle and \langle w_3, \emptyset \rangle both lead to an empty set of outputs. Intuitively, (51) is true in \textit{w_1} but false in both \textit{w_2} and \textit{w_3}. However, if we follow the standard assumption that the pronoun presupposes a d-ref, we predict that (51) is false in \textit{w_2} but undefined in \textit{w_3}.

Since the discrepancy is a more general phenomenon in dynamic semantics, I do not take it to pose

\textsuperscript{18}The discrepancy also has an effect on the interaction between negation and an output constraint, as discussed in the next subsection.

\textsuperscript{19}I thank Simon Charlow for helpful discussions on intensional dynamic semantics.
a real challenge to formulating the monotonicity requirement as an output constraint.

### 3.5.2 Negation

Negation in dynamic semantics has interesting properties. In DRT/FCS/DPL, negation is a static closure operator, which does not allow variables introduced in its scope to support anaphors outside its scope (Heim 1982, Groenendijk and Stokhof 1991, Kamp and Reyle 1993). Translating its definition into DPlLM gives rise to (52).

(52)  **Static negation**

\[ G \not\models \phi \models H = \top \text{ iff } G = H \land \neg \exists K : G \not\models \phi \models K \]

Importantly, if an indefinite occurs in the scope of negation, its dynamic effect is not accessible outside the scope of negation.\(^{20}\) For this reason, cross-sentential anaphora is predicted to be impossible, as shown in (53).

(53)  a. John does not own a car. #It’s red.

    b. Nobody talked to a man. #He left.

Given the well-documented behavior of negation in dynamic semantics, if binominal *each* indeed makes reference to (the structure of) a distributivity dependency via dynamic binding, as proposed in this study and in recent studies such as Champollion (2015) and Kuhn (2017), one wonders if it

\(^{20}\)When an indefinite is interpreted outside the scope of negation, its dynamic effect is not affected. Certain indefinites are known to resist being interpreted inside the scope of negation, such as *Some NPs* (Baker 1970, Szabolcsi 2004) and *a certain NPs* (Kratzer 1998).
interacts with negation as predicted by dynamic semantics and the static treatment of negation. This turns out to be a slightly more involved question, given the split scope behavior of a noun phrase hosting binominal *each* and the fact that the monotonic measurement constraint is defined as an output constraint. However, after unpacking the complexities, I demonstrate that binominal *each* indeed interacts with negation as predicted by the definition of negation as a static closure operator.

In a simple sentence with binominal *each* and negation like (54), there are four scope bearing elements: negation, the distributive operator, the counting quantifier (in the form of a trace), and the monotonicity constraint contributed by binominal *each*.

(54) The girls didn’t see one movie each.

In principle, there are 24 (i.e., the factorial of 4) scope permutations obtainable from rearranging these four elements. Since the goal here is to understand how the monotonicity constraint interacts with negation, I limit the discussion to a few scope configurations in which the monotonicity constraint is not independently ruled out by the scope configuration of the the distributive operator and the counting quantifier. In other words, I assume that the relative scope of the distributive operator and the counting quantifier is fixed, with the former always scoping over the latter. This helps bring down the possible scope permutations from 24 to 12.\(^{21}\) In addition, since the monotonicity constraint can only be computed when it is outside the scope of the distributive operator, we can further shrink the possible scope relations from 12 to four.\(^{22}\) The four surviving scope possibilities are represented in Figure 3.18.

We start with scope position A, i.e., when negation takes the narrowest scope. The corresponding translation, given in (55), shows that both \(x\) and \(y\) are introduced outside the scope of negation. As a result, negation has no effect on the dynamic effects of these variables.

---

\(^{21}\) Among 24 scope permutations, half of them have the counting quantifier scoping over the distributive operator and hence can be removed.

\(^{22}\) This is harder to see than the previous computation. Among the 12 possibilities, six of them have the distributive operator taking widest scope followed by random rearrangement of the rest. All of the six possibilities are ruled out. Three of them have the monotonicity constraint taking widest scope and all are reserved for further consideration. The remaining three have negation taking widest scope, following by the monotonicity constraint inside the scope of the distributive operator (2 cases) or the monotonicity constraint outside the distributive operator (1 case). Only the single case involving the monotonicity constraint outside the scope of the distributive operator is selected for further consideration. Summing up the three cases of ‘widest-monotonicity’ (see positions A, B, and C in Figure 3.18) and the one case of ‘widest-negation’ (see position D in Figure 3.18) gives us four scope possibilities.
Figure 3.18: Four scopal possibilities

Note: \( u, u' \) and \( Q \) appear unbound because their binders are elided from the tree to reduce its size.

\[
\text{(55) A. } \begin{align*}
\max_x & \left( \text{girl } x \right) \wedge \delta_x \left( \exists y \wedge \text{movie } y \wedge \\ & \mu_{\text{card}} y = 1 \wedge \neg \text{saw } y x \right) \wedge \text{dm}_{x,y}(\mu_{\text{card}}) \\
\end{align*}
\]

(ii) Distributivity: For each girl, there is a movie that she failed to see.

(iii) Monotonicity: The measurement of movies is positively correlated with the number of girls.

It is predicted that the monotonicity constraint is imposed on the output of distributivity, requiring that the number of movies the students failed to see be positively correlated with the number of students. The prediction is borne out: In a scenario in which every girl failed to see a different movie, as in (56), (54) is judged acceptable and true. However, in a scenario in which the girls saw all of the movies (out of a salient set) except for one, the sentence is judged unacceptable, as shown in (57).

(56) In a scenario in which Anna failed to see Titanic, Beth failed to see Inception, and Carol failed to see Aliens:
    The girls didn’t see one movie each.

(57) In a scenario in which Anna, Beth, and Carol saw all the movies (assigned by their teacher) except for Titanic:
    *The girls didn’t see one movie each.
Next, consider the scope positions B and C. According to the definition of static negation, there is no well-formed interpretation if negation takes scope anywhere between the higher-order dynamic GQ and the GQ trace. In other words, B and C are not possible scope positions for negation. In position B, the dynamic effect stemming from the GQ trace (more precisely, the reconstructed GQ to the trace position) is blocked outside the scope of negation, as shown in (58); in position C, the dynamic effect stemming from both the reconstructed GQ trace and the Key is blocked, as shown in (59).

\[(58) \text{ B. } \max^x (\text{girl } x) \land \delta_x \left( \neg \exists y \land \text{movie } y \land \mu_{\text{card}} y = 1 \land \text{saw } y \land x \right) \land \text{dm}_{x,y}(\mu_{\text{card}}) \]

(i) Distributivity: For every girl, there is no movie that she saw.

(ii) Monotonicity: There is a positive correlation between the the measurement of movies and the number of girls.

\[(59) \text{ C. } \neg \left( \max^x (\text{girl } x) \land \delta_x \left( \exists y \land \text{movie } y \land \mu_{\text{card}} y = 1 \land \text{saw } y \land x \right) \right) \land \text{dm}_{x,y}(\mu_{\text{card}}) \]

(i) Distributivity: Not every girl saw a movie.

(ii) The measurement of movies is positively correlated with the number of girls.

How do we know that (58) and (59) are indeed out? Ideally, we can find all the scenarios in which every girl saw no movie (position B) and not every girl saw a movie (position C), and see if they are unacceptable. However, this is not feasible because of the availability of position D, which gives rise to a reading that is weaker than the readings associated with positions B and C. Therefore, our best evidence that (58) and (59) are indeed unavailable comes from the fact that in situations where no boy read any book (B), or not every boy read a book (C), there is no pressure for the monotonicity constraint to hold. In other words, we simply judge the sentence to be true in these situations regardless of the status of the monotonicity constraint. This is suggestive that the monotonicity constraint cannot be evaluated in these scope configurations.

Lastly, consider position D, when negation scopes above both distributivity and the output constraint. The associated translation is given below:


(60) D. (i) \( \neg \left( \max^x (\text{girl } x) \land \delta_x \left( \begin{array}{c} \text{atom } x \land [y] \land \text{movie } y \land \\ \mu_{\text{card}} y = 1 \land \text{saw } y \ x \end{array} \right) \right) \overline{\text{dm}}_{x,y}(\mu_{\text{card}}) \)

(ii) It’s not true (that every boy saw two movies and by the way, the measurement of movies is positively correlated with the number of boys).

This scope configuration is particularly interesting because it informs us how the monotonicity constraint behaves inside the scope of negation. Because the constraint is formulated as a defined-ness condition on the output of distributivity, we expect it to be calculable as long as negation scopes above both distributivity and the constraint. The truth conditions of (60) is given in Table 3.5, in which \( \delta \) represents the asserted content, i.e., the distributivity update, \( \text{dm} \) represents the output constraint, i.e., \( \overline{\text{dm}} \), \( \neg \) represents the outcome of imposing \( \text{dm} \) on \( \delta \), and \( \neg \) represents the predicted outcome of negation scoping over the complex meaning:

<table>
<thead>
<tr>
<th>( \delta )</th>
<th>( \text{dm} )</th>
<th>( \overline{\text{dm}} )</th>
<th>( \neg )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>#</td>
<td>#</td>
</tr>
<tr>
<td>0</td>
<td>#</td>
<td>#</td>
<td>#</td>
</tr>
</tbody>
</table>

Table 3.5: Negation over complex meaning

Let us start with the first row. When both the distributivity evaluation and the monotonic measurement constraint evaluation are ‘true’, negation is evaluated to ‘false’, in accordance with native speakers’ intuition that the sentence in (61) is simply a false statement.

(61) **In a scenario in which every girl watched one movie and they did not all watch the same movie:**

The girls didn’t watch one movie each.

When distributivity is evaluated to ‘true’ but the monotonic measurement constraint is evaluated to ‘false’, negation is evaluated to ‘undefined’. The cases in (62) and (63) support this prediction.

(62) **In a scenario in which all the girls watched one and the same movie:**
#The girls didn’t watch one movie each.

(63)  \textit{In a scenario in which all the cocktails are exactly 60 degrees Fahrenheit.}

#The cocktails aren’t 60 degrees (Fahrenheit) each.

When distributivity is evaluated to ‘false’, the output is an empty set, so the monotonic measurement constraint cannot be tested. As a result, \(\neg\) is evaluated to ‘undefined’. Applying negation to an undefined output is also predicted to be ‘undefined’. Again, this is where native speakers’ intuition differs from the prediction. According to native speakers’ judgment, if distributivity is evaluated to ‘false’, the negated sentence is evaluated to ‘true’, rather than ‘undefined’. The following example demonstrates the judgment.

(64)  \textit{In a scenario in which not every girl watched a movie:}

The girls didn’t watch one movie each.

This is the same discrepancy we have seen in the previous section: when distributivity is evaluated to ‘false’, the output constraint is ignored. For this reason, negation acts as if the constraint is absent. It reverses the truth value (i.e., ‘false’) contributed by the distributivity update and yields a ‘true’ statement.

3.6 Comparisons with and connections to previous studies

3.6.1 Studies in the PCDRT framework

Henderson (2014) investigates distributive numerals in Kaqchikel and conclude that they should be analyzed as imposing a post-suppositional plurality condition (known as evaluation-level plurality) on the functional dependency arising from distributive quantification, which he modeled using info-states from PCDRT. Champollion (2015) later extends Henderson’s analysis to binominal each. Kuhn (2017) modifies Henderson’s analysis, replacing post-suppositions with a scope-taking mechanism, and allowing distributive numerals (noun phrases with binominal each included) to induce distributivity. However, the core of Henderson’s analysis, namely, that distributive numerals contribute an evaluation-level plurality condition, is shared in both Champollion (2015) and Kuhn (2017).
Since the evaluation-level plurality condition is a major point of departure between the present study and previous studies in the PCDRT tradition, let me introduce it with a concrete sentence like (65). This sentence can be translated into DPILM as in (66). The only bit that cannot be interpreted in DPILM is the evaluation-level plurality condition in the last conjunct. Let me define it in 28, using Henderson’s definition.

(65) The students hugged one dog each.

(66) \( \max_x (\text{student } x) \land \delta_x (\exists y \land \mu_{\text{card}} y = 1 \land \text{dog } y \land \text{hug } y x) \land y > 1 \)

Definition 28 (Evaluation plurality (cf. Definition 25 in Chapter 2))

\( G\| y > 1 \| H = T \iff G = H \text{ and } |\{h y \mid h \in H\}| > 1 \)

\( h \) in Definition 28 is a single assignment, so \( h y \) yields a single value (which can be in the form of a plural individual). Since an info-state has a set of \( h \)-assignments \((h_1, h_2, \ldots, h_n)\), we can collect a set of \( h y \) values \( \{h_1 y, h_2 y, \ldots, h_n y\} \). An evaluational-level cardinality can be computed based on how many members are in this set. If all the assignments assign to \( y \) the same value, then there is only one member in the set. Such a set does not satisfy evaluation-level plurality. However, if at least two assignments assign different values to \( y \), evaluation-level plurality is satisfied.

Since the evaluation-level plurality condition is evaluated after the distributivity quantification, each \( h \) that associates \( y \) with a value also associates \( x \) with a value, from the Key. For this reason, requiring \( y \) to exhibit evaluation-level plurality following a distributive quantification has the same effect as requiring \( y \) to depend on \( x \) at the value level. To see this, consider the definition of value dependence, repeated below from Definition 8 of Chapter 2.

Definition 8 (Value dependence)

\( y \) is value-dependent on \( x \) in an information state \( G \) iff there are \( a, b \in G x. G|_{x=a} y \neq G|_{x=b} y \)

If we set the variable \( x \) to store all the relevant values in the Key and set the variable \( y \) to stores the values introduced by a distributive numeral. The requirement that \( y \) is associated with different values for at least two distinct values in \( x \) is the same as saying that \( y \) is associated with at least two values at the evaluation level. So, we can safely conclude that evaluation-level plurality and value...
dependence are the same requirements. Evaluation-level plurality is notationally more economical as it only uses one variable, i.e., the variable introduced by a distributive numeral. The variable storing values contributed by the Key is not used explicitly. However, it is used implicitly. This is because distributive quantification delivers a set of assignments each of which stores a different value from the Key. For this reason, saying that \( y \) depends on assignments in the output of a distributive evaluation is essentially the same as saying that \( y \) depends on the Key.

Value dependence is useful for modeling the variation requirement, but does not capture the measurement-sensitivity of binominal each, which subsumes the counting quantifier requirement and the extensive measurement requirement. This is so because value dependence cannot handle requirements that track the internal structure of a functional dependency, such as dependence between individuals and measurements of individuals, i.e., degrees, as discussed in the previous chapter. By contrast, since the monotonic measurement constraint requires structural dependence, it captures both the variation requirement and the measurement sensitivity. As has been established in Section 3.2, binominal each makes crucial reference to measurement. For this reason, value dependence via evaluation-level cardinality cannot be extended to handle the measurement-sensitive nature of binominal each.

In addition to the primary difference between value dependence and structural dependence, there are a few less pronounced differences between the present study and its predecessors. First, there is a difference in the kind of meaning status given to the evaluation-level plurality requirement and the monotonic measurement requirement. In Henderson (2014) and Champollion (2015), evaluation-level plurality is analyzed as a delayed at-issue test. However, in both Kuhn (2017) and the present study, the corresponding component is analyzed as a not-at-issue meaning. The motivation for modeling it as a not-at-issue meaning is empirically driven—failure to satisfy the variation component leads to unacceptability rather than falsity. Although Kuhn (2017) calls his evaluation-level plurality constraint a ‘presupposition’ while I call the monotonicity constraint here an ‘output constraint’, the two essentially amount to the same thing. Kuhn (2017) calls the constraint a presupposition because it is placed on the ‘input’ to the constraint formula, which is precisely the output of distributive quantification. I call the constraint an output constraint because I intend for it to constrain the output of distributive quantification.
In addition, this study has adopted a higher order meaning approach to model delayed evaluation, following Charlow (to appear). Henderson (2014) follows Brasoveanu (2013) and uses a post-supposition instead and the assumption carries over to Champollion (2015). Lastly, Kuhn (2017) uses ordinary scope-taking without higher order meaning to model delayed evaluation. The merits and shortcomings of these strategies are discussed in Section 3.4 (see also Charlow, to appear).

A third difference lies in the dynamic logic in which the constraint giving rise to the variation requirement is couched. Studies in the PCDRT tradition make use of PCDRT, a dynamic plural logic with domain pluralities, dependence-introducing variable introduction, and distributive evaluation of lexical relations. The present study is couched in DPILM, a dynamic plural logic with domain pluralities and a collective evaluation of lexical relations, but crucially no dependency-introducing variable introduction. The two logics make distinct predictions regarding whether dependencies could in principle occur without distributivity. In particular, PCDRT allows it while the present framework does not. As a consequence, PCDRT-theoretic studies predict that the variation component may exist independently of distributivity, while the present framework predicts a close connection between distributivity and the variation requirement.

I must add that the monotonic measurement constraint, the constraint designed to replace evaluation-level plurality, can be reformulated with minor changes to adapt to a PCDRT-style dynamic plural logic just as easily. Ultimately, whether a PCDRT-style logic or a DPILM-style logic should be chosen to couch the monotonicity constraint should be based on empirical considerations. In particular, if the dependency introduced by random assignment turns out to be very useful, as argued in Brasoveanu (2007, 2008), then a PCDRT-style logic should be favored. However, there is at least some initial evidence, reported in Champollion et al. (forthcoming), that the original empirical motivations for the dependency-introducing random assignment considered in Brasoveanu (2007, 2008) may be accounted for without the machinery used in PCDRT.

3.6.2 Studies in static semantics

There is a vast literature on binominal each couched in static semantics. It is beyond the scope of the present chapter to offer a comprehensive review of previous studies on this topic. However, it is worth pointing out the major developments that have paved way for the ideas used in the present chapter.
An early study on binominal *each* is Link (1987). He set the stage for treating binominal *each* as a distributive operator, which is adopted in many subsequent studies, including Zimmermann (2002), Dotlačil (2012), Champollion (2010, 2017). However, these studies place their primary focus on the distributivity component and do not really recognize the variation component. As such, they differ quite drastically from the present chapter, which takes the variation component as its primary concern.

There are a few studies that take up the variation component. For example, Safir and Stowell (1988) recognize a strong form of the variation inference, and conceive binominal *each* as a one-to-one distribution function, establishing a one-to-one correspondence between elements in the Key to elements in the distributivity share. This strong form of variation is later criticized by Moltmann (1991) and Zimmermann (2002). Cable (2014) extends the semantics established for distributive numerals in Tlingit to binominal *each*, arguing that *each* is both a distributive marker and bears a variation inference. Despite recognizing the variation component, these studies either fail to account for the counting quantifier requirement and/or the extensive measurement requirement.

Despite these differences, studies in the static tradition have offered great insights to the study of binominal *each* in the present work. For one thing, it has been a longstanding puzzle how binominal *each* access the Key. The received wisdom is that there are null pronouns in the NP that hosts binominal *each* that help connect it with the Key, as suggested in Safir and Stowell (1988). This idea is further refined in Zimmermann (2002), with the pronoun treated as an anaphoric index directly borne by binominal *each*. The strategy is then imported into a dynamic framework by Dotlačil (2012) and adopted in Kuhn (2017) and the present work.\(^{23}\)

Many studies also share the intuition that *each* is a marker of quantificational dependence or subordination. Choe (1987a) and Milačić et al. (2015) are notable examples. This intuition is also relevant in the present study, albeit in a slightly different manner. In previous studies, the core contribution of binominal *each* is to signal quantificational dependence. However, in the present study, the core contribution is a variation component formalized in terms of a monotonic measurement constraint. A separate constraint is needed because quantificational dependence is a *necessary but not sufficient* condition for using binominal *each*.

\(^{23}\)The anaphoric index provided by the Key is not used in Henderson 2014, as his formulation of the evaluation-level plurality condition does not need direct reference to the Key.
As a final note, I would like to relate the present study to the idea of ‘structure-preserving binding’, developed in Jackendoff (1996) to deal with a host of phenomena ranging from telicity to quantification. Jackendoff suggests to broaden the notion of binding from a relation between two identical variables to a relation between two variables that are linked in some way. Most importantly, he argues that it is fruitful to study the links in terms of structure-preserving maps. He implements structure-preserving binding in the framework of Conceptual Semantics, which differs from the framework used in this work substantially. However, the core of the idea of structure-preserving binding resonates with the notion of the monotonicity constraints developed here.

3.6.3 Other ways to model functional dependencies

Besides using sets of assignments, a few other approaches have been developed to model functional dependencies arising from distributive quantification. I discuss two options in this section.

The first approach is to use **Skolemized choice functions**. Some notable studies arguing for a Skolem function treatment of universal quantification are Huang (1996) and Solomon (2011). According to these studies, universal quantification (closely related to distributive quantification) has the effect of functionalizing an expression in its scope. In Solomon (2011), the expression is an indefinite, while in Huang (1996), it may be an indefinite or an event. Abstracting away from the compositional details, what these scholars suggest is essentially representing a sentence like (67-a) as (67-b), where \( f \) is a Skolem function mapping each girl to a book she read.

\[
\begin{align*}
(67) & \quad \text{a. The girls each read a book.} \\
& \quad \text{b. } \exists f \forall x. x \leq \text{the.girls} & \text{atom}(x) \rightarrow \text{book}(fx) & \text{read}(fx)(x)
\end{align*}
\]

The monotonicity constraint imposed by binominal *each* can be defined using a Skolem function:

\[
(68) \quad \text{dm}(f) := \forall A, A' \subseteq \text{Dom}(f). A \subseteq A' \rightarrow \mu \bigoplus \{fx \mid x \in A\} \leq \mu \bigoplus \{fx' \mid x' \in A'\}
\]

The constraint in (68) can be directly conjoined with the contribution of distributive quantification in (67-b). \( f \) will be correctly identified to be the Skolem function contributed by the distributive quantification.
Despite its initial success, however, this approach faces some challenges. First, when the indefinite in the share happens to be a downward monotone quantifier, \( f \) may happen to be a defective function. For example, (69-a) is grammatical and compatible with a situation in which some of the girls did not read any book. Its corresponding interpretation involving a Skolem function, as formulated in (69-b), is predicted to be defective in a situation with some girls not reading any book. This is because the Skolem function \( f \) must choose some book from the set, predicting that every girl must read some book. In addition, \( f \) is not appropriately maximized, so (69-b) is true even in a situation in which the girls each read more than one book, as one can always find a (less informative) function that assigns no more than one book to each girl.

\[
(69) \begin{align*}
a. & \quad \text{The girls read no more than one book each.} \\
 b. & \quad \exists f \forall x. x \leq \mathbb{G}_{\text{girl}} \& \text{atom}(x) \rightarrow \text{book}(f(x)) \& |(f(x))| \leq 1 \& \text{read}(f(x))(x)
\end{align*}
\]

The two problems identified above can be resolved by letting the Skolem function take narrow scope relative to distributivity and a negation operator contributed by the downward monotone quantifier.

\[
(70) \quad \forall x. x \leq \mathbb{G}_{\text{girl}} \& \text{atom}(x) \rightarrow \neg \exists f. \text{book}(f(x)) \& |(f(x))| > 1 \& \text{read}(f(x))(x)
\]

However, doing so renders it impossible for the monotonicity constraint to access \( f \) in (70), as it is embedded under negation. For the constraint to access \( f \), it has to occur inside the scope of negation. However, the resulting interpretation would become: for every girl, there is no function that (i) maps the girl to more than a book she read and (ii) overall the girls read a variety of books. This is true when each girl read the same book, obviously not what (69-a) means.

Another problem with the Skolem function approach concerns the difficulty in assembling a Skolem function compositionally. Huang (1996) has to make special assumptions to fix the domain and range of the function as they are being pulled from distant parts of a sentence—the domain is pulled from the Key, and the range from the indefinite (or whatever expression that functionally depends on the Key). Solomon (2011) offers a more compositional account, at the cost of upgrading the semantics of distributive quantifiers.

\[\text{24 For concreteness, Huang (1996) has to rely on two assumptions to set the domain and range of the function: (i) an indefinite in the distributed share contributes only a variable and not existential quantification, à la DRT, and (ii) every/each binds this variable.}\]
If Skolem functions coming from distributive quantification is not ideal, one may wonder if attributing the source of Skolem functions to indefinites works any better. After all, choice functions have been widely used to analyze indefinites taking exceptional scope (Hintikka 1973, Kratzer 1998, Matthewson 1999, Winter 2002, Schwarz 2001). Their Skolemized versions, known sometimes as Skolemized choice functions, can model indefinites interpreted in the scope of another quantifier, either because they contain a pronoun bound by the the latter or because they are assumed to be functionally dependent on the latter. Relatedly, Milačić et al. (2015) have argued that Skolem functions be used for modeling the semantics of distributive numerals.

However, this proposal does not line up with empirical facts. Indefinites that have been widely accepted as having a functional interpretation, such as a certain NP, do not support binominal each, as shown in (71). By contrast, noun phrases that do not have exceptional scope properties and hence are not typically analyzed as functional indefinites, such as counting quantifiers with modified numerals (cf. Abels and Martí 2010), as shown in (72), do support binominal each.

(71) *The girls read a certain book each.

(72) The girls read at least/most two books each.

Therefore, we can conclude that Skolem functions contributed by indefinites are neither sufficient (71) nor necessary (72) for supporting binominal each.

Event Semantics is another tool that has been used to model the functional dependency of distributivity (Schein 1993, Lasersohn 1995, Landman 2000, Ferreira 2005, and Champollion 2010, 2017). One of the reasons why Event Semantics is chosen for this job is because events have (mereological) parts, which can, in principle, be used to model the structure needed for building the functional dependency of distributivity. As an illustration, I use Schein's semantics for distributive quantifiers shown in (73).

(73) Schein’s semantics for a distributive quantifier

\[
\text{every girl} := \lambda P \lambda e. \forall x \left[ \text{girl}(x) \right] \left( \exists e' \left[ e' \leq e \right] \left( P(x)(e') \right) \right)
\]

When this quantifier is used in a sentence like (74-a), it gives rise to the interpretation in (74-b).
(74) a. Every girl left.
   b. $\exists e (\forall x [\text{girl}(x)](\exists e'[e' \leq e](\text{leave}(x)(e'))))$

Here, $e$ is a sum event with parts. At least some of these parts are events in which a girl left.\(^{25}\) For each girl, her leaving event takes up a part of $e$. Because of the availability of the sum event $e$, ways can be defined to extract the girl-event dependency in (74-b), typically by resorting to the event’s agent thematic role.\(^{26}\)

I do not have a strong objection to using Event Semantics for modeling distributivity. As long as one can successfully retrieve the relevant participants that form a functional dependency from events, the main ideas developed in this dissertation can be largely transported to an event-based framework.

### 3.7 Conclusion

In this work, I have borrowed the insight from previous studies that distributivity makes available a set of functional dependencies with a nontrivial internal structure (Schein 1993, Lasersohn 1995, Krifka 1996b, van den Berg 1996, Landman (2000), Nouwen 2003, Brasoveanu 2008, Champollion 2010, 2017). Following many recent studies, this dependency is modeled with help of a dynamic plural logic. The particular version used in chapter is a hybrid of van den Berg (1996)’s DPlL and Brasoveanu (2008)’s PCDRT. More details about this logic are given in the previous chapter.

In addition, I have argued that binominal *each* piggybacks on this dependency, and introduces a monotonicity constraint requiring that the measurement of the values associated with its host tracks the part-whole relation of the dependency. I have demonstrated how the monotonicity constraint shed light on three generalizations on binominal *each*: the variation requirement, the counting quantifier requirement and the extensive measurement requirement.

Moreover, I have also shown that the monotonicity constraint can be generalized to account for

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\(^{25}\)Schein’s semantics is too weak, as pointed out in Ferreira (2005) and Champollion (2010). This is because there is no restriction placed on $e$ other than that it consists of a set of subevents each of which is a girl-leaving event. Besides these subevents, it may contain many other events that are not relevant. Both Ferreira (2005) and Champollion (2010) have provided ways to improve Schein’s event-based distributivity.

\(^{26}\)The thematic roles of $e'$ and $e$ in (74-b) are not explicitly represented. In neo-Davidsonian Event Semantics, thematic roles are argued to be explicitly represented (Parsons 1990, Landman 2000) and introduced by separate thematic heads in syntax (Kratzer 1996, Pylkkänen 2008, Champollion 2017).
the event differentiation condition associated with adverbial and determiner *each*. The generalization helps us see that even ordinary distributive markers, such as determiner and adverbial *each*, may encode constraints on distributivity.

Lastly, I have demonstrated that a dynamic treatment of binominal *each* makes correct predictions about its interactions with negation, justifying the use of dynamic semantics in this area of research.
4

CANTONESE saai: AN INDEPENDENCE CONSTRAINT

4.1 Introduction

As we have seen, previous studies have identified distributive markers that require obligatory co-variation of expressions in the distributivity share relative to the Key. Well known examples of this type of distributive markers include: markers of distributive numerals/indefinites, which are found in Georgian, Hungarian, Telegu, Kaqchikel, ASL, and many other languages (Farkas 1997, Balusu 2005, Henderson 2014, Kuhn 2017, a.o.), English binominal each (Choe 1987b, Safir and Stowell 1988, Champollion 2015, see also Chapter 3 of this dissertation), and Mandarin ge (Lin 2004, Lee et al. 2009a, Li and Law 2016, see also Chapter 5 of this dissertation).

One may wonder if there is any distributive marker that does just the opposite, namely, requiring expressions in the distributivity share to not co-vary with the Key. In this study, I show that such a distributive marker can be found in Cantonese.

The distributive marker that exhibits this property is the post-verbal distributivity suffix saai, which has been previously taken up in Lee (1994), Tang (1996) and Lee (2012). These studies have already observed that saai strongly resists indefinites showing up in the object position of a transitive verb suffixed by saai. Lee (1994) and Tang (1996) even suggest building in a definiteness or specificity requirement in the semantics of saai to explain this incompatibility.
Against this background, the contribution of this study is three-fold.

- At the empirical front, I show that *saai*'s resistance to indefinites is interpretation-based—as long as an indefinite does not co-vary with the Key, the incompatibility goes away. The resistance against co-variation is shown to generalize to disjunction (section 3.2), as well as to measure phrases (section 4.5).

- At the theoretical front, I argue that definiteness and specificity are not adequate notions for accounting for the property of *saai*-distributivity (section 4.2.2). Instead, I propose that the ban on co-variation can be understood in terms of independence. I explore two analyses, one relying on scope (section 4.3) and the other relying on an independence constraint (section 4.4), to model independence in *saai*-distributivity. The independence constraint is further argued to hold not at the value level, i.e., requiring a lack of co-variation with the Key, but at the structure level, i.e., requiring a lack of co-variation with the internal mereological structure of the Key (section 4.5).

- Finally, by identifying *saai* as a distributive marker indicating independence, I enrich the typology of distributive markers: there are distributive markers signaling (structural) dependence (as in the case of English *each* and Mandarin *ge*) as well as distributive markers signaling (structural) independence.

### 4.2 The distribution of *saai*

#### 4.2.1 Establishing *saai* as a distributive marker

*Saai* is a verbal suffix indicating distributivity, according to Tang (1996), Lee (2012), and Lei (2017). These authors have also provided a variety of evidence to support *saai*'s status as a distributive marker, including *saai*'s requirement for a plural Key, as well as its interactions with collective predicates and mixed predicates. I briefly review these pieces of evidence below to show that *saai* has typical properties of a distributive marker.

According to Tang (1996), *saai* requires a Key that exhibits ‘divisibility’, in the sense that the Key must be divisible into a plurality of proper parts. This is a common characteristic of distributive markers and can be seen as a ban against distributive quantification operating on a singleton domain.
A immediate qualification is that the Key can be in the singular form, as shown in (2). What is important is that it be divisible into proper subparts to feed distributive quantification. This property is shared by a more widely studied distributive marker *dou in Mandarin and has been formalized in terms of a cover-based semantics for distributivity in Lin (1998a) (see also Schwarzschild 1996).

Tang (1996) further suggests the contrast in (3) and (4) to establish *saai’s role as a distributive marker. According to Tang (1996), *git-zo fan ‘got married’ in (3) is ambiguous between a collective interpretation (3-a) and a distributive interpretation (3-b). However, suffixing the verb with *saai instead of *zo makes the collective interpretation unavailable, as demonstrated in (4).

A concern with using the contrast in (3) and (4) to diagnose distributivity is that the size of the plural entity that serves as the Key makes a difference for the judgment. If the plural pronoun in (4) refers to a bigger group of individuals, then the ‘collective’ interpretation is more easily acceptable. This is reminiscent of *all in English, which is compatible with a type of collective predicates called *gather-type predicates, as shown in (5).
(5) All the students gathered.

However, Champollion (2017) argues that there are two types of collective predicates: those that allow distributivity down to non-atomic units and those that resist distributivity altogether (see also Dowty 1987, Kuhn 2014). The first type of predicate is exemplified by *gather, fit together,* and *hold hands,* while the second type of predicate is exemplified by *be numerous,* and *be a large group.* Although *all* is compatible with gather-type predicates, it is incompatible with numerous-type predicates, suggesting that it is indeed incompatible with a genuine collective predicate.

(6) *All the students are numerous.*

*Saai* in Cantonese patterns like *all* in this respect. It is compatible with a *gather*-type predicate (7) and incompatible with incompatible with a *numerous*-type predicate (8).1

(7) Keoi dei zeoi-saai hai munhau.
   they gather-SAAI at entrance
   ‘The gathered at the entrance.’

(8) Keoi dei jansou do-zol/*saai.
   they number large-ASP/SAAI
   ‘They became numerous.’

Another test that can be used to establish *saai*’s role as signaling distributivity involves the use of so-called ‘mixed’ predicates, i.e., predicates that are ambiguous between a distributive and a collective interpretation. The predicate *toi jat-bou gongkam* ‘lift a piano’ in (9) is an example of a ‘mixed’ predicate. The two interpretations are included immediately below the sentence.

(9) Di-hoksaang toi-zo jat-bou gongkam.
   CL.PL-student lift-ASP one-CL piano
   a. Collective: The students lifted a piano together.
   b. Distributive: The student each lifted a piano.

1Mandarin *dou,* a marker that has been argued to be a distributive marker, is also compatible with *gather*-type predicates (Lin 1998a).
Replacing the aspectual suffix zo with saai brings about two changes. First, it makes the collective interpretation unavailable, leaving the distributive interpretation the only viable interpretation:

(10) %Di-hoksaang toi-saai jat-bou gongkam.

   CL.PL-student lift-ASP one-CL piano

   a. *Collective: The students lifted a piano together.

   b. Distributive: The student each lifted a piano.

Second, the sentence itself is slightly degraded for some speakers, even for the distributive interpretation. Lee (1994), Tang (1996), Lee (2012) and Lei (2017) attribute the degradedness to the requirement that a noun phrase following saai has to receive a specific or definite interpretation. The specificity (or definiteness) requirement, which is at the heart of this study, is discussed in more detail in the next subsection.

In summary, with help from numerous-type predicates and mixed predicates, I have shown that Cantonese saai signals distributivity. Let me immediately clarify that signaling distributivity does not equal contributing a distributive operator. Saai may merely indicate the presence of a distributive operator, as has been suggested for the case of binominal each.

4.2.2 Interactions with indefinites and disjunction

Saai is selective about what expression may follow it and the range of interpretations a post-saai expression may take. This subsection documents these selectional requirements. It incorporates observations noted in studies such as Lee (1994), Tang (1996), Lee et al. (2009a) and Lei (2017), as well as observations stemming from my own research.

It is widely noted that saai favors definite expressions as post-saai objects over indefinite expressions (e.g., Lee 1994, Tang 1996, Lee 2012). Cantonese has a few ways to form definite expressions. For example, they can be formed with a demonstrative determiner followed by a classifier and a common noun, as shown in (11), as well as with a bare classifier (i.e., without a numeral) followed by a common noun, as shown in (12) (Au Yeung 1998, Cheng and Sybesma 1999, Jiang 2012). Both expressions can follow saai without any issue.
Indefinites in Cantonese are typically introduced by numeral classifier constructions of the form \[\text{Num + Cl + N}\]. Numeral classifier constructions are similar to counting quantifiers in English. I call them cardinal indefinites to emphasize their indefinite nature. As shown in (13), saai is marked when followed by a cardinal indefinite.²

(13) \%Di-hoksaang gin-saai jat-go lousi.
   CL.PL-student see-SAAI one-CL teacher
   ‘The students all saw a teacher.’  
   Cardinal indefinite

The judgment is similar for cardinal indefinites involving a higher numeral, as shown in (14). However, sentences involving two plural arguments and saai have an extra layer of complexity, as they are ambiguous between a subject-Key reading and an object-Key reading. In the subject-Key reading, the subject is Key, whereas in the object-Key reading, the object is the Key. This flexibility is a well-documented property of distributivity with saai (Lee 2012) and is discussed as a remaining issue in Section 4.7.³

²Sentences involving saai and a post-saai cardinal indefinite may be prefixed by ‘%’ or not. When not prefixed, they are fully acceptable. When prefixed, they exhibit inter-speaker variability in judgment. Regardless of the % prefix, the cardinal indefinites in these sentences must not co-vary with distributive quantification, a generalization established in Section 4.2.3 and is the main concern of this chapter. I do not take up the question why some examples are more natural than others, but two factors are likely relevant: (i) whether or not the context can support the lack of co-variation of a cardinal indefinite, and (ii) the potential competition with a definite expression.

³Other quantifiers also exhibit similar markedness when the Key is set to be the subject:

(i) %Di-hoksaang gin-saai daboufeng lousi.
   CL.PL-student see-SAAI most teacher
   ‘The students all saw most teachers.’

(ii) %Di-hoksaang gin-saai housiu lousi.
   CL.PL-student see-SAAI few teacher
   ‘The students all saw few teachers.’

When the Key is set to be the object, the markedness disappears, because the distributed share now contains the subject and the verb. The subject is a definite expression so it is not marked.

Due to the ambiguous nature of data involving two plural arguments, I do not think they offer us the clearest clue as to what distinguishes between definite expressions and indefinite expressions in the distributed share when distributivity
Bare noun phrases are also sometimes treated as indefinites in Cantonese for they occur in existential sentences. However, *saai* is compatible with bare noun phrases. In other words, bare noun phrases pattern like definite expressions in not showing markedness effects when they follow *saai*.

No study has addressed why bare noun phrases pattern more like definite expressions in terms of their ability to appear after *saai* (but see Section 4.4.3). The contrast between (11) and (12) on the one hand and (13) one the other hand, however, has motivated the generalization that a post-*saai* expression must be definite or specific. The literature has not formally tested whether definiteness or specificity is the relevant notion. Presumably, if the relevant notion is a definiteness requirement, then the inter-speaker variability reflects whether or not a speaker can assign a definite interpretation using an indefinite form; if the relevant notion is specificity, then the variability reflects whether or not one can assign a specific indefinite interpretation to an indefinite form.

There are a few reasons to believe that definiteness is not the right notion. First, a post-*saai* expression may enter into scope interaction with negation in ways that a definite expression cannot. For example, the indefinite in (16) may be interpreted as having wide scope relative to negation (31-a), or as having narrow scope relative to negation (31-b).4

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4The narrow-scope interpretation can be further brought out by the use of *jamho* 'any' before the numeral.
However, a definite expression in the same position does not interact with negation. As a result, the following sentence only has one interpretation:

(17) Di-hoksaang mou gin-saai go-lousi.
    CL-students not see-SAAI CL-teacher
    ‘The students did not each see the teacher.’

Second, a post-*saai* indefinite must introduce a referent that is not intended to be familiar in the discourse (see also the novelty condition of Heim 1982). For example, B’s answer in (18) is infelicitous because the indefinite has the same referent as the possessive noun phrase introduced in A’s question.

(18) A: Gamjong-bun* syu hotai ma?
    Gamjong-CL new book goodread POLQ
    ‘Is Gamjong’s new book a good read?’

    B: #Hotai aa. Di-hoksaang maa-saai jat-bun syu, lai taai.
        goodread SFP CL-students buy-SAAI one-CL book to read
        ‘Yes. The students each bought a book to read.’

However, using a definite expression in the same position is acceptable:

(19) A: Gamjong-bun syu hotai ma?
    Gamjong-CL new book goodread POLQ
    ‘Is Gamjong’s new book a good read?’

    B: Hotai aa. Di-hoksaang maa-saai bun-syu lai taai.
        goodread SFP CL.PL-students buy-SAAI CL-book to read
        ‘Yes. The students all bought the book to read.’

Given the above considerations, the markedness of indefinites should not be explained in terms of a definiteness requirement on the indefinites. This leaves us with the specificity requirement. That is, the markedness of indefinites is due to the need to interpret them as specific indefinites. I provide some evidence below suggesting that specificity indeed provides a more adequate explanation. Reserving a more precise formulation of the kind of specificity involved in *saai* until Section 4.3 and 4.4, here I only use specificity in an intuitive sense: an indefinite is specific when it refers to a referent identifiable by the speaker.\(^5\)

\(^5\)I examined all instances of the post-verbal *saai* that co-occur with a cardinal indefinite in Hong Kong Cantonese
To begin with, it is known that indefinites with a more descriptive content have a better chance being interpreted as specific (Fodor and Sag 1982, Schwarzschild 2002b). Relatedly, enriching the descriptive content of an indefinite by introducing a modifier improves its ability to co-occur with saai.

(20) Di-hoksaang gin-saai jat-go san lei-ge lousi.
    CL.PL-student see-SAAI one-CL newly arrive-MOD teacher
    ‘The students all saw a newly arrived teacher.’

In addition, by comparing the interaction of indefinites with saai and the interaction of indefinites with other distributive markers, it can be shown that indefinites following saai indeed receive a specific interpretation. There are two other distributive markers in Cantonese: cyunbou ‘all, completely’ and dou.7 Indefinites co-occurring with these markers do not exhibit markedness effects and may co-vary with distributive quantification, as shown below:

(21) a. Keoidei cyunbou gin-zo jat-go lousi
    they all saw-ASP one-CL teacher
    ‘They all saw a teacher (possibly different teachers).’

Corpus (CanCorp, Luke and Wong 2005). The findings are two-fold. First, cardinal indefinites do follow saai in naturally occurring discourse. Second, all the cardinal indefinites following saai invariably have a strong specific indefinite flavor. I provide two naturally occurring data here for illustration. The data are slightly modified to reduce their length and have the dropped arguments re-introduced in parentheses to facilitate interpretation). In both sentences, the indefinite has a specific referent: a specific tank in (i) and a specific CD in (i). In other words, (i) cannot be true if the fish are in different tanks, and (i) cannot be true if the good songs are in different CDs.

(i) Dim wuzou (di-juu) dou hai-saai jat-go gong leoibin lo.
    however dirty CL-fish dou in-SAAI one-CL tank inside SFP
    ‘However dirty, (the fish) are all inside a single tank.’6
    (ID: FC-033)

(ii) Hou go dou baai-saai lok jat-zek (dip) dou aa.
    good song dou put-SAAI in one-CL CD there SFP
    ‘The good songs are all in one (CD).’
    (ID: FC-109a)

7Cantonese dou is a cognate of the more famous Mandarin dou (Cheng 1995). Dou (in both languages) signals distributivity, but whether it constitutes a distributive operator is subject to debate. While Lin (1998a) argues that dou is a generalized distributive operator, many recent studies disputed this view, including Chen (2005, 2008), Xiang (2008), Liu (2016), Xiang (2016).

I am not aware of any formal analysis of Cantonese cyunbou, other than its partial cognate quan in Mandarin taken up in Tomioka and Tsai (2005). However, the properties of cyunbou and quan are quite different. In this study, I use all to translate cyunbou. However, it must be noted that Brisson (1998, 2003) argues that all is not a distributive operator but a maximality marker for removing pragmatic slack. I have not investigated cyunbou in enough detail to determine whether it is a genuine distributive operator or a maximality operator akin to all.
b. Cyunbou jan dou gin-zo jat-go lousi
   all people DOU saw-ASP one-CL teacher
   ‘All the people saw a teacher (possibly different teachers). ’

However, indefinites following saai not only are marked for some speakers, but also may not co-vary with distributive quantification for the speakers who accept them:

(22) %Di-hoksaang gin-saai jat-go lousi.
    CL.PL-student see-SAAI one-CL teacher
    ‘The students all saw a teacher (the same teacher).’

Lastly, specificity can be extended to understand the interaction of saai and disjunction. Compare (23-a), which allows (but does not require) the disjunction to co-vary with the Key when distributivity is marked with cyunbou and dou, with (23-b), which disallows the co-variation when cyunbou and dou are replaced by saai.

(23) a. Di-hoksaang cyunbou dou maai-zo Emma waatze Jane Eyre.
    CL.PL-student all dou buy-ASP Emma or Jane Eyre
    ‘The students all bought Emma or Jane Eyre.’

b. Di-hoksaang maai-saai Emma waatze Jane Eyre.
    CL.PL-student buy-SAAI Emma or Jane Eyre
    ‘The students all bought Emma or they all bought Jane Eyre.’

In summary, when distributivity is marked with saai, the expression following saai has to assume a specific interpretation. By comparing the interpretation of indefinites co-occurring with saai and those co-occurring with other distributive markers, I have shown that the specificity comes from saai rather than from the indefinites. An important question arising from this discussion is why as a distributive marker saai carries a specificity requirement. To answer this question, it is necessary to understand the type of specificity associated with saai, a task I take up in the next subsection.

4.2.3 Independence in distributivity

As pointed out in many studies, there is no agreed-upon definition for specificity. The main reason is that there are different types of specificity, each with its own characteristics (e.g., Farkas 2002a, von Heusinger 2002). In this section, I explore three types of specificity that may be associated with
saai’s specificity effect. They are epistemic specificity, scopal specificity, and relational specificity, as classified in von Heusinger (2002). The conclusion I arrive at is that saai’s specificity effect resembles none of them, so the analyses for these types of specificity cannot be directly applied to account for the specificity effect of saai. I propose that saai’s specificity effect should be understood as a kind of specificity that targets distributive quantification. This type of specificity is referred to as independence in distributivity.

An epistemically specific indefinite refers to an individual that the speaker has in mind. Some studies take epistemic specificity to indicate that indefinites have a non-quantificational, referential use (Fodor and Sag 1982). von Heusinger (2002) uses the following example as an illustration.

(24) A student in Syntax 1 cheated on the exam.
   a. His name is John.
   b. We are all trying to figure out who it was.

In (24-a), the speaker can uniquely identify the individual the indefinite in (24) refers to, so the indefinite is said to be epistemically specific. However, if the speaker cannot uniquely identify the referent of the individual, as in (24-b), then the indefinite is said to be epistemically nonspecific.

When it comes to an indefinite following saai, it may be epistemically specific or not, as both types of follow-ups in (25-a) and (25-b) are felicitous. In other words, the specificity effect of saai is not epistemic specificity. If epistemic specificity is used to indicate that an indefinite has a referential use, then we can conclude that an indefinite following saai does not need to assume a referential use.

(25) %Di-hoksaang gin-saai jat-go lousi.
   CL.PL.-student see-ASP one-CL teacher
   ‘The students each saw a teacher.’
   a. Zauhai Lei Lousi.
      namely Lei Teacher
      ‘Namely Teacher Lei.’
   b. Dan ngo m-zi hai binggo lousi.
      but I not-know be which teacher
      ‘But I don’t know which teacher.’
There are different definitions of scopal specificity in the literature, depending on whether non-island-bound scope (i.e., exceptional scope) is taken to be a defining feature. In this study, I follow von Heusinger (2002) and take scopal specificity to refer to the ability to take scope over another scope-bearing element (regardless of the presence of syntactic islands), such as negation, modals or conditionals.\(^8\) Saaï as a verbal suffix generally cannot attach to modal verbs, so the relevant testing cases are negation and conditionals. von Heusinger (2002) cites the following example (from Karttunen 1976) to show the interaction of specificity and negation. To give rise to the interpretation in (26-a), the existential quantification is interpreted outside the scope of negation, so the indefinite is said to be scopally specific. By contrast, to give rise to the interpretation in (26-b), the existential quantification contributed by the indefinite is inside the scope of negation, so the indefinite is said to be scopally nonspecific.

(26) Bill didn’t see a misprint.
   a. There is a misprint which Bill didn’t see.
   b. Bill saw no misprints.

Similarly, indefinites in English may scopally interact with a conditional (Reinhart 1997). When it is interpreted as having wide scope relative to the conditional, as in (27-a), the interpretation is said to be specific. When it is interpreted as having narrow scope relative to the conditional, as in (27-b), the interpretation is referred to as non-specific.

(27) If a relative of mine dies, I will inherit a house.
   a. There is a particular relative of mine such that if s/he dies, I will inherit a house.
   b. If any relative of mine dies, I will inherit a house.

\(^8\)Some studies take scopal specificity to follow from epistemic specificity (Fodor and Sag 1982) but others argue that it is a separate class (Farkas 1981, Kamp and Bende-Farkas 2019). Different theories of scopal specificity for indefinites have been developed over the years. When an indefinite does not occur in a syntactic island, the use of ordinary scope-taking suffices (Montague 1974, May 1977). When an indefinite occurs in an island, exceptional wide scope has been argued to come from (i) the use of (skolemized) choice functions (Reinhart 1997, Kratzer 1998, Matthewson 1999), (ii) existential quantification with a singleton domain (Schwarzchild 2002b), (iii) anaphora to a quantificational structure (Brasoveanu and Farkas 2011, DeVries 2016), and (iv) the use of dynamic alternatives semantics (Charlow 2014). The analysis developed in this dissertation for saai is closest to the anaphora approach. I reserve a reincarnation of the present study in other frameworks for future research.
There are languages with morphology that gives rise to scopal specificity. For example, the determiner *ti* in St’át’imcets marks an indefinite that has to take wide scope relative to negation and conditional.

(28) cwʔaʔoz kw-s áz’-en-as [ti stš’úqwaz’-a] kw-s Sophie
NEG DET-NOM buy-3ERG DET fish-DET DET-NOM Sophie
‘There is a fish which Sophie didn’t buy.’ (Matthewson 1999: (21))

(29) cuz’ tsaʔcw kw-s Mary lh-t’íq-as *ti* qelhmémen’-a
going.to happy DET-NOM Mary HYP-arrive-3CONJ DET old.person(DIMIN)-DET
‘Mary will be happy if a particular elder comes.’ (Matthewson 1999: (16))

When *ti* appears with two scope bearing elements, it has to out-scope both, as shown in the following example:

(30) [tákem i waʔ tsunám’-cal] cuz’ waʔ qwenúxw-alhts’aʔ
all DET.PL PROG teach-INTR going.to PROG sick-inside
lh-k’aw-lec-as [ta twíw’t-a]
HYP-far-INTR-3-CONJ DET child-DET
‘Every teacher will be sad if a child quits.’ (Matthewson 1999: (60))

a. Accepted in context: There is one child, who every teacher doesn’t want to leave. (Widest scope)

b. Rejected in context: For each teacher, there is one child who s/he doesn’t want to leave. (Intermediate scope)

c. Rejected in context: Every teacher will be said if any child leaves. (Narrow scope)

As pointed out earlier in the discussion of (16) (repeated below), a post-*saai* indefinite may scopally interact with negation:

(31) Di-hoksaang mou giʔ-saai jat-go lousi.
CL.PL-students not see-SAAL one-CL teacher
a. There is a teacher that the students did not all see.

b. There is no teacher that the students all saw.

Moreover, a post-*saai* indefinite may scopally interact with a conditional:
if CL.PL-fish in-SAAI one-CL tank inside I will very happy

a. There is a tank such that if all the fish are inside that tank, I’ll be very happy.
b. If all the fish are in a single tank (regardless of which tank), I’ll be very happy.

What this tells us is that the specificity effect of saai is not one that fully resembles widest scope indefinites. In particular, an indefinite following saai may freely interact with other scope bearing elements such as negation and conditionals. It looks, quite interestingly, that if any scopal effect is relevant, a post-saai indefinite is only required to be interpreted outside the scope of distributivity, not any other operator.

Finally, we test relative specificity. von Heusinger (2002), attributing the identification of this type of specificity to Enç (1991), uses the following example (due to Hintikka 1986) to illustrate it:

(33) According to Freud, every man unconsciously wants to marry a certain woman—his mother.

What is interesting is that there is no particular scope configuration of the indefinite that would give the sentence the intended interpretation. If the indefinite takes wide scope, we end up with a truth condition that is too strong: there is a particular woman that every man wants to marry. If the indefinite takes narrow scope, then the truth condition is too weak: as long as every man wants to marry some woman or other, the sentence is true. Rather, what the sentence requires is a specific relation linking the man and the woman—the woman is the man’s mother.

Is the specificity effect of saai reducible to relational specificity? I think not. (34) cannot refer to a situation in which each fish is in a different tank even if the tanks happen to be the respective favorite tanks of the fish’s. The only interpretation that is available is that the fish are all in the same tank and the tank is their favorite one.

(34) Di-juu hai-saai jat-go gong japmin, jiuhai keoidei zeoi zungji-ge gong.
CL.PL-fish in-SIDE one-CL tank inside namely their most like-MOD tank.
‘The fish are all in a tank, namely, their favorite tank.’

What arises from this discussion is that the specificity effect of saai cannot be captured by epistemic specificity or relational specificity, and it only partially resembles scopal specificity. The kind of specificity effect we need is one that is intimately tied to distributivity. Let me suggest that this type
of specificity be understood as **independence** relative to distributivity, formulated as a generalization below for easy reference:\(^9\)

\[(35) \quad \text{The Independence Generalization of } saai\text{-distributivity} \]

The evaluation of a post-\textit{saai} expression is independent of the evaluation of distributive quantification.

All that \textit{saai} requires is that an expression following it remain constant relative to distributive quantification. The expression may scopally interact with any other scope-bearing element as long as the independence generalization is satisfied. The rest of the chapter is devoted to two different accounts that derive the Independence Generalization. I ultimately argue in favor of the second account. However, exploring the first account offers useful preparation for the second account.

The first account is outlined in section 4.3. According to this account, \textit{saai} as a suffixal distributive marker always combines with a verb and introduces distributivity that scopes only over the verbal predicate. A post-\textit{saai} constituent, in this case, is naturally interpreted outside the scope of \textit{saai}-distributivity.\(^{10}\) I call this a **scope** account, for it derives the Independence Generalization by forcing distributivity contributed by \textit{saai} to take narrow scope relative to post-\textit{saai} nominals.

The second account is introduced in 4.4. In this account, \textit{saai} is allowed to introduce distributivity that freely scopally interact with other scope expressions. However, a separate mechanism (formulated as a constraint) ensures that the post-\textit{saai} constituent is interpreted as if it is outside the scope of distributivity. I call this a **pseudo-scope** account. The pseudo-scope account is very similar to the scope account, but the there are empirical and conceptual differences that tell it apart from the former.

\(^9\)I chose the term ‘independence’ rather than ‘specificity’ because the phenomena include both independence of individuals and independence of degrees (see Section 4.5). While independence of individuals can be think of as specificity, independence of degrees is much harder to think in terms of specificity.

\(^{10}\)However, it may enter into scopal interactions with other sentential operators, and hence does not enjoy widest scope.
4.3 A scope account in terms of narrow-scope distributivity

The scope account relies on the key syntactic assumption that *saai* as a verbal suffix introduces distributivity that always takes narrow scope relative to other nominals. A concrete structural illustration is given in Figure 4.1. Since the scope account does not need to appeal to dynamic semantics or plural logic, I return to a basic static semantics (with domain pluralities) throughout this subsection.

\[
\begin{align*}
\text{the students} & \quad \Lambda u' \\
\text{one teacher, Emma or Jane Eyre} & \quad \Lambda u \\
\text{buy} & \quad \text{saai}
\end{align*}
\]

Figure 4.1: Narrow-scope distributivity

In this analysis, *saai* has the definition in (36). It takes a relation provided by a transitive verb, such as *see* in (37), and returns another relation. The newly returned relation is just like the original one except for the fact that the relation no longer holds between a subject and an object, but between the atomic parts of the subject and an object, as shown in (38).

\[(36) \quad \text{saai} := \lambda R \lambda y \lambda x. \forall z [z \leq_A x](R y z), \text{where } \leq_A \text{ is the ‘atomic part-of’ relation}\]
gin 'see' := λy.λx.see y x

(38)  gin-saai := λy.λx.∀z(≤A x)(see y z)

Saai is flexible with the arity of its relational argument, as it can also combine with an intransitive verb, as shown in (39).  

(39)  Keoidei zau-saai la.
they leave-SAAI SFP
‘They each left.’

For this reason, saai’s argument structure should be generalized. Instead of taking a two-place relation (e → e → t) and returning a two-place relation, it should be able to take a relation of arbitrary arity and return a relation of the same arity. A type-flexible definition of saai is offered below:

(40)  saai := λαλy.λx.∀z(≤A x)(α y z), where ≤A is the ‘atomic part-of’ relation, α a n-nary predicate, and y is a sequence of n − 1 variables.

4.3.1 Cardinal indefinites

A cardinal indefinite denotes a generalized quantifier, as shown in (41) (e.g., Montague 1974, Barwise and Cooper 1981). Following Montague (1974), a plural definite subject can also be modeled as a generalized quantifier, as shown in (42).

(41)  jat-go lousi ‘one teacher’ := λP.∃y[book y ∧ µCARD y = 1](P y)

(42)  Di-hoksaang ‘the students’ := λP.P(⊕ stds)

Folding in the lexical ingredients in (38), (41) and (42), a sentence with a cardinal indefinite following saai is then interpreted as follows:

(43)  a.  the students (λx. one teacher (λy. see-saai y x))

11The fact that saai can be suffixed to an intransitive verb indicates that it cannot be analyzed as originating from a noun phrase.
b. \[ \exists y [\text{book } y \land \mu_{\text{CARD}} = 1] (\forall z [z \leq A \oplus \text{stdts}] (\text{read } y z)) \]

The cardinal indefinite naturally takes wide scope over the distributive quantification introduced by \textit{saai}. This is because the distributive quantification is introduced, in the first place, as only having scope over an individual, i.e., the ‘trace’ of a quantificational object. So, it is not surprising that the cardinal indefinite ends up taking wider scope over distributivity. This also essentially ensures that the cardinal indefinite is interpreted as independent of, i.e, not co-varying with, distributivity.

For ease of comparison, let me illustrate the range of possible interpretations for a sentence with a cardinal indefinite and a distributive marker like \textit{cyubou} or \textit{dou}. Assume that these distributive markers contribute a standard VP-level distributive operator, which can scopally interact with cardinal indefinites. The scope interaction then gives rise to two interpretations: the “distributivity > indefinite” interpretation in (44) and the “indefinite > distributivity” interpretation in (45).

(44) a. \textbf{the students (cyunbou/dou (λx. one teacher (λy. see y x)))}
   b. \[ \forall z [z \leq A \oplus \text{stdts}] (\exists y [\text{teacher } y \land \mu_{\text{CARD}} = 1] (\text{see } y z)) \]

(45) a. \textbf{one teacher (λy. the students (cyunbou/dou (λx. see y x)))}
   b. \[ \exists y [\text{teacher } y \land \mu_{\text{CARD}} = 1] (\forall z [z \leq A \oplus \text{stdts}] (\text{see } y z)) \]

### 4.3.2 Disjunction

Disjunction has long been noted to participate in scopal interactions (e.g., Larson 1985). Since proper names can be lifted to generalized quantifiers (Partee 1986; see also Montague 1974), a disjunction involving two proper names can be treated as a disjoined generalized quantifier following Rooth and Partee (1982), as shown in (46). This generalized quantifier occupies the same position as a cardinal indefinite. For this reason, a sentence with a disjunction following \textit{saai} also naturally has the disjunction out-scoping the distributivity, as shown in the LF in (47-a) and the semantic translation in (47-b).

(46) \textbf{Emma or Jane Eyre} := λP.P e ∨ P je

(47) a. \textbf{the students (λx. Emma or Jane Eyre (λy. read-saai y x))}
   b. \[ \forall z [z \leq A \oplus \text{stdts}] (\text{read } e z) \lor \forall z [z \leq A \oplus \text{stdts}] (\text{read } je z) \]
If distributivity is introduced not by *saai* but by an adverbial distributive marker capable of scopally interacting with disjunction, such as *cyunbou* or *dou*, then the corresponding sentence is ambiguous, as we have seen in the case of cardinal indefinites.

\[(48)\]
- a. *the students* (*cyunbou/dou* (*λx. Emma or Jane Eyre* (*λy. read* *y x*)))
- b. \(∀z[z ≤ A \bigoplus \text{stdts}]\)(read je \(z\) \(\lor\) read e \(z\))

\[(49)\]
- a. *Emma or Jane Eyre* (*λy. the students* (*cyunbou/dou* (*λx. read* *y x*)))
- b. \(∀z[z ≤ A \bigoplus \text{stdts}]\)(read e \(z\) \(\lor\) \(∀z[z ≤ A \bigoplus \text{stdts}]\)(read je \(z\))

### 4.3.3 Bare noun phrases

Treating bare NPs requires some caution. If we assume that bare NPs are existential quantifiers like cardinal indefinites, then the prediction is that they pattern like cardinal indefinites in their interactions with *saai*. This is not in accordance with the empirical generalization. As reported in Section 3.2, bare noun phrases are allowed to have witnesses that co-vary with distributivity. To model the interactions between bare noun phrases and *saai*, I suggest we exploit a longstanding tradition in semantics to treat bare noun phrases as proper names of kinds (Carlson 1977a,b, Chierchia 1998b, Dayal 2004, 2011a). On this view, they are not scope-bearing elements and may directly serve as an argument for a predicate that has composed with *saai*. As an example, the bare noun phrase *syu* ‘book(s)’ is translated and abbreviated as follows:

\[(50)\]
- *syu* ‘book(s)’ := \(λs. \bigoplus \text{bks}_s\)
  \(=\) bk-kind

Plugging in this bare noun phrase into the structure in Figure 4.1 yields the LF (51-a) and its semantic translation in (51-b).

\[(51)\]
- a. *the students* (*λx.see-saai book(s)* *x*)
- b. \(∀z[z ≤ A \bigoplus \text{stdts}]\)(read bk-kind \(z\))

---

12 Even if they are lifted to generalized quantifiers, they behave like proper names and do not enter into scope interactions with other operators.

13 Bare noun phrases are number-neutral in Cantonese.
It is well known in the literature on bare noun phrases that a sortal repair strategy is needed to compose an object-level predicate (like read) and a kind term contributed by a bare noun phrase. In this study, I adopt Derived Kind Predication (DKP), as proposed in Chierchia (1998b) to repair the sortal mismatch. This sortal repair strategy is defined as follows ($\cup$ shifts a kind to a property)$^{14}$:

\begin{equation}
(52) \text{DKP}
\end{equation}

If $R$ is an $n$-place relation over individuals and $k$ a kind term, then:

$$R(k) := \lambda x_1,\ldots,\lambda x_{n-1}.\exists y[\cup y \cdot (R y x_1,\ldots,x_{n-1})]$$

Note that the existential quantification introduced by the sortal repair strategy always takes the narrowest scope (Carlson 1977a,b, Chierchia 1998b). Therefore, applying DKP to (51-b) yields a narrow scope existential interpretation, as shown in (53), which is compatible with witness variation.

\begin{equation}
(53) \forall z [z \leq \bigoplus \text{stdts}] (\exists y[\cup y \cdot (\text{bk-kind} y, (\text{read} y z)])
\end{equation}

In short, according to this analysis, saai contributes narrow-scope distributive quantification. As a result, the distributive quantification fails to interact with other scopal expressions, such as cardinal indefinites and disjunction, giving rise to a wide-scope interpretation of these scopal expressions. Since a wide scope indefinite or disjunction, without additional assumptions, do not co-vary with distributivity, this analysis can then predicts the lack of co-variation of these expressions with distributivity. Bare noun phrases are exceptions because they induce existential quantification as a sortal repair strategy, which always takes the narrowest scope. As a result, bare noun phrases behave as if they can co-vary with distributivity.

4.3.4 Multiple post-saai constituents

A very nice prediction of the narrow scope distributivity account is that any cardinal indefinite introduced following saai has to not co-vary with distributivity. This is because saai is stipulated to introduce distributivity scoping only over the verbal relation, hence any number of cardinal indefinites (or disjunction) should be interpreted outside the scope of distributivity. This prediction is

---

$^{14}$This DKP is a relational version suggested in Chierchia (1998:fn.16). Note also that $\cup$ also needs to provide a situation argument as $k$ is of type $s \rightarrow e$ (I thank Simon Charlow (p.c.) for pointing this out to me).
borne out by the following examples:

(54) Keoidei song-saai jat-bun syu bei jat-go hoksaang
they give-SAAI one-CL book to one-CL student
‘They all gave a particular book to a particular student.’

(55) Keoidei giu-saai jat-go jan heoi maai jat-bun syu
they ask-SAAI one-CL person buy one-CL book
‘They all asked a particular person to buy a particular book.’

The narrow-scope distributivity analysis is straightforward and accounts for the data set introduced in section 3.2. However, it runs into a few empirical issues, which are discussed in the following subsections.

4.3.5 Empirical problem 1: cardinal indefinites with bound pronouns

When a cardinal indefinite contains a pronoun in the common noun restriction bound by the Key, as shown in (97) (the entire cardinal indefinite is enclosed in “[...]”), the cardinal indefinite can co-vary with the Key.15

(56) Di-hoksaang^[\textit{zigei}_x \textit{jungji-ge} jat-bun syu].
CL-PL-students buy-SAAI self like-MOD one-CL book
‘The students bought a book they like.’

The behavior of this type of cardinal indefinites cannot be accounted for by simply letting the cardinal indefinites be interpreted outside the scope of distributivity. To see this, I first translate the cardinal indefinite as an existential quantifier with the pronoun interpreted as a free variable.

(57) \textit{zigei}_x \textit{jungji-ge} jat-bun syu ‘a book they like’
\[ := \lambda P. \exists y[\text{book } y \land \mu_{\text{CARD}} y = 1 \land \text{like } x](P y) \]

After plugging the indefinite into the structure in Figure 4.1, we obtain the LF in (58-a) and its semantic translation in (58-b). However, this interpretation does not allow different students buying different books. What it allows is every student reading a single book that they collectively like.

15While the use of a reflexive pronoun \textit{zigei} readily facilitates co-variation of a cardinal indefinite that contains it, a non-reflexive pronoun, such as \textit{keoidei} ‘they’, is not as effective in facilitating co-variation. I leave the contrast for future research.
Even if we assume that ‘collective liking’ gets resolved in the same way as distributive liking, (58-a) and (58-b) are still inadequate because they do not allow the books to co-vary with the students.

\[(58)\]
\begin{align*}
&\text{a. the students } (\lambda x. \text{a book they}_x \text{ like } (\lambda y. \text{buy-saai } y x)) \\
&\text{b. } \exists y [\text{book } y \land \mu_{\text{CARD}} y = 1 \land \text{like } y \oplus \text{stdts}] (\forall z [z \leq A \oplus \text{stdts}] (\text{buy } y z))
\end{align*}

The behavior of indefinites with bound pronouns can be accounted for if we assume that an indefinite with a bound pronoun falls inside the scope of the distributive quantification introduced by saai. This is because the pronoun will be bound by the universal quantifier that quantifies over the atomic parts of the plurality denoted by the Key. The corresponding semantic interpretation is given below:

\[(59)\]  
\[
\forall z [z \leq \oplus \text{stdts}] (\exists y [\text{book } y \land \mu_{\text{CARD}} y = 1 \land \text{like } y z] (\text{read } y z))
\]

However, there is no way to sneak a cardinal indefinite back into the scope of the distributive quantification introduced by saai, given that the distributive quantification is formulated to only scope over individuals, i.e., which are traces of quantifiers like cardinal indefinites. If we are to assume that saai has an alternative lexical entry allowing the distributive quantification it introduces to scopally interact with cardinal indefinites with a bound pronoun, we then need to justify what bans this lexical entry in the cases of simple cardinal indefinites and disjunction.

In short, a cardinal indefinite with a bound pronoun imposes conflicting requirements on the relative scope of the cardinal indefinite and the distributive quantification introduced by saai: to maintain the integrity of the narrow-scope distributivity analysis, the indefinite should be interpreted outside the scope of distributivity; however, to model the co-variation induced by the bound pronoun, the indefinite has to be interpreted inside the scope of distributivity. Without further assumptions, it is not clear how an indefinite can be inside and outside the scope of distributivity at the same time.

### 4.3.6 Empirical problem 2: scope interference from other distributive markers

As we have seen, there are two other distributive markers in Cantonese: cyumbou (60-a) and dou (61-a). Saai can co-occur with both without inducing ungrammaticality, as evidenced by (60-b) and (61-b).
There are two reasons why co-occurring distributive markers are of interest to this study. First, if saai, cyunbou and dou all contribute genuine distributive quantification targeting the same plural subject, it is unclear why they do not give rise to vacuous distributive quantification, which is banned in languages like English.¹⁶

*Every student each saw Miss Carla.*

Admittedly, co-occurring distributive markers is a poorly understood phenomenon. What it challenges is the practice of translating every instance of markers of distributivity as an independent distributive operator, rather than the proposal that saai contributes narrow-scope distributivity.¹⁷

For this reason, it is useful to consider another interesting pattern resulting from co-occurring distributivity: scope interference. Concretely, when cyunbou and dou occur without saai, they are capable of scopally interacting with an indefinite, as pointed out in Section 3.2. For example, both (63) and (64) allow a narrow-scope interpretation of the cardinal indefinite in the object position.

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¹⁶ Many languages allow more than one distributive marker to occur in a sentence without inducing double distributivity. For example, Kaqchikel (Henderson 2014), Hungarian (Kuhn 2017), and American Sign Language (Kuhn 2017) allow the co-occurrence of a distributive quantifier and a distributive numeral. Relatedly, Szabolcsi (2010) reports that to some (but not all) speakers of English, distributive quantifiers may co-occur with binominal each, as in (i) (cited from Kuhn 2017: (15)).

(i) %Every job candidate was in the room for fifteen minutes each.

¹⁷ That being said, the analysis developed in Section 4.4 does address the co-occurrence puzzle of distributive markers to some extent. The spirit of the analysis is that saai does not in fact introduce distributive quantification. Rather, it imposes an ‘independence constraint’ on the functional dependency resulting from distributive quantification.
(63)  Di-hoksaang cyunbou gin-zo jat-go loui.
   CL.PL-students all see-ASP one-CL teacher
   ‘The students each saw a teacher (possibly different ones).’

(64)  Cyun-ban hoksaang dou gin-zo jat-go loui.
   whole-class student DOU see-ASP one-CL loui
   ‘Each student in the class met a teacher (possibly different ones).’

However, when saai surfaces in these sentences, cyunbou and dou fail to scopally interact with the cardinal indefinites, as shown in (65) and (66). It is as though saai’s presence interferes with the scope interactions between cyunbou/dou and other scopal expressions.

(65)  Di-hoksaang cyunbou gin-saai jat-go loui.
   CL.PL-students all see-SAAI one-CL teacher
   ‘The students each saw a teacher (the same teacher).’

(66)  Cyun-ban hoksaang dou gin-saai jat-go loui.
   whole-class student DOU see-SAAI one-CL loui
   ‘Each student in the class met a teacher (the same teacher).’

Treating saai as merely contributing narrow-scope distributivity does not account for its ability to induce scope interference.

4.3.7 Interim summary

Given the empirical challenges faced by the narrow-scope distributivity account, I do not think it holds the ultimate key to analyzing saai distributivity. However, there is no denying that interpreting indefinites and disjunction outside the scope of distributivity does offer a relatively natural and simple analysis for their lack of co-variation with distributive quantification. In the formulation of an alternative analysis to address the empirical issues, it is worth preserving the simplicity of the narrow-scope distributivity account.

In the next section, I offer a pseudo-scope account that mimics the narrow-scope distributivity account very closely in terms of the predictions for cardinal indefinites, disjunction, and bare noun phrases in saai-distributivity. The account crucially relies on the use of an independence constraint, imposed on the functional dependency arising from distributive quantification. To model the fact that distributive quantification contributes a functional dependency that can be subject to
further constraints, I use the framework developed in Chapter 2.

4.4 A pseudo-scope account in the framework of DPILM

4.4.1 Proposal: an independence constraint

The pseudo-scope account has the following main ingredients:

- *Saai* is not treated as distributive operator. Rather, I argue that it imposes a constraint on the functional dependency arising from distributive quantification.

- Distributivity, as contributed by *cyunbou*, *dou*, or a null distributive operator, is allowed to freely scopally interact with any post-*saai* expressions. In other words, the assumption that *saai* contributes narrow-scope distributivity is removed.

- The constraint contributed by *saai* requires that values introduced inside the scope of distributivity by a post-*saai* expression remains independent of distributivity. More precisely,
  
  - if a post-*saai* expression is interpreted outside the scope of distributive quantification, nothing happens to it; however,
  
  - if a post-*saai* expression is interpreted inside the scope of distributive quantification, then it is required to have a constant witness relative to, i.e., not co-vary with, distributive quantification.

The last point is particularly important. It amounts to giving an expression inside the scope of distributive quantification pseudo wide-scope. In fact, quite a number of indefinites have received a pseudo-scope account using choice functions, such as indefinites marked by *(t)i- in Sta’át’imcets (Matthewson 1999) and indefinites marked by the suffix *-khí* in Tiwa (Dawson 2018).

While analyzing indefinites marked by *saai* as choice function indefinites seems like a plausible option, it does not account for why the wide-scope behavior of *saai* is inherently tied to distributivity. In other words, *saai* does not mark the wide scope status of an indefinite when there is no distributivity, as shown in (67), unlike the wide-scope markers in Sta’át’imcets (68) or Tiwa (69).

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18For additional challenges faced by the choice-function approach to indefinites, see Geurts (2000), Schwarz (2001), Brasoveanu and Farkas (2011), Charlow (2014).
In this paper, I pursue a pseudo-scope account couched in DPILM as developed in Chapter 2. The primary reason for using this logic is because it allows distributive quantification to contribute functional dependencies that can be passed down from context to context. In other words, distributivity is fully dynamic in this logic. As a result, we can talk not only about introducing distributivity into context, but also retrieving it from context. The latter is an important component in the semantics of saai, which is analyzed as imposing a constraint on the distributivity dependency it accesses anaphorically. Using DPILM to model the independence constraint of saai has an extra benefit: it allows us to directly compare saai’s independence constraint and binominal each’s monotonic measurement constraint. The comparison is offered in Section 4.6 in this chapter.

At the core of saai’s contribution is an independence constraint, formulated as in (70).

\[
G \langle \text{ind}_{x, y} \rangle H := G = H \text{ and for all } a, b \in G(x) : G|_{x=a}(\overline{y}) = G|_{x=b}(\overline{y})
\]

‘y’s value is constant relative to x’s value.’

Saai anaphorically accesses the functional dependency introduced by distributive quantification with the help of the first index, i.e., the variable x that stores the values contributed by the Key. Then, it accesses all the new variables \(y_1, y_2, \ldots, y_n\) introduced in the scope of distributive quantification, and requires that each variable stores values that are constant relative to the values in the Key. In other words, although distributive quantification allows a variable introduced in its scope to exhibit dependence with the variable storing the values associated with the Key, saai effectively forbids the dependence. What this amounts to is that the variable is introduced as if it is outside the scope of distributivity. A very similar account is offered in Brasoveanu and Farkas (2011) for indefinites that
seem to take exceptional scope. This is how the independence constraint mimics the scope account.

### 4.4.2 Proper names, definite descriptions, Cardinal indefinites, and disjunction

In this section, I discuss four types of expressions that are interpreted as independent of the Key. The first two types, proper names and definite expressions, naturally do not induce variation in the scope of distributive quantification. So, the independence constraint, when applied to them, is trivially satisfied. The remaining two types of expressions, namely, cardinal indefinites and disjunction, may induce variation in the scope of distributive quantification. However, the independence constraint forces them to lack co-variation.

A notational note before proceeding. To differentiate between the static semantics used in the narrow scope distributivity analysis and the constraint-based analysis, slightly different symbols are used to translate lexical entries and phrases in the two types of semantics.

**Proper names and definite expressions**  Proper names and definite expressions may not co-vary with distributivity, so the independence constraint has no effect on them. In particular, both types of expressions introduce into an info-state a variable storing a fixed set of values that do not change in the course of evaluating distributive quantification. This can be shown with the definition of the proper name Mingzai in (71-a) and the definition of the definite expression di-lousi ‘the teachers’ in (72-a).

**(71) a.** Mingzai$^y := \lambda P. \exists y \land y = m \land P y$

\[ \text{b. } \max^x(\text{stdts } x) \land \delta_x(\exists y \land y = m \land \text{see } y x) \land \text{ind}_x,y \]

**(72) a.** di lousi$^y$ ‘the teachers’ := $\lambda P. \max^y(\text{teachers } y) \land P y$

\[ \text{b. } \max^x(\text{stdts } x) \land \delta_x(\max^y(\text{teachers } y) \land \text{see } y x) \land \text{ind}_x,y \]

In (71-a), variable introduction introduces the variable $y$ and $y = m$ ensures that $y$ is associated with only one value, namely, $m$. Evaluating $\exists y$ in the scope of the distributive operator $\delta_x$ in (71-b) may give rise to covariation between the students stored in $x$ and the random values stored in $y$, but the next conjunct $y = m$ makes sure to remove all the info-states in which $y$ has any value other than $m$. This essentially ensures that for all values in $x$, the corresponding $y$-value can only be
m, i.e., the individual Mingzai. Similarly, in (72-a), maximization over y makes sure that y stores all the teacher values in the model. So, even if the maximization over y falls inside the scope of the distributive operator $\delta_x$ in (72-b), there is no co-variation between the students and the teachers they saw. Therefore, when a post-saai expression is a proper name or a definite expression, the independence constraint is guaranteed to be satisfied as long as the distributivity contribution is true.

**Cardinal indefinites**  
Cardinal indefinites are translated as dynamic generalized quantifiers, as shown in (73), with $\exists y$ understood as variable introduction.

(73)  
*Jat-go louisi* ‘one teacher’ := $\lambda P. \exists y \land \text{book } y \land \mu_{\text{CARD}} y = 1 \land P y$

A definite plural like *di-hoksaang* ‘the students’ is also treated as a dynamic generalized quantifier, as shown in (74). It introduces into an info-state a d-ref associated with the maximal plural individual that satisfies the common noun restriction. The maximal plural individual is obtained in the manner stated in (75) (see Chapter 2, Definition 17 for how maximization works).

(74)  
*Di-hoksaang* ‘the students’ := $\lambda P. \max^y (\text{stdts } y) \land P y$

(75)  
$G[\max^x(P x)]H = T$ iff $G[\exists x \land P x]H = T$ and there is no $H'$, such that $\bigoplus H'(x) > \bigoplus H(x)$ and $G[\exists x \land P x]H' = T$

To see how the independence constraint contributed by saai constrains the scope interaction between a distributive operator and a cardinal indefinite, let us consider a concrete sentence with three elements: (i) distributive quantification induced by cyunbou or dou, (ii) a cardinal indefinite, and (iii) saai.

As discussed in Section 4.3, the distributive operator and the cardinal indefinite may enter into scope interactions. For our purpose, let us first zoom into the LF in (76-a), in which the distributive operator takes wide scope over the cardinal indefinite. The resulting interpretation is given in (76-b).

(76)  
a.  
the students$^x$ (cyunbou/dou$_x$ ($\lambda u. \text{one book}^y$ ($\lambda u'. \text{read } u' u$)))

b.  
$\max^x (\text{stdts } x) \land \delta_x (\exists y \land \text{book } y \land \mu_{\text{CARD}} y = 1 \land \text{buy } y x)$
Evaluation of such a formula in DPLLM against an input info-state gives rise to a set of info-states. Suppose $H$ and $H'$ in Figure 4.2 are two info-states in the output set. In both info-states, $x$ stores the student values contributed by the Key, and $y$ stores a set of book values contributed by the cardinal indefinite. In addition, each $x$-value bought the corresponding $y$-value as instructed by the assignment functions. The two info-states differ in the values associated with d-ref $y$ introduced by the cardinal indefinite. In $H$, $y$ stores a singleton set of values, while in $H'$, $y$ stores a set of three values.

$$
\begin{array}{ccc}
\hline
 & x & y \\
\hline
h_1 & s1 & bk1 \\
h_2 & s2 & bk1 \\
h_3 & s3 & bk1 \\
\hline
\end{array}
$$

Info-state $H$

$$
\begin{array}{ccc}
 & x & y \\
\hline
h'_1 & s1 & bk1 \\
h'_2 & s2 & bk2 \\
h'_3 & s3 & bk3 \\
\hline
\end{array}
$$

Info-state $H'$

Figure 4.2: $H$ satisfies the independence constraint of $saai$ while $H'$ does not
Now, we are ready to add the contribution of *saai*. *Saai*, together with the d-refs it anaphorically accesses, contributes an independence constraint, i.e., the last conjunct in (77). Recall that evaluation of the first two conjuncts in (77) returns the set of info-states in Figure 4.2. The independence constraint is imposed on this output set. For each info-state in Figure 4.2, we check to see if it satisfies the independence constraint. If it does, the info-state is kept; otherwise, it is discarded. If after evaluating the constraint the final output has at least one info-state, the sentence is true. If the final output is an empty set, the sentence is not false, but undefined. Recall from the definition of “⊼” in Chapter 3 that two propositions connected by “⊼” yields undefinedness if the evaluation of the second proposition yields an empty set. “⊼” is defined again in (100) in this chapter.

\[(77) \quad \text{max}_x (\text{stdts} x) \land \delta_x (\exists y \land \text{book} y \land \mu_{\text{CARD}} y = 1 \land \text{buy} y x) \quad \text{ind}_{x,y} \]

The independence constraint checks the values stored in two d-refs. The first d-ref is the variable storing the values associated with the Key. The second d-ref is the variable introduced inside the distributive scope of the first variable that stores the values introduced by a post-*saai* expression. The constraint requires that the latter be independent of the former. When the independence constraint is imposed on \(H\), it is satisfied. However, when the same constraint is imposed on \(H'\), it is not satisfied. This is because not all \(x\)-values are associated with the same \(y\)-value in \(H'\). As long as there is an info-state like \(H\) in the output that can satisfy the independence constraint, the sentence with *saai* followed by a cardinal indefinite is judged to be true.

We have seen what happens when a cardinal indefinite is interpreted *inside* the scope of distributivity. However, this is not the only possible scope configuration. We need to consider what happens when a cardinal indefinite is interpreted *outside* the scope of distributive quantification, as indicated in the LF in (78-a) and the formula in (78-b).

\[(78) \quad \text{a. the students}^x (\lambda u. \text{one book}^u (\lambda u'. \text{cyunbou/dou}_u) (\text{bought} u' u)).
\quad \text{b. max}_x (\text{stdts} x) \land \exists y \land \text{book} y \land \mu_{\text{CARD}} y = 1 \land \delta_x (\text{buy} y x)\]

It is clear that the wide scope indefinite configuration renders the independence constraint vacuous—since a wide scope indefinite already fails to co-vary with the Key, the independence constraint’s
effect cannot be felt. This seemingly correct analysis, however, turns out to be a problem for indefinite containing bound pronouns, which are allowed to co-vary with the Key. For this reason, I assume that *saai* does not track the independence of a d-ref if it is introduced outside the scope of distributivity. This assumption may seem stipulative at first, but it is closely related to how indefinites receive their interpretation in Brasoveanu and Farkas (2011). In their study, an indefinite chooses its (in)dependence, by tracking and relating to variables introduced by structurally more dominant quantifiers. In this study, a distributive marker chooses the (in)dependence of an indefinite, by tracking and relating to variables dominated by them, i.e., variables introduced in their scope. Since there is no indefinite introduced in the scope of distributivity, there is no need to track its (in)dependence.

Disjunction The same account can be straightforwardly extended to disjunction, which also scopally interacts with distributivity. First, let us define a disjunctive DP in DPILM:

\[
(79) \quad \text{Emma waatze}^{y} \text{ Jane Eyre} \quad \text{‘Emma or Jane Eyre’} := \lambda P. \exists y (y = e \lor y = je) \land P y
\]

A disjunctive DP is a dynamic generalized quantifier, just like an ordinary, non-disjunctive DP. The only difference is that it introduces a d-ref that stores values corresponding to either of the disjoined DPs. For simplicity, I only illustrate the interpretation when disjunction takes narrow scope relative to distributivity, as indicated in the LF in (80-a) and the formula in (80-b).

\[
(80) \quad \begin{align*}
\text{a. the students}^{x} & \quad \text{(cyunbou/dou (} \lambda u. \text{ Emma or}^{y} \text{ Jane Eyre (} \lambda u'. \text{ read } u' u))) \\
\text{b. max}^{x}(\text{stdts } x) & \land \delta_{x}(\exists y (y = e \lor y = je) \land \text{read } y x) \land \text{ind}_{x,y}
\end{align*}
\]

The independence constraint forces the disjunction to be associated with a set of values fixed relative to the Key.

4.4.3 Bare noun phrases

Bare noun phrases in a post-*saai* position have interesting properties. First, unlike cardinal indefinites, bare noun phrases are not marked. More importantly, bare noun phrases are allowed to co-vary with distributivity. In other words, they seem to be immune to the independence constraint. The
following examples help illustrate these two properties:

(81) Di-hoksaang maaï-saai syu.
CL.PL-student buy-SAAI book
‘The students each bought one or more books (and they possibly bought different books).’

(82) Di-jyu hai-saai gong japmin.
CL.PL-fish in-SAAI tank inside
‘The fish each are in a tank (and they are possibly in different tanks).’

Since Carlson (1977a,b), it is widely recognized that ordinary indefinites and bare noun phrases are semantically quite different. So, it is not entirely surprising that bare noun phrases do not pattern like cardinal indefinites in Cantonese with respect to the independence constraint. That said, it is still desirable to have a concrete way to model the differences between bare noun phrases and cardinal indefinites that are responsible for the distinct interactions with the independence constraint. I explore a possibility below.19

I propose that a bare noun phrase is like a proper name for the purpose of the independence constraint. Carlson (1977a,b) has explicitly argued to treat bare plurals in English as proper names of kinds. This analysis has been extended to bare noun phrases in Cantonese by Cheng and Sybesma (1999) and Jiang (2012). Following their analysis, a bare noun phrase like syu ‘book(s)’ in Cantonese can be translated as follows into DPILM:

(83) \textit{book(s)} := \lambda P. \exists y^k \land y^k = \text{book-kind} \land P y^k

A bare noun phrase is very similar to a proper name, with the exception that the d-ref being introduced is a kind-level d-ref. To distinguish between an individual-level d-ref as well as a kind-level d-ref, I notate the latter as \(x^k\). This notation is borrowed from the literature of kinds terms (Carlson 1977a,b, Yang 2001, Dayal 2011a). Carlson (1977b) and Dayal (1999) have demonstrated that pronominal anaphora to kinds are acceptable in English and Hindi. (84) shows that pronominal anaphora to a kind is also possible in Cantonese.

---

19 Another possibility I have not explored is to treat bare noun phrases in Cantonese as semantically incorporated expressions that do not introduce d-refs (see also Dayal 1999, 2011b, Farkas and de Swart 2003, Krifka and Modarresi 2016) for bare singulars in languages whose bare noun phrases may bear number morphology.
The intended interpretation for (84) is for the plural pronoun to refer to squirrels in general rather than to the particular squirrels the speaker saw. The fact that this type of anaphora is possible indicates that the antecedent bare noun phrase songsyu ‘squirrel(s)’ introduces a kind-level d-ref that can be anaphorically accessed later by a plural pronoun.

An immediate merit of the kind-based analysis is that it allows the independence constraint to be satisfied with use of the kind-level d-ref. This is shown in (85). After all, the kind-level d-ref is just like a d-ref storing a proper name, which does not vary with distributivity.

There are two challenges for only recognizing the kind-level contribution of a bare noun phrase. The first one is that an additional mechanism is needed to evaluate a lexical relation involving a kind-level d-ref. In other words, a mechanism is needed for properly interpreting buy $y^k x$ in (85).

The literature has offered a few sortal repair strategies, including the stage predication proposed in Carlson (1977b) and Derived Kind Predication proposed in Chierchia (1998b) (cf. Dayal (2013)). Despite their differences, they both share the effect of turning a kind into a (possibly plural) concrete individual. Since we have seen Chierchia (1998b)’s Derived Kind Predicate in the discussion of bare noun phrases in Section 4.3, I use it as a basis for formulating a special evaluation rule involving an individual-level relation and a kind-level d-ref:

(86) **DKP** (in DPlLM)  
If $R$ is an 2-place relation over individuals and $y^k$ a kind-level variable, then:

\[
G[R y^k x]H = T \text{ iff } G = H \text{ and there is a possibly plural } z \text{ such that } z \in \bigcup \bigoplus [y^k]G \text{ and } \langle \bigoplus [x]G, z \rangle \in I(R)
\]
Formulating DKP as an evaluation rule linking a dynamic proposition to its truth condition essentially makes it a static procedure. The existential quantification over individuals in the instantiation set of the kind is an ordinary static existential quantifier and cannot introduce discourse referents into an info-state. Without a d-ref storing the individuals (in addition to the kind) it is harder to model anaphoric reference to individuals. As shown in (87), anaphoric reference to individuals appears to be possible with a bare noun phrase antecedent in Cantonese. I do not have a satisfactory account for how to model pronominal anaphora involving bare noun phrases. Some proposals targeting this phenomenon have been developed by Dayal (2011b) and Krifka and Modarresi (2016).

(87) Mingzai hai gongjuan gindou siupangjau. Keoidei wan-dak hou hoisam. Mingzai in park see child they play-RES very happy
‘Mingzai saw some children in the park. They were playing happily.’

Before ending this section, I would like to note a difference between indefinites and bare noun phrases that is indicative of their different discourse status (p.c. Simon Charlow and Veneeta Dayal). The difference lies in the so-called uniqueness implication. A hallmark property of anaphora involving indefinites is that it may (but not necessarily) lack a uniqueness implication (Heim 1982, 1990, Kadmon 1987; cf. Evans 1977). An example showing this is given below:

(88) There once was a doctor in London. He was Welsh. (Heim 1982:27)

Since variable introduction (i.e., ∃x) brought about by the indefinite is non-deterministic, the pronoun in the second clause only refers to the non-deterministically introduced doctor value. There is no implication that London only had a doctor, who happened to be Welsh.

By contrast, a bare noun phrase must give rise to a uniqueness implication. For example, the first clause in (89), necessarily makes relevant all the children that Mingzai saw in the park (see also Cohen and Erteschik-Shir (2002) for a similar finding for English bare plurals). The plural pronoun in the second clause then refers to this maximal set of children. As a result, it is implicated that all the children were swinging.

(89) Mingzai hai gongjuan gindou siupangjau. Keoidei wan-gan cincau. Mingzai in park see child they play-PROG swing
‘Mingzai saw children in the park. They were swinging.’
The uniqueness effect is not a special property of the plural pronoun. If a singular pronoun is used, a similar uniqueness implication is still observed. Consider (90). The singular pronoun gives rise to the uniqueness implication that Mingzai only saw one child and the child was playing swing.

(90) Mingzai hai gongjuan gindou siupangjau. Keoi wan-gan cincau.
Mingzai in park see child he play-PROG swing
‘Mingzai saw one or more children in the park. He was playing the swing.’

The uniqueness implication associated with bare noun phrases not only indicates that the discourse status of bare noun phrases is different from that of indefinites, it also points to a plausible way to analyze the felicitous anaphora involving bare noun phrases. In particular, Chierchia (1992) observes that an indefinite in a donkey sentence is ambiguous between a strong (i.e., unique) and weak (i.e., non-unique) reading. He further proposes to distinguish between two types of anaphora. The non-unique anaphora can be derived via standard use of d-refs whereas the unique anaphora can be derived via a E-type strategy not involving the use of d-refs (see also Heim 1990). Given that anaphora involving bare noun phrases pattern like the strong reading in terms of the uniqueness implication, it is possible to extend the E-type strategy formulated for the latter to the former. However, I reserve the precise analysis for another study.

4.4.4 Indefinites with a bound pronoun

Recall from the discussion in the previous section that the presence of saai-distributivity and cardinal indefinites with a bound pronoun results in conflicting scopal requirements. In order to satisfy the narrow-scope distributivity requirement, the indefinite must be interpreted outside the scope of distributivity. However, in order for the pronoun to be properly bound and co-vary with distributivity, it must be inside the scope of distributivity.

Interestingly, the fact that DPlL is plural and dynamic provides a way to resolve this dilemma. Recall that the former trait allows it to represent dependencies using a plurality of assignments while the latter allows it to pass those dependencies from context to context. Now, if we let indefinites with a bound pronoun zigei ‘self’ be interpreted outside the scope of a distributive operator but allow it to introduce its own dependency, then we can account for the fact that a pronoun-containing indefinite may co-vary with the Key. More concretely, a pronoun bound by the Key may induce a dependency
between the Key and the indefinite that contains the pronoun. Since DPIL is a dynamic logic, the dependency is passed down the stream of interpretation. When a distributive operator is evaluated, the dependency induced by the pronoun is preserved. However, since the indefinite is introduced outside the scope of the distributive operator, saai spares it for the independence constraint.\footnote{One may wonder why plain cardinal indefinites and disjunction cannot take advantage of the availability of relational assignment in DPlLM (Lucas Champollion, p.c., Veneeta Dayal, p.c.). I side with van den Berg (1996) in assuming that relational assignment needs to be supported lexically or contextually. I discuss some examples of contextually salient relations in connection with \textit{respective} distributivity in Section 5.3.5 Chapter 5. Note that the assumption that dependencies are introduced only in limited cases is also shared by Henderson (2014).}

As a first step of the illustration, let us define an indefinite containing \textit{zigei} ‘self’ as in (91), following van den Berg (1996)’s relational assignment $\exists y_{Rx}$.

\begin{equation}
\text{(91) } \text{\textit{zigei}$_x$\textit{zungji-ge jat-bun syu}’a book$^y$ they$_x$ like’}
\end{equation}

\begin{equation}
\begin{split}
&:= \lambda P. \exists y_{Rx} \land \text{bk } y \land \mu_{\text{CARD}} y = 1 \land \text{like } y \land P y \\
&\text{iff } G\{y_{Rx}\}H := \bigcup_{a \in G(x)} \{g^{y \rightarrow d} \mid g \in G \land R(d, a) \land d \in D_e\}
\end{split}
\end{equation}

\begin{equation}
\begin{split}
&\text{iff } G(x) = H(x) \land \forall a \in G(x).G|_{x=a}\{y \land R y x\}H|_{x=a}
\end{split}
\end{equation}

Relational assignment is formally defined in (92), which is equivalent to distributively introducing a new variable $y$ by splitting the input info-state along the $x$-dimension and checking that $x$ and $y$ stand in a certain relation $R$. As already pointed out in Section 2.3 of Chapter 2, distributively evaluating variable introduction brings about dependencies between variables. Therefore, relational assignment is technically equivalent to a PCDRT-type variable introduction. The two types of variable introductions only differ conceptually: Relational assignment in DPlL/DPlLM is only used when there is a clear relational cue, such as the presence of a bound pronoun or a relational noun. However, PCDRT generalizes it to all standard cases of variable introduction.\footnote{Given the close resemblance between relational assignment and a PCDRT-type variable introduction, it is not surprising that lexical relations following relational assignment should be evaluated distributively. Concretely, in (91), ‘bk $y \land \mu_{\text{CARD}} y = 1 \land \text{like } y x’ should fall inside the scope of distributive evaluation. Otherwise, the newly introduced relation between $x$ and $y$ will be lost, and the cardinality test will cause a problem.}

There are two sources of support for treating \textit{zigei} as inducing relational assignment. First, although there has not been any study showing that the reflexive pronoun \textit{zigei} may introduce distributivity, its close correlate in Mandarin, i.e., \textit{ziji}, has been argued, by Huang (2002), to introduce distributivity into plural predication, based on the contrast between English reflexive pronouns and...
ziji. According to Huang (2002), plural reflexive pronouns like *themselves* in (93) are compatible with a group-praising scenario (in which the boys as a group praised the group but no individual boy praised himself) and a self-praising scenario (in which every boy praised himself). However, *ziji* in (94) is only compatible with the self-praising scenario. For this reason, Huang (2002) argues that that *ziji* is inherently distributive.  

(93) The boys praised themselves.

(94) Nanhai-men kuaijiang-le ziji.
     boy-PL praise-ASP self
     ‘Each boy praised himself.’

Cantonese *zigei*, when used in a plural predication, as in (95), behaves just like its Mandarin correlate *ziji* in being only compatible with the self-praising scenario.

(95) Di-Nanzai zan-zo zigei.
     boy-PL praise-ASP self
     ‘Each boy praised himself.’

Second, previous studies that use choice functions to analyze indefinites have suggested using skolemization to treat indefinites with bound pronouns (e.g, Kratzer 1998). Although I treat indefinites as dynamic generalized quantifiers rather than choice functions, relational assignment can be seen as the correlate of skolemization in DPIL/DPILM: a variable may be introduced to stand in a certain relation with another variable when there is explicit relational information.  

Combining (91) and (92), we get the definition in (96) for a cardinal indefinite with a bound pronoun:

---

22To express a group praising scenario, Mandarin can generally resort to a regular, non-reflexive pronoun, such as *tamen* ‘they’ (Huang 2002). However, since a regular pronoun cannot be bound by its co-argument, a group-praising has to be expressed with an embedded clause, as in the following example:

(i) Nanhai-men juede tamen hen bang.
     boy-PL think they very good
     ‘They boys think they are very good.’

23The dependency-introducing random assignment defended in Brasoveanu (2007, 2008) can be seen as an unrestricted use of relational assignment.
Figure 4.3: Indefinites with a bound pronoun

(96)  \textbf{a book}^{y} \text{ zigei}_{x} \text{ like} := \lambda P, \exists y_{R}x \land bk y \land \mu_{\text{CARD}}y = 1 \land \text{like} y x \land P y

Now, we are ready to feed this cardinal indefinite back into a sentence with a distributive operator and \textit{saai}. There are two positions for interpreting the cardinal indefinite, inside the scope of the distributive operator or outside of it, as shown in the two LF configurations in Figure 4.3.

If the cardinal indefinite is interpreted \textit{outside} the scope of distributivity (i.e., the second $\delta_{x}$ in (97)), \textit{saai} ignores it for the purpose of the independence constraint. Since the indefinite has a reflexive pronoun inside it, it introduces its own distributivity (the first $\delta_{x}$ in (97)), scoping over the restrictor of the indefinite. This ensures that the variable introduced by the cardinal indefinite is distributively evaluated and hence may co-vary with the Key.

(97)  \text{max}^{x}(\text{stds } x) \land \exists y_{R}x \land bk y \land \mu_{\text{CARD}}y = 1 \land \text{like} y x \land P y \land \delta_{x}(\text{buy } y x)

If the cardinal indefinite signaling relational assignment is interpreted \textit{inside} the scope of distributivity, the distributivity contributed by the indefinite would be vacuous, as it falls inside the scope of another distributive operator targeting the same variable.
4.4.5 Compositional implementation

I assume that distributivity in a sentence with saai is contributed by a distributive operator $\delta_x$ adjoined at the sentence level. The domain of the distributive quantification is determined anaphorically, by the subscripted index. This distributive operator may be realized covertly or overtly as cyunbou or dou. It may enter into scope interactions with other noun phrases in the VP, as shown in Figure 4.4.

$Saai$ is modeled as another sentence-level operator, located immediately above the distributive operator, as shown in 4.5. It takes a dynamic proposition as its argument and imposes an independence constraint on this dynamic proposition, as shown in (99).

\[(98) \quad \max^x(\text{stds } x) \land \delta_x(\exists y \land \text{bk } y \land \mu_{\text{CARD}} y = 1 \land \text{like } y x) \land \text{buy } y x) \land \text{ind}_{x,y}\]

In summary, I have argued that $saai$ imposes an independence constraint on the functional dependency induced by distributive quantification. I have shown how such a constraint can be couched in DPILM, a semantics that represents the functional dependencies engendered by distributive quantification. In the next subsection, I discuss how the analysis proposed in this subsection can be compositionally implemented.
\[ (e \to t) \to t \]

the students

\[ \Lambda u \]

\[ saai_{x,y} \]

\[ \delta x \]

a book \((\lambda u'. \text{ bought } u' t_u)\)

**Figure 4.5:** *Saai* is structurally higher than the distributive operator

\[
\text{(99)} \quad saai_{x,y} := \lambda \phi. \phi \overset{\text{ind}}{\wedge} \] } 

Note that the constraint is imposed on a dynamic proposition with distributivity. Using \( \phi \) as a stand-in for a distributive sentence and \( \psi \) as a stand-in for the independence constraint, we can formulate their combined contribution as follows:

\[
\text{(100)} \quad G[\phi \wedge \psi]H = T \quad \text{if} \quad G[\phi]H = T \quad \& \quad H[\psi]H = T, \quad \text{undefined otherwise.}
\]

### 4.5 Extension: Measurement sensitivity

#### 4.5.1 Value independence vs. structure independence

The independence constraint is formulated in direct opposition to the well-known dependence requirement of distributive numerals and dependent indefinites. For a distributive numeral, it is important that it does not introduce the same value relative to the Key (Farkas 1997, Farkas 2002a,b Henderson 2014, Kuhn 2017); however, for *saai*, it is important that a post-*saai* expression introduces the same value relative to the Key.\(^\text{24}\)

\(^{24}\)While distributive numerals and dependent indefinites are widespread in natural language, *saai*-like distributivity markers are much more limited. Cantonese is the only language I am aware of that has a distributivity marker indicating independence. Interestingly, Cantonese also does not have any distributive marker that forces co-variation, unlike many other languages.
As discussed in Chapter 3, a subclass of distributive numerals, as represented by noun phrases that host binominal each, is sensitive to type of measurement (see also Zhang 2013). For example, binominal each requires its host to contribute an extensive measure function rather than an intensive one.

(101) The boxes are 10 pounds each.

(102) *The drinks are 90 degrees (Fahrenheit) each.

This finding has been used to argue, in Chapter 3, for an analysis in which the dependence condition of binominal each manifests as a monotonicity constraint checked relative to the internal mereological structure of distributivity. This type of dependence is termed structure dependence, to distinguish it from value dependence, which is checked in relation to a functional dependency but not necessarily its internal mereological structure.

One may justly wonder if saai’s independence constraint is one of value independence or structure independence. I address this question in three steps.

• First, I show that when a discourse variable stores ordinary individuals, whether the independence constraint is stated relative to the functional dependency of distributivity or its internal, mereological structure does not make a difference.

• Second, I show that when a discourse variable stores values of degrees, then value dependence and structural dependence yield distinct predictions.

• Third, I draw on Cantonese data to show that measure phrases following saai are required to contribute an intensive measure function rather than an extensive one, in contrast to the requirement of binominal each.

The independence constraint formulated in the previous section is re-stated in (103) with the prefix V indicating that it expresses value independence. It is checked by making reference to $x$ and $y$, the former allows it to associate with distributivity, and the latter allows it to target a potential $y$ in the distributive scope of $x$. 
\[(103) \quad \mathcal{G} \| \text{V-ind}_{x,y} \| H := G = H \land \forall a, b \in G(x) : G|_{x=a}(\bar{y}) = G|_{x=b}(\bar{y})
\]

\[\text{‘y’s value is constant relative to x’s value.’}\]

This constraint can be upgraded so that it is checked in association with the internal structure of distributivity. To do so, we just need to project \(x\) stored in increasingly bigger sub-info-states, and check if the corresponding values stored in \(y\) remain constant in these sub-info-states, as demonstrated in (104) (the prefix \(S\) signals that that the independence is stated in terms of structural independence).

\[(104) \quad \mathcal{G} \| \text{S-ind}_{x,y} \| H := G = H \land \forall X, X' \in G(x) : G|_{x \in X}(\bar{y}) = G|_{x \in X'}(\bar{y})
\]

\[\text{‘y’s value is constant relative to x’s size.’}\]

It is easy to see that (104) entails (103) when \(y\) stores individual values: if \(y\) stores the same individual(s) regardless of \(x\)’s value, then \(y\) stores the same individual(s) regardless of how many values are assigned to \(x\). For concreteness, let me illustrate their equivalence using the info-states in Figure 4.6.

\[\mathcal{G} \quad x \quad y \quad \mathcal{G}' \quad x \quad y
\]

\[
\begin{array}{ccc}
g_1 & \text{child1} & \text{book1} \\
g_2 & \text{child2} & \text{book1} \\
\end{array}
\]

\[
\begin{array}{ccc}
g'_1 & \text{child1} & \text{book1} \\
g'_2 & \text{child2} & \text{book2} \\
\end{array}
\]

\[G|_{x=\text{child1}}(y) = \{\text{book1}\} \quad G'|_{x=\text{child1}}(y) = \{\text{book1}\}
\]

\[G|_{x=\text{child2}}(y) = \{\text{book1}\} \quad G'|_{x=\text{child2}}(y) = \{\text{book2}\}
\]

\[G|_{x \in \{\text{child1, child2}\}}(y) = \{\text{book1}\} \quad G'|_{x \in \{\text{child1, child2}\}}(y) = \{\text{book1, book2}\}
\]

**Figure 4.6:** \(G\) satisfies both value independence and structural independence, while \(G'\) does not satisfy either.

The info-state \(G\) satisfies both value independence and structural independence: it satisfies the former because \(g_1\) and \(g_2\) assign different values to \(x\) but the same value to \(y\), and it satisfies the latter because when \(G\) has more assignments assigning values to \(x\), the values \(G\) assign to \(y\) remain unchanged. The info-state \(G'\) fails to satisfy either value independence or structural independence: it fails the former because \(g'_1\) and \(g'_2\) do not assign the same value to \(y\), and it fails the latter because \(G'\) assigns more values to \(y\) when there are more assignments assigning values to \(x\). Because of this
equivalence, it is impossible to tell apart value independence and structural independence if we only consider variables of individuals.

When the information stored in a discourse variable concerns **degrees** rather than individuals, then it makes a difference whether it is required to be independent at the value level or at the structure level. The reason, as we have seen in Chapter 2, is because depending on the type of measurement that produces a degree, a degree may or may not track the internal mereological structure of the individuals being measured. In the case of an extensive measurement, the corresponding degree tracks the mereological structural of the individual it measures. However, in the case of an intensive measurement, the corresponding degree does not.

Consider the two info-state in Figure 4.7, where \( d \) is a discourse variable storing degrees resulting from an extensive measurement (**volume**), and \( d' \) is one storing degrees from an intensive measurement (**temperature** (in Fahrenheit)). Degrees are modeled as triples, as discussed in Chapter 2.

\[
\begin{array}{|c|c|c|c|}
\hline
G & x & d & d' \\
\hline
\{g_1\} & \text{drink1} & \langle 60\text{oz}, \text{vol}, \text{drink1} \rangle & \langle 60\text{F}, \text{temp}, \text{drink1} \rangle \\
\{g_2\} & \text{drink2} & \langle 60\text{oz}, \text{vol}, \text{drink2} \rangle & \langle 60\text{F}, \text{temp}, \text{drink2} \rangle \\
\hline
\end{array}
\]

\[G_{|x=\text{drink1}}(d) = \langle 60\text{oz}, \text{vol}, \text{drink1} \rangle\quad G_{|x=\text{drink1}}(d) = \langle 60\text{F}, \text{temp}, \text{drink1} \rangle\]
\[G_{|x=\text{drink2}}(d) = \langle 60\text{oz}, \text{vol}, \text{drink2} \rangle\quad G_{|x=\text{drink2}}(d) = \langle 60\text{F}, \text{temp}, \text{drink2} \rangle\]
\[G_{|x=\{\text{drnk1,drnk2}\}}(d) = \langle 12\text{oz}, \text{vol}, \text{drnk1@drnk2} \rangle\quad G_{|x=\{\text{drnk1,drnk2}\}}(d) = \langle 60\text{F}, \text{temp, drnk1@drnk2} \rangle\]

**Figure 4.7:** Extensive vs. intensive measurement

Intuitively, when asked how much collective volume the two drinks have and what collective temperature they have, the answer should be ‘12oz’ and ‘60F’ (or ‘not sure’), respectively. This is because two weights can be added up straightforwardly to a bigger weight but two temperatures cannot be added up straightforwardly to a bigger temperature. As already discussed in Chapter 2, this intuition can be captured by modeling degree projection not using sets, just operations on sets.

We can check that \( x \) and \( d \) in the info-state in Figure 4.6 satisfy value dependence:

\[(105)\quad \text{V-ind}_{x,d} := \forall a, b \in G(x) : G|_{x=a}^{i=1}(d) = G|_{x=b}^{i=1}(d)\]
\[
\text{‘}d\text{’s value on the first coordinate is fixed/constant relative to } x\text{’s value.}\]
However, they do not satisfy structure independence:

\[(106) \quad S\text{-ind}_{x,d} := \forall X, X' \subseteq G(x) : G|_{x \in X}^{i=1}(d) = G|_{x \in X'}^{i=1}(d)\]

‘d’s value on the first coordinate is fixed/constant relative to different sizes of x.’

This is because although different assignments in G assign the same value to d, collectively they assign a different value (a larger value, to be more precise) to d.

The situation is reversed with intensive measurement. Consider the relationship between x and d’ in the info-state in Figure 4.7. They satisfy both value independence (107) and structure independence (108).

\[(107) \quad V\text{-ind}_{x,d} := \forall a, b \in G(x) : G|_{x = a}^{i=1}(d) = G|_{x = b}^{i=1}(d)\]

‘d’s value on the first coordinate is fixed/constant relative to x’s value.’

\[(108) \quad S\text{-ind}_{x,d} := \forall X, X' \subseteq G(x) : G|_{x \in X}^{i=1}(d) = G|_{x \in X'}^{i=1}(d)\]

‘d’s value on the first coordinate is fixed/constant relative to different value sizes of x.’

What the discussion in this subsection amounts to is that only by testing discourse variables storing degrees do we stand a chance for testing whether the independence constraint of saai is one of value independence or structural independence. This is because degrees is the sort of information that may receive the same value from every assignment but get a different value when more than one assignment is considered. An immediate question, however, is whether or not it is reasonable to assume that measure phrases introduce discourse variables over degrees. This next subsection is devoted to demonstrating that measure phrases do make dynamic contribution of degrees.

4.5.2 The dynamics of degrees

To test the dynamic contribution of measure phrases, we can test if they license anaphora involving degrees. Noun phrases have been argued to make dynamic contributions because they license anaphora involving individuals. The following data show that ordinary indefinites and proper names support anaphoric pronouns:
Measure phrases in Cantonese also support anaphoric reference to measurement:

(111) Jyugwo nei sik jat-bong⁴ jeok, nei jat-ding jiu sik-faan gam.do,d coi.  
if you eat one-CL meat you necessarily must eat-ALSO that.much vegetables  
‘If you eat a pound of meat, you must eat that much vegetables.’

(112) Jyugwo nei zou jat-go zung⁴ wundong, nei jau jiu tau-faan gam.leoi,d.  
if you do one-CL hour exercise you then must rest-ALSO that.long  
‘If you exercise for an hour, you need to rest that long.’

(113) Jyugwo nei haang saam-gonglei⁴ heoi hokhaau, nei jaujuu haang-FAAN gam.juan,d  
If you walk three-kilometer to school, you need walk-ALSO that.far  
faan ukkei return home  
‘If you walked three kilometers to go to school, you have to walk that far to get back home.’

(114) Mingzai sik-zo leong-go⁴ pingguo. Siufan dou sik-zo gam.duo,d pingguo.  
Mingzai eat-ASP two-CL apple. Siufan also eat-ASP that.much apple  
‘Mingzai ate two apples. Siufan also ate that many apples.’

(115) Mingzai diu-zo saam-jat⁵ jyu. Siufan dou diu-zo gam.leoi,d jyu.  
Mingzai fish-ASP three-day fish Siufan also fish-ZO that.long fish  
‘Mingzai fished for three days. Siufan also fished for that long.’

(116) Mingzai paau-zo saam gonglei⁴. Siufan dou paau-zo gam.juan,d.  
Mingzai run-ASP three km Sifan also run-ASP that.far  
‘Mingzai ran three kilometers. Sifan also ran that much.’

Given the parallelism between degrees and individuals with respect to their ability to support anaphora,
we have reasons to believe that measure phrases and ordinary noun phrases both make dynamic contributions. Consequently, we also have reasons to suspect that the independence constraint of *saai* may interact with measure phrases. In the next subsection, I provide data from Cantonese showing that the dynamic effects of measurement indeed interact with *saai*.

### 4.5.3 Interactions of *saai* and measure phrases

To ease into the interaction of *saai* and measure phrases, first observe that that extensive measure phrases may occur following a verb and a verbal suffix, as shown in (117).

(117) Di-gunzong haam-zo jat-ci.  
`CL.PL-audience cry-ASP one-time`  
‘The audience cried once.’

(118) Di-bengjan faan-zo leong-go jung gaau  
`CL.PL-patients take-ASP two-CL hour sleep`  
‘The patients slept for two hours.’

Post-verbal extensive measure phrases may fall inside the scope of a distributive operator, such as *cyunbou* (*dou*):

(119) Di-gunzong cyunbou (dou) haam-zo jat-ci.  
`CL.PL-audience all DOU cry-ASP one.time`  
‘The audience all cried once.’

(120) Di-bengjan cyunbou (dou) faan-zo leong-go jung gaau  
`CL.PL-patients all DOU take-ASP two-CL hour sleep`  
‘The patients all slept for two hours.’

However, when the distributive marker is replaced by *saai*, the sentences become ungrammatical,

---

Note that the above data in this section only suggest that degrees can be referred to. To establish that degree anaphora show hallmark properties of *donkey anaphora*, i.e., anaphora with an antecedent introduced into the discourse non-deterministically, we need data of the following kind:

(i) Mingzai sik-zo gei-wan faan. Wo dou sik-zo gam.do faan.  
Mingzai eat-ASP several-bowl rice. I also eat-ASP that.much rice  
‘Mingzai ate several bowls of rice. I also ate the same amount of rice.’

Here, the quantity of rice that Mingzai ate is not fully specified, so the context is compatible with a variety of quantities, which can be introduced non-deterministically and capable of binding the pronoun. I thank Simon Charlow (p.c.) for prompting me to clarify this point.
unless the measure phrases are removed:

(121) Di-gunzong\textsuperscript{x} haam-saai\textsubscript{x,d} (*jat-ci\textsuperscript{d}).  
CL.PL-audience cry-SAAI one-time  
‘The audience all cried once.’

(122) Di-bengjan\textsuperscript{x} faan-saai\textsubscript{x,d} (*leong-go jung\textsuperscript{d}) gaau  
CL.PL-patients take-SAAI two-CL hour sleep  
‘The patients all slept for two hours.’

This is unexpected if the independence constraint is stated at the value level. This is because the first coordinate of each $d$-value remains constant relative to increasingly more $x$-values in these two examples. However, once we take the independence constraint to be stated at the structural level, then the ungrammaticality falls out: the first coordinate of $d$ indeed changes (i.e., increases) with more $x$ values.

What about intensive measure phrases associated with an intensive measurement? Ideally, we should show that saai is fully compatible with intensive measure phrases and use this fact to further support that the independence constraint is stated at a structural level. However, there are a few complications in making this argument.

First, let us first establish some cases of intensive measurement in Cantonese. The following examples show that intensive measure phrases typically occur as modifiers inside noun phrases (the relevant noun phrases are enclosed in “[...]”):

(123) Siufan maai-zo [18K-ge gaaizi].  
Siufaan buy-ASP 18K-MOD ring  
‘Siufan bought (one or more) 18-Karat gold ring(s).’

(124) Mingzai jam-zo [sei-dou-ge binseoi].  
Mingzai drink-ASP 4-DEGREE-MOD icy.water  
‘Mingzai drank 4-degree icy water.’

When zo is replaced by saai, the examples are indeed still fully acceptable:

(125) Di-guhaak maai-saai [18K-ge gaaizi].  
CL-customers buy-SAAI 18K-MOD ring  
‘The customers all bought (one or more) 18-Karat ring(s).’
(126) Di-siupangjau jam-zo [sei-dou-ge binseoi].
   CL-children drink-ASP 4-degree-MOD icy.water
   ‘The children all drank 4-degree icy water.’

However, we cannot directly conclude, based on the above data, that intensive measurement supports *saai*, because the relevant noun phrases are also bare noun phrases. It is possible that they contribute kind terms and the intensive measure phrases only serve to modify the kind terms. Given that we have already seen that bare noun phrases may satisfy the independence constraint of *saai* by contributing a kind-level d-ref, we cannot be entirely sure that the degree information plays a decisive role.

To draw a more convincing conclusion, it is necessary to consider intensive measure phrases that do not serve as a modifier of a bare noun phrase. Fortunately, Cantonese has a class of measure predicates that may directly take a measure phrase as its argument. These measure predicates may take an extensive measure phrase, as in the case of *sau* ‘lose, be thin’, which takes an extensive measure phrase *m-bong* ‘5 pounds’ in (127). Other measure predicates, such as *siu-dou* ‘heat to’ in (128), may take an intensive measure phrase, such as *100-du* ‘100 degrees’.

(127) Leidi wuijyun cyunbou dou *sau-zo* m-bong la.
    these member all dou lose-ASP five-pounds SFP
    ‘These members all lost five pounds.’

(128) Leidi sui cyunbou dou *siu-dou* 100-du la.
    these water all dou heat-to 100-degrees SFP
    ‘These units of water have all been heated to 100 degrees.’

When *saai* attaches to these measure predicates, there is a contrast between an extensive measure phrase and an intensive measure phrase:

(129) *Leidi wuijyun cyunbou dou sou-saai m-bong la.*
    these member all dou lose-SAai five-pounds SFP
    ‘These members all lost five pounds, so they can go to the next class.’

(130) Leidi sui cyunbou dou *siu-dou-saai* 100-du la.
    these water all dou heat-TO-SAai 100-degrees SFP
    ‘These waters have all been heated to 100 degrees.’

26 Not all speakers perceive the contrast. In particular, speakers who judge cardinal indefinites following *saai* to be unacceptable even under a specific interpretation do not accept either sentence.
To summarize, I have shown that *saai* can be followed by intensive measure phrases but not extensive measure phrases, indicating that it exhibits measurement-sensitivity, just like binominal *each*. For this reason, I have argued that the independence constraint of *saai* should be understood as requiring the independence of a d-ref relative to the internal mereological structure of the functional dependency of the Key.

### 4.6 Comparison with the monotonic measurement constraint

The independence constraint of *saai* is both similar to and different from the monotonic measurement constraint of binominal *each*. The crucial feature they share lies in their reference to the internal mereological structure of a distributivity dependency. This feature explains why both distributive markers are sensitive to type of measurement (albeit in distinct ways). As already pointed out in Chapter 2, to model measurement sensitivity it is necessary to make reference to the mereological structure of a distributivity dependency.

Although Cantonese *saai* and English binominal *each* both make reference to the mereological structure of a dependency, the constraints they impose on it differ in important ways. First, the monotonic measurement constraint of binominal *each* requires that a targeted expression (i.e., the host) tracks the size of the Key whereas the independence constraint of Cantonese *saai* requires that a targeted expression (i.e., a post-*saai* noun phrase) not to track the size of the Key. In addition, the monotonic measurement constraint requires access to a measure function provided by the target expression to construct degrees whereas the independence constraint has no requirement on the presence of a measure function in the target expression. Due to these differences, *saai* and binominal *each* require different types of expressions to satisfy their respective constraints. For the reader’s reference, I summarize below how different types of expressions fare with *saai* and binominal *each* and where to find the relevant discussions. The summary is presented in Table 4.1 at the end of this section.

Counting quantifiers like *two books* in English and *jat-bun syu* ‘one book’ in Cantonese (referred to as a cardinal indefinite) can satisfy both the independence constraint (see Section 4.4.2 of this chapter) and the monotonic measurement constraint (see Section 3.4.3 of Chapter 3) but are subject to distinct interpretive requirements. Assuming that a counting quantifier contributes both
an individual d-ref and a degree d-ref, the independence constraint requires them both to be structurally independent. What this means is that the individual d-ref should lack co-variation and the degree d-ref should store a set of degree names that do not depend on the size of the Key. Since the degree d-ref stores the measurement information of the individual d-ref, when the individual d-ref lacks co-variation, the degree d-ref will remain constant regardless of the size of the Key. This is how the two variables work together to satisfy the independence constraint. The situation is very different with the monotonic measurement constraint, which requires the individual d-ref to exhibit co-variation. Although binominal each is not posited to access the degree d-ref anaphorically, it accesses the same measurement information by compositionally retrieving a measure function from its host and applying the measure function to the individual d-ref provided by the host.

Measure phrases can satisfy both constraints, too, for they contribute a measure function to the monotonic measurement constraint (Section 3.4.3 of Chapter 3) and a degree variable to the independence constraint (Section 4.5 of this chapter). However, it must be made clear that it is extensive measure phrases that satisfy the monotonic measurement constraint but intensive measure phrases that satisfy the independence constraint.

Quantifiers that are hypothesized to lack an appropriate measurement component, such as some NPs, few NPs and most NPs, do not support the monotonic measurement constraint (see Section 3.3.2 of Chapter 3). However, the Cantonese correlates of these quantifiers can support the independence constraint, as long as they receive a specific interpretation (see Section 4.4.2 and footnote 3 of this chapter). In other words, they pattern like counting quantifiers with respect to the independence constraint. These quantifiers are acceptable because the independence constraint may, but need not, make use of measurement information, unlike binominal each.

Relatedly, disjunction such as John or Mary does not support the monotonic measurement constraint for its lack of a measure function. However, its correlate in Cantonese may satisfy the independence constraint as long as disjunction does not co-vary with distributivity (see Section 4.4.2 of this chapter).

Proper names such as John in English and Mingzai in Cantonese and definite expressions such as the (two) books in English and go-bun syu ‘that book’ in Cantonese satisfy the independence constraint (see Section 4.4.2 of this chapter) but not the monotonic measurement constraint (see Section 3.2.2 of Chapter 3). This is because these expressions do not co-vary with distributivity. The lack
Table 4.1: Expressions (in the distributed share) that support and do not support Cantonese *saai* and English binominal *each*

<table>
<thead>
<tr>
<th>Expression</th>
<th><em>Saai</em></th>
<th><em>Each</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting quantifier</td>
<td>✓ (no co-variation)</td>
<td>✓ (co-variation)</td>
</tr>
<tr>
<td>Extensive measure phrases</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>Intensive measure phrases</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Non-counting quantifiers</td>
<td>✓ (no co-variation)</td>
<td>✗</td>
</tr>
<tr>
<td>Disjunction</td>
<td>✓ (no co-variation)</td>
<td>✗</td>
</tr>
<tr>
<td>Proper names</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Definite expressions</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Bare noun phrases</td>
<td>✓</td>
<td>✗</td>
</tr>
</tbody>
</table>

Co-variation allows a definite expression to readily satisfy the independence constraint. However, since co-variation is a necessary (but not sufficient) condition for the monotonic measurement constraint, the lack of co-variation makes it impossible for these expressions to satisfy the monotonic measurement constraint.27

Lastly, bare noun phrases do not satisfy the monotonic measurement constraint but satisfy the independence constraint. Bare noun phrases are modeled as proper names of kinds. As such, they pattern like proper names for their inability to satisfy the monotonic measurement constraint (see Section 3.2.2 in Chapter 3 for the data) and their ability to satisfy the independence constraint (see Section 4.4.3 of this chapter).

4.7 Remaining issues

4.7.1 Flexible Key

There are a few properties of *saai* that have not been addressed in this dissertation. I document them in this section to facilitate future research. To begin with, events in the scope of distributivity are not required to lack co-variation with the Key. More concretely, in a sentence like (131), there is no

27Relatedly, since the cumulative reading does not exhibit co-variation, it also does not support the monotonic measurement constraint, as pointed out in Chapter section 2.3 of Chapter 2.
requirement that there is only a single leaving event.

(131)  Di-hoksaang zau-saai.
       CL.PL-student leave-SAAI
       ‘The students each left.’

If (131) has the LF in (132-a) and the interpretation in (132-b), the independence constraint predicts that the event d-ref introduced inside the scope of the distributive operator should only have a single value relative to different values in the Key. In other words, all the students left in the same leaving event. However, the empirical fact is that there is no requirement on what values the event d-ref may be associated with.

(132)  a.  the students\(^x\) (cyunbou/dou\(_x\) (\(\lambda u.\) leave \(u\)))

       b.  max\(^x\)(stdts \(x\)) \(\land\) \(\delta_x(\exists e \land\) leave \(e \land ag e = x) \land\) ind\(_x\),e

There are a few plausible explanations. First, the constraint may be ruled out for pragmatic reasons. If the independence constraint in (132-b) took effect, it would lead to a contradiction. As pointed out in Carlson (1998) and subsequent studies, events with distinct participants are distinct events.\(^{28}\)

In (132-b), the events stored in \(e\) each has a different student as its agent. For this reason, the events cannot be independent of the size of the Key—the number of events depend precisely on the number of agents found in the Key. Since imposing the independence constraint on an event variable in the scope of a distributive operator always leads to a contradiction, a pragmatic mechanism may prevent the constraint from applying to an event variable. Alternatively, saai may be only sensitive to d-refs introduced by noun phrases (including individuals and degrees) and event d-refs are simply ignored. Finally, saai’s surface position may play a more prominent role in determining what is tracked for the independence constraint. For example, saai may only track the d-refs introduced by a post-saai expression. Since verbal predicates linearly precede saai, the event variables presumably introduced by them are spared.

\(^{28}\)I thank Robert Henderson (p.c.) for pointing this out to me.
Another interesting feature of distributivity with *saai* is that *saai* can signal distributive quantification over any grammatical position (Lee 1994, Lee 2012). For example, the distributive quantification is over the plural subject in (133-a). As shown in the interpretation, the subject noun phrase introduces the individuals being distributed over. However, when the plural noun phrase is placed in the object position, as in (134-a), *saai* can readily induce distributive quantification over the object, as shown in the interpretation in (134-b). There is no need to move *saai* to a different position to indicate the change of the Key from the subject to the object.

(133) a. Di-hoksaang gin-saai Miss Cheung.
   CL.PL-student see-SAAI Miss Cheung
   ‘Each student saw Miss Cheung.’
   b. max*(stdts x) ∧ δx(∃ y ∧ y = miss-cheung ∧ saw y x) ⊭ indx,y

(134) a. Miss Cheung gin-saai di-hoksaang.
   Miss Cheung see-SAAI CL.PL-student
   ‘Miss Cheung saw each student.’
   b. max*(stdts x) ∧ δx(∃ y ∧ y = miss-cheung ∧ saw x y) ⊭ indx,y

Other distributive markers, such as *cyunbou* and *dou*, are more restricted, as they can only signal distributive quantification over an expression they follow. It is possible that the independence constraint has a connection with *saai*’s flexibility with the Key. I reserve this potential connection for future research.

4.8 Conclusion

In this chapter, I have used *saai* as a case study to show that there are distributive markers that require a lack of co-variation between expressions in the distributed share and the Key. Although at first glance, *saai* seems to be an entirely different beast from markers of distributive numerals, I have argued that their semantics in fact have a common core. Both types of distributive markers impose constraints on the functional dependencies arising from distributive quantification. The major difference is that *saai* requires a functional dependency to lack co-variation, while markers of distributive numerals require a functional dependency to exhibit co-variation.

In addition, I have shown that the independence requirement of *saai*, just like the dependence
requirement of *each*, should both be understood at the structural level, rather than at the value level.
Mandarin ge and other uses of each: generalized monotonicity

5.1 Introduction

This chapter is primarily devoted to the Mandarin distributive marker ge. However, the conclusions drawn from the investigation of ge in Section 5.2 and Section 5.3 can be extended to understand determiner and adverbial each, which I take up in Section 5.6.

Although ge is widely regarded as a distributive operator (Lin 1998b, Lee et al. 2009a), it exhibits two families of properties that pose a challenge to this view. The first family of properties, reported in Section 5.2.1, suggests that ge lacks its own distributive force, as it may co-occur with distributive markers that encode different types of distributivity. The second family of properties, reported in Section 5.2.2, shows that ge, like English binominal each and Cantonese suffix saai, imposes restrictions on what expressions can show up in the Share and how they are interpreted (Lin 1998b, Soh 2005, Lee et al. 2009a, Tsai 2009, Li and Law 2016). It is also shown that although these restrictions do not fully overlap with binominal each’s licensing conditions, ge and binominal each share important similarities that warrant a unified analysis.

Based on these properties, I propose (in Section 5.3) that ge, like binominal each, is a marker
requiring a monotonic mapping from one structure (i.e., the mereological structure of the plurality provided by the Key) to another structure (i.e., the mereological structure of the plurality obtained by distributively evaluating a relevant part of the Share). This requirement is formulated as a monotonicity constraint, which accesses the relevant mereological structures using d-refs and quantificational subordination (van den Berg 1996, Brasoveanu 2008, Brasoveanu and Farkas 2011; see also Roberts 1989, Krifka 1996a). However, unlike binominal each, which is only acceptable when a monotonic mapping is from a structure consisting of individuals to a structure consisting of degrees, ge is compatible with more than one type of monotonic mapping. In particular, ge allows mappings from individuals to degrees as well as mappings from individuals to individuals. The cross-categorial nature of ge’s monotonicity constraint is formally captured by allowing ge to use both degree d-refs and individual d-refs to retrieve the mereological structures for building the monotonicity constraint.

In Section 5.6 I show that the monotonicity constraint can be further generalized to model the so called ‘event differentiation’ condition of determiner each, which is first discussed in detail by Tunstall (1998) (see also Vendler 1962, Brasoveanu and Dotlačil 2015). I show that by allowing a monotonicity constraint to make use of event d-refs and pragmatically available thematic functions, the event differentiation condition is just a special case of the monotonicity constraint.

In Section 5.5 I discuss some intriguing locality conditions of ge not taken up in this study. I conclude this chapter in Section 5.7.

5.2 The distribution of ge

In this section, I discuss two puzzling properties of ge: its co-occurrence with other distributive markers, as well as the licensing requirements it imposes on the Share. These properties are collectively taken to challenge the standard view that ge is a distributive marker (Lin 1998b, Lee et al. 2009a; cf. Tsai 2009).

5.2.1 Co-occurrence with other distributive markers

Ge co-occurs with two types of distributive markers, those marking ordinary distributivity, i.e., the type of distributivity canonically associated with the Mandarin adverb dou or the English adverb
each, and those marking ‘respective’ distributivity, i.e., distributivity associated with the Mandarin adverb fenbie ‘alternately or respectively’ or the English adverb respectively. I discuss these two types of distributivity and their compatibility with ge below.

**Compatibility with dou**

The first piece of evidence that calls into question ge’s role as a distributive operator comes from the fact that ge may co-occur with another distributive marker, as noted in Tsai (2009), Lee et al. (2009b), and Li and Law (2016). An example from Tsai (2009) is given in (1).

(1) Tamen dou ge mai-le yi-ben shu.
    they DOU GE buy-ASP one-CL book
    ‘They each bought a book.’ (Tsai 2009:162)

If dou is a distributive operator, as argued in Cheng (1995) and Lin (1998a), or a distributive universal quantifier, as argued in Lee (1986), it begs the question what role ge plays. If ge is also a distributive operator contributing distributive quantification, then its distributivity should at least trigger some type of vacuous quantification effect, which is presumably responsible for the ill-formedness of (2) in English.

(2) *Every boy each left.

Of course, the acceptability of data points like (1) must be used with caution, as many studies have taken dou to not be a distributive operator, but an operator with a semantics closer to even in English (Chen 2008, Xiang 2008, Liu 2016, Xiang 2016). Given the functional multiplicity of dou and ge’s rather stable connection with distributivity, it could be argued that ge is the distributive operator and dou is merely there to perform another grammatical function. For this reason, it is useful to look at another type of distributive marker that can co-occur with ge. This is done in the next subsection.

**Compatibility with ‘respective’ distributivity**

When two (or more) coordinated phrases with an equal number of conjuncts co-occur in a sentence, a special type of distributive interpretation arises, as exemplified by (3).
With this type of distributivity, each conjunct in the coordinated subject in (3) is paired with a conjunct in the coordinated verb phrase and vice versa. Since this type of distributivity can be optionally marked in English with the adverb *respectively*, I refer to it as ‘respective distributivity’ in this study. According to the analysis advanced in Gawron and Kehler (2004) (see also Kubota and Levine 2016), this type of distributivity involves a covert distributive operator $\text{RESP}_f$ (I will return to this operator in Section 5.3.5). This operator takes two pluralities at a time, break them into parts, pairs the parts using a pragmatically available sequencing function $f$, and performs a pair-wise evaluation facilitated by $f$.

Although $\text{RESP}_f$ is a covert operator, there are lexical items such as English *respectively* that can be added to force respective distributivity. In Mandarin, the adverb *fenbie* ‘separately, respectively’ can be used for this purpose. When there is only one coordinated phrase, *fenbie* is interpreted as *separately* (see Lasersohn 1995, 1998 for English adverb *alternately*, which has a related interpretation):

(4) Zilu he Ziyou fenbie chang-le ge.
Zilu and Ziyou separately sing-ASP song
‘Zilu and Ziyou sang a song separately.’

(5) Zilu fenbie chang-le ge he tiao-le wu.
Zilu separately sing-ASP song and jump-ASP dance
‘Zilu sang and danced separately.’

When there is more than one coordinated phrase, *fenbie* serves to mark respective distributivity, as shown in (6).\(^1\)

(6) Zilu he Ziyou fenbie change-le ge he tiao-le wu.
Zilu and Ziyou respectively sing-ASP song and dance-ASP dance
‘Zilu and Ziyou sang and danced, respectively.’

\(^1\)The *separately* interpretation is still available but less preferred.
Since the respective distributive operator $\text{RESP}_f$ is incompatible with the ordinary distributive operator contributed by $\text{dou}$, the respective interpretation vanishes when $\text{dou}$ co-occurs with $\text{fenbie}$.\footnote{The incompatibility is due to the fact that $\text{dou}$ breaks down one coordinated phrase, i.e., the coordinated subject, for establishing distributive quantification. $\text{RESP}_f$, which falls inside the scope of $\text{dou}$’s distributive quantification, only has access to one plurality, i.e., the plurality contributed by the coordinated VP. The first plurality is no longer available since it is inside the scope of $\text{dou}$. Since the respective distributivity interpretation must be built with two pluralities, it is hence not available inside the scope of $\text{dou}$.}

As shown in (7), when the two co-occur $\text{fenbie}$ can only take up the separately interpretation.

\example{Zilu he Ziyou (dou) fenbie (dou) change-le ge he tiao-le wu}{
Zilu and Ziyou DOU separately DOU sing-ASP song and dance-ASP dance
‘Each of Zilu and Ziyou sang and danced separately.’}

However, when $\text{ge}$ and $\text{fenbie}$ co-occur, as shown in (8), the respective distributivity interpretation is still available. In other words, $\text{ge}$, unlike $\text{dou}$, does not introduce a distributive operator that would interfere with respective distributivity.

\example{Zilu he Ziyou (ge) fenbie (ge) change-le ge he tiao-le wu.}{
Zilu and Ziyou GE respectively GE sing-ASP song and dance-ASP dance
‘Zilu and Ziyou sang and danced, respectively.’}

There is more than one way to interpret the behavior of $\text{ge}$. We may take $\text{ge}$ to contribute a distributive operator, as do in Lin (1998b) and Lee et al. (2009a), and devise a mechanism to deactivate the operator when another one is present. Alternatively, we may take $\text{ge}$ to embody the respective distributive operator $\text{RESP}_f$ and reduce all distributivity with $\text{ge}$ to respective distributivity, a line of research explored in Tsai (2009). Lastly, we may take $\text{ge}$ to not contribute a distributive operator at all. On this view, $\text{ge}$ is compatible with different types of distributivity because it does not contribute a distributive operator of its own. However, for the last view to have any traction, it is necessary to clarify a few questions: if $\text{ge}$ does not contribute a distributive operator, why does it always show up in conjunction with distributivity? What functions does it serve in a distributively interpreted sentence?

To answer these important questions, I turn to another set of $\text{ge}$’s distributional properties in the next subsection. These properties show that $\text{ge}$ is not compatible with just any sentence with a distributive interpretation. In particular, $\text{ge}$’s presence needs to be licensed by certain morphosyntactic and interpretive properties of the expressions in the Share.
5.2.2 Ge’s licensing requirements

In this section, I discuss ge’s licensing requirements. The term ‘licensing’ is borrowed pre-theoretically from the literature on negative polarity items to describe the fact that ge is only felicitously used when certain factors are met. I discuss the range of conditions that licenses ge and the generalizations we can draw from these licensing conditions. Many of the licensing conditions have been reported in the literature, in Kung (1993), Lin (1998b), Soh (2005), Lee et al. (2009a), Tsai (2009), and Li and Law (2016). Moreover, based on these licensing conditions, Lin (1998b), Lee et al. (2009a), Tsai (2009), and Li and Law (2016) have developed analyses of ge that are closely related to the generalized monotonicity constraint developed in later parts of this study. I will offer a review of these studies in Section 5.4, after developing my own analysis in Section 5.3.

A point of clarification. To highlight ge’s licensing requirements, I use an unmarked distributive marker, namely, dou, as a reference point. For the purpose of this study, I follow Lee (1986), Cheng (1995), and Lin (1998a) in treating dou as a distributive marker. However, I do not rule out the possibility that dou is merely compatible with distributivity, rather than contributing distributivity (Chen 2008, Xiang 2008, Liu 2016, Xiang 2016). If dou turns out to not be a distributive marker, the differences between ge and dou will be attributed to the differences between ge and whatever mechanism gives rise to a distributive interpretation, such as the use of a null distributive operator.

Licensing by counting quantifiers and measure phrases

The first category of expressions that licenses ge is counting quantifiers (Kung 1993, Lin 1998b, Tsai 2009, Lee et al. 2009a, Li and Law 2016). Observe that while a counting quantifier is obligatory when distributivity is marked by ge, as shown in (9), it is optional when distributivity is marked by dou, as shown in (10).

(9) Zhe-xie haizi ge kan-le *(liang-chu) dianyin.
    this-CL.PL child GE see-ASP two-CL movie
    ‘These children saw two movies each.’

(10) Zhe-xie haizi dou kan-le (liang-chu) dianyin.
    these-CL child DOU see-ASP two-CL movie
    ‘These children each saw two movies.’
Recall, from Chapter 3, that counting quantifiers are also required to license distributivity with binominal \textit{each}:

(11) The girls saw *(two) movies each.

The parallelism between binominal \textit{each} and \textit{ge} regarding licensing by counting quantifiers goes beyond the morphosyntactic requirement of a counting quantifier in the Share. In fact, these two distributive markers also share an important interpretive property, namely, that a counting quantifier in their Share must co-vary with the Key. For this reason, the wide scope, specific interpretation of a counting quantifier is unacceptable:

(12) ??The girls saw two movies each, namely, \textit{Avatar} and \textit{Ice Age}.

(13) ??Zhe-xie haizi ge kan-le liang-chu dianyin, jiushi Afanda he Bingheshiji
    these-CL child GE see-ASP two-CL movie, namely Avatar and Ice.Age
    ‘The children saw two movies each, namely, Avatar and Ice Age.’

Unsurprisingly, \textit{dou} is not subject to the same requirement, as evidenced by the acceptability of (14).

(14) Zhe-xie haizi dou kan-le liang-chu dianyin, jiushi Afanda he Bingheshiji
    these-CL child GE see-ASP two-CL movie, namely Avatar and Ice.Age
    ‘The children each saw two movies, namely, Avatar and Ice Age.’

Closely related to counting quantifiers are \textbf{measure phrases}. Recall that measure phrases with an extensive measure function can license the use of binominal \textit{each} but those with an intensive measure function cannot. This contrast is shown in (15-a) and (15-b). The measure phrase in the former provides an extensive measure function, i.e., \textit{volume} (in ounce), but the measure phrase in the latter provides an intensive measure function, i.e., \textit{temperature}.

(15) a. The drinks are six ounces each.
    b. ??The drinks are sixty degrees each.

The same contrast holds for \textit{ge}. The extensive measure phrase with the measure function \textit{volume}
(in mililiter) in (16-a) licenses ge but the intensive measure phrase with the measure function \textit{temperature} in (16-b) does not.

(16)  
\begin{enumerate}[a.]
\item Zhe-xie yinliao ge (you) 200 haosheng.  
\textit{These drinks are 200 ml each.}  
\item Zhe-xie yinliao ge (you) 60 du.  
\textit{Each of these drinks is 60 degrees.}  
\end{enumerate}

The same contrast is not observed when distributivity is introduced by \textit{dou}—both extensive measurement and intensive measurement are compatible with \textit{dou}:

(17)  
\begin{enumerate}[a.]
\item Zhe-xie yinliao dou (you) 200 haosheng.  
\textit{These drinks are 200 ml each.}  
\item Zhe-xie yinliao dou (you) 60 du.  
\textit{Each of these drinks is 60 degrees.}  
\end{enumerate}

Based on the data presented so far, it may seem to the reader that ge is a Chinese variant of binominal \textit{each}: It can be licensed by expressions like counting quantifiers and extensive measure phrases because they have a measure function component that ge needs in order to construct a monotonicity constraint. However, simply equating ge with binominal \textit{each} is premature, as ge differs from binominal \textit{each} in two important respects.

First, ge does not need to be adjacent to the expression that licenses it. (18) shows that ge can be separated from the counting quantifier that licenses it by a main verb (and an aspectual suffix). (19) and (20) show that there can also be additional adverbials between ge and the counting quantifier.

(18)  
\textit{These children each saw two movies.}

(19)  
\textit{These children each saw two movies on Sunday.}
(20) Zhe-xie haizi zai xinqitian toutou-de kan-le liang-chu dianyin.
these-CL child GE on Sunday sneakily see-ASP two-CL movie
‘These children each saw two movies on Sunday sneakily.’

The lack of an adjacency requirement stands in contrast to the distribution of binominal each, which, according to Stowell (2013), has a strong preference to immediately follow the counting quantifier that hosts it. The examples below serve to show the adjacency requirement of binominal each.

(21) a. The boys carefully read one book each.  
   (Stowell 2013: (56c))
   b. The boys read one book carefully each.  
   (Stowell 2013: (56e))

The lack of an adjacency requirement for ge has implications on ge’s compositional semantics. If we are to explain ge’s sensitivity towards measurement type along similar lines as the monotonic measurement constraint of binominal each, it is necessary to find a way to extract the measure function from the measure phrase. In the case of binominal each, the extraction is done syntactically. This is possible because each forms an immediate constituent with its licensor (Safir and Stowell 1988). Since ge does not form an immediate constituent with its licensor, we need to find ways for ge to gain access to a measure function, which I assume is inside a noun phrase.3

There are at least two hypotheses that we can entertain. For one thing, ge may be underlingly more similar to binominal each in being adjacent to its licensor.4 It may undergo movement to the boundary of a verb phrase for syntactic reasons. Although such an analysis has not been proposed in the literature, previous studies have argued that ge may adjoin to different verb phrases when there is more than one verb phrase available (Lin 1998b, Soh 2005). If the movement analysis turns out to be correct, then ge can access a measure function syntactically, in the same way that binominal each accesses one. Alternatively, ge may extract the measure function at a distance using discourse anaphora. In Section 5.3.1, I propose an analysis in terms of discourse anaphora involving a dependent degree variable to achieve this effect.

The second difference between ge and binominal each lies in their licensing conditions. While binominal each is only licensed by counting quantifiers and extensive measure phrases, ge admits

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3There is some initial evidence suggesting that ge may be syntactically related to it licensor. I discuss the evidence in Section 5.5.
4I thank Veneeta Dayal (p.c.) for encouraging me to explore this possibility.
a wider range of licensors. In addition to counting quantifiers and measure phrases, it can be li-
censed when the Share contains any of the following expressions: a pronoun bound by the Key, a quantifier-internal adjective like butong ‘different’, and a coordinated phrase participating in re-
spective distributivity. I discuss these licensing conditions in turn below.

**Lack of licensing by bare noun phrases**

We have seen, in (9), that a bare noun phrase does not license ge. Since a bare noun phrase has a range of interpretations, it is worth checking whether this generalization holds for all the interpre-
tations of a bare noun phrase. To begin with, observe that a bare noun phrase oscillates among an existential, definite, and kind interpretation, depending on the environment in which it occurs, as shown in the following examples (Cheng and Sybesma 1999, Yang 2001, Dayal 2013, a.o.). Note that (24) is more complex than it needs to be in anticipation of the addition of ge.

(22) Zhe-xie haizi kan-le shu.
    this-CL.PL child see-ASP shu
    ‘The children read one or more books.’ (Existential)

(23) Zhe-xie haizi ba shu kan-le.
    this-CL.PL child BA book see-ASP
    ‘The children read the book (or the books).’ (Definite)

(24) Zhe-xie haizi xiwang konglung juezhong.
    this-CL.PL child wish dinosaur extinct
    ‘The children wish dinosaurs to be extinct.’

Under none of these interpretations can a bare noun phrase support ge:

(25) ??Zhe-xie haizi ge kan-le shu.
    this-CL.PL child GE see-ASP shu
    ‘The children read books each.’ (Existential)

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5Imagine a context in which kids were told that dinosaurs were extremely dangerous but were not told that they were already extinct.

6According to Tsai (2009), sentences like (25) become acceptable if the aspectual marker is changed to guo, which marks a perfective aspect of a repeatable event. The speakers I have consulted do not find the aspect marker guo a useful aid. A potential factor for the different judgments lies in the type of Mandarin being studied in Tsai (2009) and in the present study. While the present study is based on Mandarin as spoken in Mainland China, Tsai (2009) is likely based on Mandarin spoke in Taiwan.
(26) ??Zhe-xie haizi ge ba shu diu-le.
   this-CL.PL child GE BA book discard-ASP
   ‘The children throw away books each.’
   (Definite)

(27) ??Zhe-xie haizi ge xiwang konglong juezhong.
   this-CL.PL child GE wish dinosaur extinct
   Intended: ‘The children each wish dinosaurs to be extinct.’
   (Kind)

The uniformity in the patterns is not surprising, given that bare noun phrases in Mandarin have been argued to be kind terms, regardless of their interpretations (Yang 2001, Trinh 2011, Jiang 2012). In section 4.4.3, I address why kind terms fail to support ge.

**Licensing by bound pronouns**

Adding a pronoun (but not a proper name) as part of the modifier of a bare noun phrase significantly improves the above sentences, as shown in (28) and (29) (see also Lee et al. 2009a, Tsai 2009).

(28) Zhe-xie haizi ge kan-le ziji/*Zilu dailiang-de shu.
    these-CL.PL child GE read-ASP self/Zilu bring-DE book
    ‘These children each read the book(s) they/??Zilu brought.’

(29) Zhe-xie haizi ge kan-le ziji-de/*Jinyong-de shu.
    these-CL.PL child GE read-ASP self-POSS/Jinyong-POSS book
    ‘These children each read their/Jinyong’s book.’

The form of the pronoun is relatively flexible. It may be in the form of a reflexive pronoun ziji, as in (28) and (29), a third person plural pronoun tamen ‘they’, as in (30), or a reciprocal pronoun duifang ‘the other/each other’, as in (31).7

(30) Zhe-xie haizi ge kan-le tamen-de shu.
    these-CL.PL child GE read-ASP they-DE book
    ‘These children each read their book.’

(31) Zilu he Ziyou ge kan-le duifang-de shu.
    Zilu and Ziyou GE read-ASP the.other-DE book
    ‘Zilu and Ziyou each read the other’s book.’

7Similar to English, a third person singular pronoun in Mandarin cannot take a morphologically plural noun phrase as its antecedent, even when the pronoun is in the scope of a distributive operator.
However, the interpretation of the pronoun is subject to restrictions. In particular, the pronoun must co-vary with the Key. This generalization can be most readily verified when the pronoun involved is a third person plural pronoun, which is ambiguous between a so-called ‘free’ interpretation (referring to a contextually salient individual) and a so called ‘bound’ interpretation (co-varying with a distributive quantifier in the same sentence). As shown in (32), when \textit{dou} is used to give rise to distributivity, the pronoun in the Share may receive a bound interpretation or a free one. However, when \textit{ge} is used to mark distributivity, as in (33), the pronoun may only receive a bound interpretation.

(32) \textit{Zhe-xie haizi} x dou kan-le tamen\textsubscript{x/y}-de shu.  
\textit{These-CL.PL child DOU read-ASP they-DE book}  
‘These children\textsuperscript{x} each read their\textsubscript{x/y} book(s).’ \textbf{Bound/Free}

(33) \textit{Zhe-xie haizi} x ge kan-le tamen\textsubscript{x/y}-de shu.  
\textit{These-CL.PL child GE read-ASP they-DE book}  
‘These children each read their\textsubscript{x/y} book(s).’ \textbf{Bound/#Free}

With some care, it is possible to verify the bound pronoun generalization by using a reciprocal pronoun or a reflexive pronoun. The reciprocal pronoun \textit{duifang} is also ambiguous between a free interpretation (34) and a bound one (35), when distributivity is marked by \textit{dou}. However, when distributivity is marked by \textit{ge}, only the bound interpretation survives.

(34) \textit{Zilu he\textsuperscript{x} Ziyou dou kandao-le duifang\textsubscript{x/y}-de lian.}  
\textit{Zilu and Ziyou DOU see-ASP the.other-DE face}  
‘Zilu and\textsuperscript{x} Ziyou saw each other\textsubscript{x}’s face/the other\textsubscript{y} person’s face.’ \textbf{Bound/Free}

(35) \textit{Zilu he\textsuperscript{x} Ziyou ge kandao-le duifang\textsubscript{x/y}-de lian.}  
\textit{Zilu and Ziyou GE see-ASP the.other-DE face}  
‘Zilu and\textsuperscript{x} Ziyou saw each other\textsubscript{x}’s face/#the other\textsubscript{y} person’s face.’ \textbf{Bound/#Free}

The reflexive pronoun \textit{ziji} must be bound by an antecedent introduced within the same sentence (Huang 1982, Tang 1989, Pan 1998, a.o.), so a ‘free’ interpretation is independently unavailable. However, since \textit{ziji} may take on different antecedents that precede it (see also Huang 1982, Huang and Liu 2001, a.o.), the flexibility can be used to corroborate the bound pronoun generalization. Observe first that the antecedent of \textit{ziji} is ambiguous in (36) when distributivity is marked with \textit{dou}: \textit{ziji} may refer to the higher subject \textit{Zhang Laoshi} ‘Teacher Zhang’ or the lower subject \textit{zhe-xie haizi}
‘these children’, which is also the Key.

(36) Zhang Laoshi\textsuperscript{x} rang zhe-xie haizi\textsuperscript{y} dou kan-le ziji\textsubscript{x/y} dailai-de shu.
Zhang Teacher ask these-CL.PL child DOU read-ASP self bring-DE book
‘Teacher Zhang\textsuperscript{x} asked these children\textsuperscript{y} to each read the book(s) they\textsubscript{y}/she\textsubscript{x} brought.’

However, once \textit{ge} is used in place of \textit{dou}, the ambiguity disappears. In (37), \textit{ziji} can only refer to the Key \textit{zhe-xie haizi} ‘these children’.

(37) Zhang Laoshi\textsuperscript{x} rang zhe-xie haizi\textsuperscript{y} ge kan-le ziji\textsubscript{x/y} dailai-de shu.
Zhang Teacher ask this-CL.PL child GE read-ASP self bring-DE book
‘Teacher Zhang\textsuperscript{x} asked these students\textsuperscript{y} to each read the book(s) they\textsubscript{y}/she\textsubscript{x} brough.’

The bound pronoun generalization suggests that \textit{ge} requires the Share to contain a constituent that stands in relation to the Key. I will show that this intuition is essentially correct and underlies all the other licensing conditions discussed below.

\textbf{Licensing by sentence-internal readings}

In addition to bound pronouns, \textit{ge} can be licensed by expressions in the Share that induce a so-called sentence-internal interpretation. An example is given in (38), which shows that the adjective \textit{butong} ‘different’ licenses \textit{ge}. Similar expressions like \textit{buyiyang} ‘different’ have the same licensing effect.

(38) Zhe-xie haizi ge kan-le butong-de shu.
this-CL.PL child GE read different-DE book
‘These children read different books.’

\textbf{Carlson (1987)} points out that English \textit{different} has a sentence-internal interpretation as well as a sentence-external interpretation (see also Beck 2000, Brasoveanu 2011). These interpretations are similar to the bound and free interpretations of reciprocal pronouns discussed earlier.

In a sentence with distributivity marked by \textit{dou}, such as (39-b), \textit{butong-de shu} ‘different book(s)’ may refer to a single book that is different from a salient sentence-external antecedent, i.e., \textit{Emma} introduced in (39-a). This is the external interpretation indicated in (39-b-i). Alternatively, it may refer to a set of different books that different children read, as shown in (39-b-ii). This is the sentence-internal interpretation.
(39)  a. Laoshi kan-le *Emma.
teacher read-ASP Emma
‘The teacher read Emma’

b. Zhe-xie haizi dou kan-le butong-de shu
this-CL.PL child DOU read-ASP different-DE book
(i) These children each read some book(s) that differed from *Emma.  *External
(ii) These children each read some book(s) that differed from the books the rest of
the children read.  Internal

When *dou* is replaced by *ge*, however, the external interpretation becomes unavailable:

(40)  a. Laoshi kan-le *Emma.
teacher read-ASP Emma
‘The teacher read Emma’

b. Zhe-xie haizi *ge* kan-le butong-de shu
this-CL.PL child GE read-ASP different-DE book
(i) These children each read some book(s) that differed from *Emma.  *External
(ii) These children each read some book(s) that differed from the books the rest of
the children read.  Internal

Also relevant is the fact that not just any sentence-internal adjective can support *ge*. Only those
adjectives that induce co-variation have a licensing effect, as evidenced by the fact that the sentence-
internal reading of an adjective like *tong* ‘same’, offers no rescue:

(41) *Zhe-xie haizi ge kan-le tong yi-ben shu
this-CL.PL child GE read-ASP same one-CL book
‘These children all read the same book.’

Licensing by sentence-internal readings has a similar flavor as licensing by bound pronouns as both
classes of expressions help induce co-variation between the Key and part of the Share.

**Licensing by respective distributivity**

As I have already mentioned in Section (2), *ge* is licensed when two conjunctions give rise to
respective distributivity (see also Lee et al. 2009a, Tsai 2009). An example involving respective
distributivity is given below:
Zilu and Ziyou sing a song and perform a dance, respectively.

There is evidence showing that it is the respective interpretation that licenses *ge*, but not just the use of two plural noun phrases. (43) shows that when the two coordinated phrases are replaced by two definite plurals, *ge* becomes highly marked.

(43) ??Zhe-xie haizi ge kan-le na-xie shu.

this-CL.PL child GE read-ASP that-CL.PL book

Intended: ‘These children read those books, respectively.’

Pragmatically providing a pairing between the children and the books does not provide much help to (43). To make a respective interpretation fully acceptable with two definite plurals, the respective number of the entities contributed by the plurals have to be specified and matched, and preferably the adverb *fenbie* is also used. When these ingredients are present, as in (44), the use of *ge* is licensed.

(44) Zhe san-ge haizi fenbie ge kan-le na san-ben shu.

this three-CL child respectively GE read-ASP that three-CL book

‘These three children read those three books, respectively.’

Since the concern of the present study is on licensing *ge* rather than on licensing respective distributivity, I do not explore why there is a difference between (43) and (44) in supporting respective distributivity. I take the generalization to be that whenever respective distributivity is available, *ge* is licensed.

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8Chaves (2012) reports a similar contrast in the licensing of *respectively* in English:

(i) a. The three best students received the three best scores respectively.
   b. #The students were pleased by their scores, respectively. (Chaves 2012:(7a-b))
Interim summary

In summary, *ge* is licensed by counting quantifiers, extensive measure phrases, bound pronouns, adjectives with a sentence-internal reading, as well as respective distributivity. A challenge that lies in front of us is how to make sense of this conglomerate of licensing conditions. Since *ge* patterns like binominal *each* with regard to licensing by counting quantifiers and extensive measure phrases, it is reasonable to assume that *ge* also bears some form of monotonic measurement constraint. However, since *ge* can also be licensed without the presence of any measurement, the constraint must be more general than the monotonic measurement constraint.

In the next section, I develop a generalized monotonicity constraint on the basis of the monotonic measurement constraint. The idea is that *ge* is similar to binominal *each* in requiring a monotonic mapping between two pluralities living in a distributive dependency. However, while binominal *each* only allows a mapping from individuals to measurements of individuals, i.e., degrees, to satisfy the monotonic measurement constraint, *ge* also allows a mapping from individuals to individuals to satisfy its monotonicity constraint. Because of the generality of *ge*’s constraint, I call it a generalized monotonicity constraint. I discuss this constraint in more detail in the next section.

5.3 Proposal: a generalized monotonicity constraint in DPLM

At the heart of a monotonicity constraint on distributivity is a monotonic mapping between two mereological structures: the mereological structure contributed by the Key and the mereological structure contributed by a dependent expression after it has been distributively evaluated. We have seen, from Chapter 3, that the monotonic measurement constraint of binominal *each* requires a monotonic mapping from the Key to a dependent measurement either contributed by a measure phrase or a counting quantifier.

(45) The drinks are 6oz each.

(46) The students bought three books each.

Using (45), the mapping can be illustrated with the help of Figure 5.1.\(^9\)

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\(^9\)Since degrees are modeled as triples in this study, this mapping is actually from individuals to the first coordinates of a set of degrees. The first coordinate of each degree stores a degree name, which is a point on a fully ordered scale.
We have also seen that quantifiers that do not bear a measure function component, such as *some NPs, most NPs, and few NPs, do not support binominal *each. An example is given below:

(47)  *Every child bought some books/a certain book each.

The unacceptability is attributed to the lack of a measure function in these quantifiers: while these quantifiers contribute a mereological structure based on individuals, their lack of measure function does not support the building of a degree mereology. In other words, binominal *each is selective about the type of monotonic mapping between two mereological structures: while an individual-degree mapping is acceptable, an individual-individual mapping is rejected. An example of an individual-individual mapping is given in Figure 5.2.

Mandarin *ge is less selective than binominal *each. It is compatible with both types of monotonic mappings: an individual-degree mapping, as well as an individual-individual mapping. I argue that recognizing these two types of mappings is all we need to account for the conglomerate of licensing conditions of *ge reported in the previous section.
Translating ge’s generalized monotonicity constraint into the DPILM framework is quite transparent. We just need to make reference to a pair of d-refs. The first d-ref stores values contributed by the Key. The second d-ref stores values contributed by a dependent expression. The constraint, as given in (48), then requires that the mereological structures computable from the values stored in the two variables observe monotonicity. The flexibility in the type of the second d-ref is the formal reflex of the generality of the monotonicity constraint.

\[(48) \quad \text{dm}_{x,u}, \text{ where } u \text{ may be a degree variable or an individual variable.}^{10}\]

Let us first consider the case when \( u \) is resolved to a degree variable \( d \). I have argued, in Chapters 2 and 3, that a monotonic mapping from a set of individuals to a set of degrees should be understood as structural dependence, whose definition is given below:

\[(49) \quad G\llparenthesis \text{dm}_{x,d} \rrparenthesis H = T \iff\]

\[\text{a. \ there are distinct nonempty sets } A, B \subseteq G(x): G\llparenthesis x \in A \rrparenthesis (d) \neq G\llparenthesis x \in B \rrparenthesis (d)\]

\[\text{b. \ for all distinct nonempty sets } A, B \subseteq G(x): \text{if } A \subseteq B, \text{ then } G\llparenthesis x \in A \rrparenthesis (d) \leq G\llparenthesis x \in B \rrparenthesis (d).\]

Recall that a degree is modeled as a triple. The first coordinate of the triple stores a degree name. A degree name can be retrieved with help of a parameterized projection function \( G\llparenthesis x \rrparenthesis_i (d) \), where \( i \) is the coordinate to be projected. The second coordinate of a degree stores a measure function (retrieved using \( G\llparenthesis x \rrparenthesis_i (d) \)), and the third coordinate stores the individual being measured (retrieved using \( G\llparenthesis x \rrparenthesis_i (d) \)). The complex structure is motivated by the need to concatenate degrees in the building of a degree mereology. A degree name is derived by applying a measure function to an individual. Without the accompanying information about the measure function and the individual being measured, it is very hard to determine how two degrees are to be concatenated only by looking at the degree names.\(^{11}\)

Concretely, to compute the first coordinate of a degree, i.e., \( G\llparenthesis x \rrparenthesis_i (d) \), which is a degree name, we

\(^{10}\)Note that I am not assuming that ge is ambiguous between imposing a degree-based monotonicity constraint and an individual-based monotonicity constraint. Rather, it is underspecified for the types elements that can satisfy its constraint.

\(^{11}\)This is regardless of whether we allow an intensive measure function to apply to two non-overlapping objects or not. Even if we disallow an intensive measure function to generate a defined measurement for non-overlapping objects, we still need access to such a measure function to know that the effect of concatenating two intensive measurements is undefined.
take the measure function stored in the second coordinate \((\bigoplus G^{i=2}(d))\) and apply it to the (possibly plural) individual stored in the third coordinate \((\bigoplus G^{i=3}(d))\). The summation operator \(\bigoplus\) is used for different reasons here. Since the second coordinate of a degree stores the same measure function for all assignments, the \(G^{i=2}(d)\) is a singleton set containing one measure function. The summation operator simply removes the set and returns the measure function. However, \(G^{i=3}(d)\) returns a set of individuals. The summation operator sums together all of these individuals in the set, allowing a measure function to apply to the sum individual. The following notation is used to compute a degree name from a degree d-ref in a plural info-state.

\[ G^{i=1}(d) := \text{the first coordinate of } \bigoplus G^{i=2}(d)(\bigoplus G^{i=3}(d)) \]

The constraint in (49) differs from the monotonic measurement constraint of binominal *each* only in compositionality. Specifically, binominal *each* assembles a monotonic measurement constraint \(dm_{x,y}(\mu)\) by gaining syntactic access to the measure function \(\mu\), then applying the measure function to the dependent individual variable \(y\) to yield a set of degree information (i.e., \(\mu(y)\) for each combination of values in \(G(x)\)). The monotonicity requirement ultimates holds between a mereological structure of individuals and a mereological structure of degrees. To satisfy the monotonicity requirement of binominal *each*, the same conditions in (49) have to be satisfied.

If \(u\) is resolved to an individual variable, such as \(y\), then the monotonicity constraint requires structural dependence between the Key variable and the dependent individual variable.

\[ G[dm_{x,y}]H = T \text{ iff } \]

\[ \]

\[ a. \text{ there are distinct nonempty sets } A, B \subseteq G(x): G|_{x \in A}(y) \neq G|_{x \in B}(y) \]

\[ b. \text{ for all distinct nonempty sets } A, B \subseteq G(x): \text{ if } A \subseteq B, \text{ then } G|_{x \in A} y \subseteq G|_{x \in B} y. \]

Recall from Chapter 2 that when a monotonic mapping is from individuals to individuals, then it can be recast in terms of value dependence or co-variation.\(^\text{12}\) However, I maintain an analysis in terms of monotonicity to reflect that *ge* is still sensitive to the extensive-intensive distinction of measurement. The fact that when a monotonic mapping only involves individuals it can be reduced

\(^{12}\) Evaluation plurality, first proposed in Henderson (2014), and later adopted in Champollion (2015) and Kuhn (2017), is also a type of value dependence.
to value dependence or co-variation follows straightforwardly from the parthood relation defined on a join semi-lattice of individuals: in plain words, the sum of two distinct, non-overlapping individuals is strictly bigger than its parts.

In the next section, I discuss how the generalized monotonicity constraint of ge accounts for its distribution reported in the previous section.

5.3.1 Accounting for licensing by counting quantifiers and measure phrases

Ge is licensed by measure phrases when they provide an extensive measure function, as shown in (52-a) and (52-b).

(52) a. Zhe-xie yinliao x ge (you) 200 haosheng.
    these-CL drink GE have 200 mililiter
    Literally. ‘These drinks each are 200 ml.’

b. *Zhe-xie yinliao x ge (you) 60 du.
    these-CL drink GE have 60 degree
    ‘These drinks each are 60 degrees.’

The dependent expressions in these examples are measure phrases. I assume that the measure phrases (you) 200 haosheng ‘be 200ml’ and (you) 60 du ‘be 60C’ are used predicatively, so they are functions from individuals to dynamic propositions. Their dynamic contribution is the introduction of a degree d-ref, which is linked to an individual by a measure function \( \mu \).

(53) \( \text{(be) 200ml} \ := \lambda x. \exists d \wedge d = \langle 200 \text{ml}, \text{vol}, x \rangle \wedge \mu x = d \)

(54) \( \text{(be) 60C} \ := \lambda x. \exists d \wedge d = \langle 60 \text{C}, \text{temp}, x \rangle \wedge \mu x = d \)

The plural demonstrative phrase zhe-xie yinliao ‘these drinks’ in (52-a) and (52-b) is treated as a dynamic generalized quantifier:

(55) \( \lambda P. \max^x (\text{drink } x) \wedge P x \)

Distributivity is introduced by a covert distributive operator:

(56) \( \text{Dist} := \lambda P \lambda x. \delta_x (P x) \)
Combining the generalized quantifier, the distributive operator, and the predicative measure phrases in (53) and (54) in the manner in (57-a) and (58-a) yields (57-b) and (58-b), respectively:

\[(57)\]
a. **these drinks** \(\text{Dist} (\lambda y. \exists d \land d = \langle 200 \text{ml}, \text{vol}, x \rangle \land \mu y = d)\)
b. \(\max^x(\text{drink } x) \land \delta_x (\exists d \land d = \langle 200 \text{ml}, \text{vol}, x \rangle \land \mu x = d) \land \text{dm}_{x,d}\)

\[(58)\]
a. **these drinks** \(\text{Dist} (\lambda y. \exists d \land d = \langle 60 ^\circ \text{C}, \text{temp}, x \rangle \land \mu y = d)\)
b. \(\max^x(\text{drink } x) \land \delta_x (\exists d \land d = \langle 60 ^\circ \text{C}, \text{temp}, x \rangle \land \mu x = d) \land \text{dm}_{x,d}\)

As already discussed in connection with the monotonic measurement constraint of binominal *each*, the monotonicity constraint in (57-b) can be satisfied while the same constraint in (58-b) cannot. This is because extensive measurement yields degree names that track the mereological structure of the Key, i.e., the drinks in this case. However, intensive measurement fails to yield degree names that have the same effect.

Counting quantifiers are taken to contribute both individual d-refs and degree d-refs. If all *ge* needs is one d-ref (whichever d-ref) to satisfy its monotonicity constraint, then counting quantifiers are predicted to be acceptable, as long as the individual d-ref they contribute co-varies with distributivity. Since degree names are derived from measuring individuals, if a counting quantifier does not contribute individuals that co-vary with distributivity, then the degree names derived from measuring these individuals will also remain constant relative to distributivity, in violation of the definition of the monotonicity constraint in (49).

### 5.3.2 Failure of licensing by bare noun phrases

Recall that *ge* is unacceptable when the Share only contains a transitive verb and a bare noun phrase, as illustrated in the following example (repeated from (25)):

\[(59)\]  *Zhe-xie haizi ge kan-le shu.*  
  this-CL.PL child GE see-ASP shu  
  ‘*The children read (the) books each.*’

Bare noun phrases in Mandarin may receive a definite interpretation or an existential, indefinite interpretation (Cheng and Sybesma 1999, Yang 2001, Trinh 2011, Dayal 2011a, Jiang 2012). As
discussed earlier, both interpretations fail to license ge. In rest of this section, I first discuss why the existential interpretation fails to satisfy the monotonicity constraint, and then move on to discuss why the definite interpretation also fails to do so. The key argument is that they fail for the same reason: they do not co-vary with the Key.

We have seen, in Chapter 4, that existentially interpreted bare noun phrases can satisfy the independence constraint of Cantonese saai.\textbf{13} Based on that, I have argued that existentially interpreted bare noun phrases are kind terms and they only introduce a kind-level d-ref (see also Cheng and Sybesma (1999) and Jiang (2012), who argue that bare noun phrases in Cantonese are kinds). The existential interpretation comes from the interpretation procedure, which has a mechanism akin to Derived Kind Predication (Chierchia 1998b, see also Carlson 1977a).\textbf{14}

Following the treatment of bare noun phrases in Cantonese in Chapter 4 and the widely accepted view that bare noun phrases in Mandarin have the core semantics of kind terms (Chierchia 1998b, Yang 2001, Trinh 2011, Jiang 2012), I take an existentially interpreted bare noun phrase in Mandarin as a kind term. Since a kind term is similar to a proper name and does not co-vary with distributive quantification, the translation of a bare noun phrase like shu ‘books’ in (60) is a similar the translation of a proper name like (61):

\begin{align}
(60) & \quad \text{shu}^y \ ‘\text{books}’ := \lambda P. \exists y^k \land y^k = \text{bk-kind} \land P y \\
(61) & \quad \text{Zilu}^y := \lambda P. \exists y \land y = z \land P y
\end{align}

The kind-level individual bk-kind is modeled, following Chierchia (1998b), as a function from a world to a plurality consisting of all the instantiations of the book kind in that world.\textbf{15} The function is in generated by applying a nominalization operator to a property:

\begin{align}
(62) & \quad \text{bk-kind} := \lambda s. \bigoplus \text{book}_s = \bigcap \lambda s \lambda x. \text{book}_s x
\end{align}

\textbf{13} According to Cheng and Sybesma 1999, bare noun phrases in Cantonese cannot receive a definite interpretation (see also Jiang 2012). So, it is the existential interpretation of bare noun phrases in Cantonese that satisfies the independence constraint.

\textbf{14} Farkas and de Swart (2003) also uses a similar interpretive mechanism to derive the existential force of a semantically incorporated bare singular (i.e., bare noun phrase that lacks plural morphology), which they analyze as uninstantiated arguments rather than kinds.

\textbf{15} An intensional compositional semantics is needed to compose a kind term and a predicate.
Note that $\lambda s.\oplus^s_{\text{book},s}$ can return different individuals in different worlds. However, in any one world $s$, the value of $\oplus^s_{\text{book},s}$ is fixed. For this reason, in any world the plurality stored in $y^k$ in (60) does not change in the course of distributive quantification. The kind-based analysis predicts that bare noun phrases do not license $ge$, because the kind-level d-refs introduced by bare noun phrases are just like proper names and cannot induce co-variation. Concretely, translating (59) into DPLM gives rise to (63). Interpreting (63) against a set of input info-states yields a set of output info-states. A member in the output is given in Figure 5.3 as an illustration. In this info-state, there is no co-variation between the children and the kind of object they read, as they all read the same kind of object, namely, books. If there is any additional info-state in the output, they all agree on this feature: $y^k$ stores a unique kind that fails to co-vary with distributivity.

$$\text{(63)} \max^x (\text{child } x) \land \delta_x (\exists y^k \land y^k = \text{book-kind} \land \text{read } y^k x) \land \text{dm}_{x,y^k}$$

### Figure 5.3: An sample info-state after interpreting (63)

<table>
<thead>
<tr>
<th>$H$</th>
<th>$x$</th>
<th>$y^k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>$c_1$</td>
<td>bk-kind</td>
</tr>
<tr>
<td>$h_2$</td>
<td>$c_2$</td>
<td>bk-kind</td>
</tr>
<tr>
<td>$h_3$</td>
<td>$c_3$</td>
<td>bk-kind</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I follow Dayal (2011a) and Trinh (2011) and assume that when a bare noun phrase receives a definite interpretation, there is an extensional operator $\text{EXT}$ that saturates the world argument of a relevant kind term and as a result yields a maximal individual (not an individual concept) that is a member of the kind. Accordingly, a bare noun phrase with a definite interpretation is treated in the same way as a definite noun phrase, which introduces a maximal individual into context:

16Since maximization does not guarantee uniqueness, the output set here will have more than one info-state if distributive quantification is assumed to allow non-atomic distributivity (also known as cover-based distributivity). For example, another info-state in the output can have the following form:

<table>
<thead>
<tr>
<th>$H$</th>
<th>$x$</th>
<th>$y^k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$</td>
<td>$c_1 \oplus c_2$</td>
<td>bk-kind</td>
</tr>
<tr>
<td>$h_3$</td>
<td>$c_3$</td>
<td>bk-kind</td>
</tr>
</tbody>
</table>

If non-atomic distributivity is ruled out, then $H$ will be the only output. A requirement I have not discussed in this study is the fact that $ge$ strongly favors atomic distributivity, in contrast to $dou$, which is compatible with both atomic and non-atomic distributivity (Lin 1998a.)
This quantifier introduces a maximal individual that has the book property. This individual, even if interpreted inside the scope of distributivity, does not co-vary with distributivity, as shown in (65). For this reason, the monotonicity constraint of ge cannot be satisfied by a bare noun phrase receiving a definite interpretation.

\begin{equation}
\max^x (\text{child } x) \land \delta_x (\max^y (\text{book } y) \land \text{read } y x) \land \text{dm}_{x,y}
\end{equation}

Given the contrasting requirements of the monotonicity constraint of ge and the independence constraint of Cantonese saai, the fact that bare noun phrases pattern differently with respect to these two constraints is a welcome result.

5.3.3 Licensing by bound pronouns

I have suggested that allowing ge’s monotonicity constraint to be satisfied by either a dependent degree variable or a dependent individual variable suffices to account for all of ge’s licensing conditions that do not involve a measurement component. In this section, I show why a bound pronoun can license ge, as exemplified in (66-a), and why proper name cannot, as exemplified in (66-b) (repeated from (28) and split into two sentences to facilitate reference to each condition of the example):

\begin{enumerate}
\item a. Zhe-xie haizi\textsuperscript{x} ge yaoqing-le ziji\textsubscript{x} xihuan-de tongxue\textsubscript{y/y^k}.
\text{These children each invited the classmate(s) they like.}
\item b. ??Zhe-xie haizi\textsuperscript{x} ge yaoqing-le Zilu xihuan-de tongxue\textsubscript{y/y^k}.
\text{These children each invited the classmate(s) Zilu likes.}
\end{enumerate}

To interpret these sentences, we need to decide how to interpret complex bare noun phrases like ziji\textsubscript{x}-xihuan-de tongxue ‘classmate(s) x likes’ and Zilu xihuan-de tongxue ‘classmates Zilu likes’. It turns out that we cannot make a uniform decision for all bare noun phrases with a relative clause modifier. Some of them are better treated as definite expressions while others are better treated as kinds terms. The decision is a delicate one. However, since the primary focus of this chapter is
on the licensing effects of bound pronouns rather than on how to analyze bare noun phrases with a modifier, I do not try to resolve the matter here. What I will do is to demonstrate that whether a bare noun phrase with a relative clause is analyzed as a definite expression or a kind term, the presence of a pronoun bound by the Key is predicted to help satisfy the monotonicity constraint of $ge$. I demonstrate the definite analysis first and turn to the kind analysis next.

To begin with, I take Zilu/ziji xihuan-de tongxue `classmate(s) Zilu/self likes` to be a definite expression, akin to the classmate(s) that Zilu/they like(s) in English. Accordingly, ziji xihuan-de tongxue and Zilu xihuan-de tongxue are translated as follows:

\[
(67) \text{classmates-ziji-likes} := \lambda P.\text{max}^y(\text{classmate } y \land \text{like } x) \land P y
\]

\[
(68) \text{classmates-Zilu-likes} := \lambda P.\text{max}^y(\text{classmate } y \land \text{like } z) \land P y
\]

Note that when the bare noun phrase contains a pronoun (modeled as a variable), as in the case of (67), the maximal individual picked out by the definite expression and introduced into an info-state depends on the value of the pronoun. When the pronoun co-varies with the Key, the maximal individual introduced into an info-state also may (but need not) co-vary with the Key. For example, (69-a) is a model in which not all the children like the same classmate, while (69-b) is a model in which all the children like the same classmate.

\[
(69)\begin{align*}
a. \text{In Model A: } & \text{like} := \{\langle \text{john, sue} \rangle, \langle \text{mary, emily} \rangle, \langle \text{paul, sue} \rangle\} \\
b. \text{In Model B: } & \text{like} := \{\langle \text{john, sue} \rangle, \langle \text{mary, sue} \rangle, \langle \text{paul, sue} \rangle\}
\end{align*}
\]

If we are in model A, then the definite expression introduces a d-ref that stores classmates that co-vary with the Key, i.e., the children. For this reason, in Model A the monotonicity constraint of $ge$ can be satisfied in (70-a), which is the translation of the Mandarin sentence in (66-a) with the distributivity and the monotonicity constraint marked by $ge$. However, in Model B the monotonicity constraint cannot be satisfied and the sentence is unacceptable.

\[
(70)\begin{align*}
a. \text{The children } & \text{ge invited classmates they like.} \\
b. \text{max}^x(\text{child } x) \land \delta_x(\text{max}^y(\text{book } y \land \text{bring } y x) \land \text{read } y x) \land \text{dm}_{x,y}
\end{align*}
\]
When the pronoun is replaced by a proper name, co-variation between the Key and the classmate(s) is no longer available. This is because the individual picked out by a proper name cannot co-vary with the Key. As a result, the maximal classmate plurality that this individual likes also cannot co-vary with the Key, making it impossible to satisfy the monotonicity constraint in (71-b). The sentence represented by the translation in (71-a), i.e., (66-b), is hence unacceptable.

(71)  a. ??The children $ge$ invited classmates $Zilu$ likes.
    b. $\max^x (\text{child } x) \land \delta_x (\max^y (\text{book } y \land \text{bring } y z) \land \text{read } y x) \land \text{dm}_{x,y}$

Now, we can take up a different type of bare noun phrase modified by a relative clause, i.e., one that may assume a kind interpretation. Like modified bare noun phrases with a definite interpretation, they also license ge when they contain a pronoun bound by the Key, as shown in (72). When the pronoun is replaced by a proper name, the licensing effect is no longer available.

(72) Zhe-xie $\text{haizi}^x$ $ge$ $\text{kanjian-le } ziji_x \text{xihuan-de } \text{dongwu}^{y^k}$.
     ‘These children each saw the animal(s) they like.’

(73) ??Zhe-xie $\text{haizi}^x$ $ge$ $\text{kanjian-le } Zilu \text{xihuan-de } \text{dongwu}^{y^k}$.
     ‘These children each saw the animal(s) Zilu likes.’

Recall that a kind is modeled as a proper name, following Carlson (1977a). A proper name is in turn modeled as a dynamic generalized quantifier in the present dynamic framework. Accordingly, the interpretations of $ziji_x \text{xihuan-de } \text{dongwu}$ ‘animals $x$ likes’ and $\text{Zilu xihuan-de } \text{dongwu}$ ‘animals Zilu likes’ are given below:

17 Carlson 1977a is perhaps the first one to note that bare noun phrases with a modifier can still function like a kind term, using the following example:

(i) Alligators in the New York sewer system... such alligators survived by eating rodents and organic debris. (Carlson 1977a)

According to Carlson (1977a), the fact that such can be used to refer to the alligators shows that the modified bare noun phrase is a kind term.

18 In static logic, proper names may be analyzed as an individual (e.g., $\text{john}$) or a generalization quantifier (e.g., $\lambda P. P \text{john}$). In a dynamic logic, proper names have to be modeled as generalized quantifiers in order for them to interact with context and have their dynamic contribution recorded.
The difference between (74) on the one hand and (75) on the other hand lies in whether \( y^k \) is allowed to store values that may co-vary with another variable. Recall that a kind is formed by applying the nominalization operator \( \cap \) to a predicate (Chierchia 1984, 1998b). If the predicate is complex, as in the case of \( zijixi\text{huan-de dongwu} \) ‘animals \( x \) likes’ and \( Ziluxi\text{huan-de dongwu} \) ‘animals Zilu likes’, I assume that \( \cap \) applies after the modification. From the definitions below, we can see the difference a variable in the modifier makes:

\[
\text{animals-x-like} := \lambda P. \exists y^k \land y^k = \text{animal-x-like} \land P y^k
\]

\[
\text{animals Zilu like} := \lambda P. \exists y^k \land y^k = \text{animal-z-like} \land P y^k
\]

In (76), the predicate being nominalized may have different members depending on the value of \( x \). As a result, the kind derived from this predicate may also vary depending on the value of \( x \). However, in (77), the predicate subject to nominalization is a fixed set, i.e., the set of things that are animals and liked by Zilu. Accordingly, the kind derived from this predicate is also fixed. For this reason, in (74) \( y^k \) can (but need not) store different kinds of animals depending on the values assigned to \( x \). However, in (75) \( y^k \) can only store a single kind of animals, namely, the animals that Zilu likes. In this sense, a bare noun with a modifier that does not contain a variable, as in the case of (75), is similar to a plain, unmodified bare noun phrase in not being able to induce co-variation.

The contrast can be more clearly illustrated when (74) and (75) are used in sentences like (72) and (73). The corresponding translations are given below, again, with \( ge \) standing in for distributivity and the monotonicity constraint:

\[
\text{(78) a. The children} \ \text{ge read books they brought.}
\]

\[
\max^x(\text{child } x) \land \delta_x(\exists y^k \land y^k = \text{animal-x-like} \land \text{see } y^k x) \land \text{dm } x, y^k
\]

\[
\text{(79) a. ??The children} \ \text{ge read books} \ Zilu \text{brought.}
\]

\[
\max^x(\text{child } x) \land \delta_x(\exists y^k \land y^k = \text{animal-z-likes} \land \text{see } y^k x) \land \text{dm } x, y^k
\]
In (78-b), \( y^h \) may vary depending on the value assigned to \( x \). For this reason, the monotonicity constraint can be satisfied. By contrast, in (79-b), \( y^h \) cannot vary relative to \( x \) and hence the monotonicity constraint cannot be satisfied.

Before leaving this section, I would like to point out two caveats. First, when a pronoun is not bound by the Key, it is not always straightforward to determine whether a noun phrase containing the pronoun may or may not co-vary with the Key. Typically, a free pronoun is not allowed, as reported earlier in Section (37). The relevant example is repeated below.

\[(80) \text{Zhe-xie haizi}\text{ge kan-le tamen}_x/y\text{-de shu.} \]
\[\text{these-CL.PL child GE read-ASP they-DE book} \]
\[\text{‘These children each read their}_x/y\text{-book.’} \]

However, if there is enough contextual support for a dependency between the Key and the plural individual the plural pronoun refers to, a similar sentence can be felicitous:

\[(81) \text{a. Context: Each child } x \text{ asked a friend } y \text{ to bring a book for them.} \]
\[\text{b. Zhe-xie haizi}\text{ge kan-le tamen}_x/y\text{-de shu.} \]
\[\text{these-CL.PL child GE read-ASP they-DE bring-DE book} \]
\[\text{‘These children each read books they}_x/y\text{-brought.’} \]

The improved judgment is not entirely unexpected. Although the pronoun is not directly bound by the Key, it does co-vary with the distributivity. In particular, the pronoun refers to a different friend for each child. Since the friends stand in a dependency with the children, the books the friends brought may also co-vary with the children.

Second, a bound pronoun is only useful when it is part of a noun phrase, as demonstrated by the contrast in (82-a) and (82-b).\(^{19}\)

\(^{19}\)Relational nouns such as mama ‘mother’ and dangdi ‘local’ can receive a bound interpretation without the use of a pronoun (see also Asudeh 2005):

(i) \[\text{Meige haizi dou yaoqing-le mama chuxi biyedianli.} \]
\[\text{every child DOU invite-ASP mother attend graduation.ceremony} \]
\[\text{‘Every child invited his mother to attend the graduation ceremony.’} \]

(ii) \[??Meige youke dou hui dao dangdi\text{-de jiaotang canguang.} \]
\[\text{every tourist DOU will to local-DE church visit} \]
\[\text{‘Every tourist visits the local church.’} \]
This can be accounted for if the dependent variable is required to be disjoint from the independent variable. In (82-a) the dependent variable introduced by the noun phrase  
\[ \text{ziji-de mama} \]
'x’s mother' is disjoint from the independent variable introduced by the Key. However, in (82-b) there is no dependent variable disjoint from the independent variable, other than the potentially available event variable contributed by the verb. However, for unknown reasons  \( \text{ge} \) cannot utilize dependent event variables.\(^{20}\)

### 5.3.4 Licensing by sentence-internal adjectives

Intuitively, the licensing effect of  \( \text{butong} \) ‘different’ can be understood in similar terms as the licensing effect of a bound pronoun:  \( \text{butong} \) makes a difference because it interacts with distributivity to help give rise to co-variation. This is the intuition I will pursue here. In particular, (83) is acceptable because  \( \text{butong} \) is able to interact with distributivity to give rise to co-variation between the variable introduced by the Key (i.e., the conjoined subject) and the variable introduced by the noun phrase in the Share.

\[
\begin{align*}
\text{(83)} & \quad \text{Zilu he} & \quad \text{Ziyou ge} & \quad \text{kan-le} & \quad \text{butong-de} & \quad \text{shu.} \\
& \quad \text{Zilu and Ziyou GE read-ASP different-MOD book} & \quad \text{‘Zilu and Ziyou each read different books.’}
\end{align*}
\]

However, they do not license  \( \text{ge} \) in the same way that a noun containing a bound pronoun does:

\[
\begin{align*}
\text{(iii)} & \quad \text{Meige haizi} & \quad \text{ge yaoqing-le} & \quad \text{mama chuxi biyedianli.} \\
& \quad \text{every child DOU invite-ASP mother attend graduation.ceremony} & \quad \text{‘Every child invited his mother to attend the graduation ceremony.’}
\end{align*}
\]

\[
\begin{align*}
\text{(iv)} & \quad \text{Meiyouke ge} & \quad \text{hui dao dangdi-de} & \quad \text{jiaotang canguang.} \\
& \quad \text{every tourist GE will to local-DE church visit} & \quad \text{‘Every tourist visits the local church.’}
\end{align*}
\]

It is unclear to me why relational nouns do not pattern like noun phrases with a pronoun.\(^{20}\) Recall from Chapter 4 that Cantonese  \( \text{saai} \) also cannot target an event variable.

---

\(^{20}\)Recall from Chapter 4 that Cantonese  \( \text{saai} \) also cannot target an event variable.
There are a few ways to formalize this intuition. I start with the simplest option, namely, assimilating *butong-de shu* ‘different books’ with its indefinite variant *yi-ben butong-de shu* ‘a different book’, which also supports *ge*, as shown below:

(84)  Zilu he Ziyou kan-le *yi-ben butong-de shu.*
      Zilu and Ziyou read-ASP one-CL different-MOD book
      ‘Zilu and Ziyou read different books.’

By analyzing *butong-de shu* ‘different books’ as an indefinite, we have essentially opened a door for it to exhibit co-variation. On this view, the adjective *butong* has the capacity to turn a bare noun phrase from a kind term, which is unable to co-vary with distributivity, to an indefinite, which may co-vary with distributivity. If we further assume that indefinites with an adjective like *butong* or *different* have a higher-order meaning akin to counting quantifiers hosting binominal *each*, we can easily model the contribution of the adjective as a test on the output of distributive quantification. The translation of *yi-ben butong-de shu* ‘a different book’ and *butong-de shu* ‘different books’ are given in (85) and (86), and the interpretation of the *diff_{x,y}* test is given in (87).\(^{21}\) I discuss them in turn below.

(85)  \(a \text{ different}_{x,y} \text{ book}^y := \lambda c(\lambda P. \exists y \ W \ y \ W \ |y| = 1 \ W \ P \ y) \ W \ \text{diff}_{x,y}\)

(86)  \(\text{different}_{x,y} \text{ books}^y := \lambda c(\lambda P. \exists y \ W \ y \ W \ P \ y) \ W \ \text{diff}_{x,y}\)

(87)  \(G][\text{diff}_{x,y}][H = T \text{ iff for all } a, b \in G(x) : G|_{x=a}(y) \not= G|_{x=b}(y)\)

(85) and (86) are identical except for the cardinality test, which is contributed by the numeral classifier.\(^{22}\) The indefinites are of type \((Q \rightarrow t) \rightarrow t\). Their scope argument \(c\) (of type \(Q \rightarrow t\)) is a function from dynamic quantifiers to dynamic propositions. The higher-order indefinites, as we have seen in Chapter 3, have just the right quantifier component to feed to their scope argument and generate a dynamic proposition. In a sentence with distributivity, the dynamic proposition encodes distributive

---

\(^{21}\)Kuhn (2017) suggests a similar analysis for the adjective *same* in English. However, he does not use higher-order meaning. The analyses for ‘plural different’ provided in Beck (2000) and Brasoveanu (2011) are also similar in spirit, in the sense that distributivity is evaluated before the contribution of different.

\(^{22}\)The numeral classifier in (85) additionally introduces a degree d-ref, which is not available in a noun phrase without a numeral classifier, such as (86). I abstract away from this difference to focus on the role of *butong.*
dependencies in the output. The contribution of the adjective butong is a test \( \text{diff}_{x,y} \), which is evaluated after the dynamic proposition is formed by combining \( c \) and the quantifier component provided by the higher-order indefinite. The monotonicity constraint of \( \text{ge} \) is evaluated after distributivity. However, it may be evaluated before or after the \( \text{diff}_{x,y} \) test. For concreteness, I have chosen to evaluate it after the \( \text{diff}_{x,y} \) test in (88-b) ( (88-a) represents the relative scope relations between the various constituents that give rise to this interpretation).

(88)  

a. \( \text{ge}_{x,y} \) (different\(_{x,y} \) books\(_y^y \) (\( \lambda Q \).these children (Dist (\( \lambda u.Q(\lambda u'.\text{read} \ u') \))))

b. \( \text{max}^x(\text{children } x) \land \delta_x(\exists y \land \text{book } y \land \text{read } y \ x) \land \text{diff}_{x,y} \land \text{dm}_{x,y} \)

For this analysis to be explanatory, we need to explain why using an adjective like butong turns a kind term into an indefinite. In fact, there is an intuitive explanation for it: if butong-de shu introduce a proper name storing bk-kind, and the semantics of butong is to require co-variation, then the co-variation requirement will never be satisfied, as demonstrated in (89-b). This is because bk-kind does not co-vary with distributivity, as demonstrated in Section 5.3.2. Therefore, the felicitous use of butong entails that something other than bk-kind is available to establish co-variation.

(89)  

a. different\(_{x,y^k} \) books\(_y^k \) (\( \lambda Q \).these children\(_x \) (Dist (\( \lambda u.Q(\lambda u'.\text{read} \ u') \))))

b. \( \text{max}^x(\text{children } x) \land \delta_x(\exists y \land y^k = \text{bk-kind} \land \text{read } y^k \ x) \land \text{diff}_{x,y^k} \)

If we are to keep butong-de shu as close as possible to the bare noun phrase shu ‘books’, then a natural option is to treat butong-de shu ‘different books’ also as a kind. However, instead of treating it as introducing bk-kind, which is destined for a contradiction, we can allow butong-de shu to introduce sub-kinds.

(90) different\(_{x,y} \) books\(_y^y \) := \( \lambda c.\lambda P.\exists y^k \land y^k \leq \text{bk-kind} \land P \ y^k \land \text{diff}_{x,y^k} \)

This, too, allows co-variation between the Key and the values associated with a noun phrase bearing butong. Adding the contribution of ge to (90) predicts that its monotonicity constraint can be satisfied.

Some may be concerned that the two analyses suggested above for bare noun phrases modified by butong ‘different’ are not fully parallel to how bound pronouns turn a bare noun phrase from
something incapable of co-variation to something capable of it. Recall that a bound pronoun makes a difference because the variable it contributes is interpreted inside the scope of distributive quantification and co-varies with the Key. However, although butong also bears variables, the variables are evaluated outside the scope of distributive quantification. In other words, they are not doing anything inside the scope of distributivity to give rise to co-variation. What is providing the needed help is an extra mechanism that prevents a bare noun phrase from behaving like a proper name of a kind, either in the form of an existential quantification over individuals or an existential quantification over sub-kinds. If we are serious about assimilating the helping effects of bound pronouns and adjectives like butong, we need to clarify the relationship between this extra mechanism and the presence of bound variables.

There is, in fact, an intimate relationship between them. The fact that a variable outside the scope of distributivity gets to determine what happens inside the scope of distributivity is an artifact of the assumption that butong’s contribution is evaluated outside of distributivity. This assumption was made to allow butong access to all of the values introduced by the Key and the corresponding values introduced by the noun phrase modified by butong, which is important to satisfying the $\text{diff}_{x,y}$ test. However, the present semantics of distributivity only allows access to these values when butong is interpreted outside the scope of distributivity. If butong is interpreted inside the scope of distributivity, the variables on it can never find all of the values in the Key nor all of the values associated with the noun phrase that contains it. It can only access a single pair of these values as evaluation is split up by distributive quantification.\(^{23}\) Ideally, if we can devise a semantics for distributive quantification that allows butong to access values inside the scope of distributive quantification as if it is outside, then we can evaluate butong inside the scope of distributive quantification and have itself be the extra mechanism that gives rise to co-variation. Although I will not try to upgrade the present semantics for distributive quantification to achieve this effect, I will sketch below how to analyze the sentence-internal reading of butong in a closer fashion as bound pronouns with the help of butong’s sentence-external reading. The reasoning follows largely the discussion in Brasoveanu (2011).

Let us go back to how a pronoun interacts with kind formation to give rise to sub-kinds. Observe

\(^{23}\) A similar problem is pointed out by Nouwen (2003) regarding the use of collectively interpreted pronouns inside the scope of distributive quantification.
that when nominalization applies to a predicate with a variable, the kind hence formed depends on the value assigned to the variable.

(91) \[ \text{anml-kind-x-likes} = \cap y. \text{animal} \land \text{like} \land y \]

When this kind-level object falls inside the scope of distributivity, as in (92), it may receive a plurality of values, giving rise to a plurality of kinds:

(92) \[ \delta_x(\exists y^{k} \land y^{k} = \text{anml-kind-x-likes}) \]

The above formula returns, for every value assigned to \( x \), a potentially different kind of animal stored in \( y^{k} \). Now, if we assume this is precisely what happens with kind formation involving an adjective like Mandarin \textit{butong} or English \textit{different}, we need to interpret the adjective with its bound variables inside kind formation. Let me exemplify this first with a sentence-external reading below and later generalize the idea to the internal reading. The main assumption, following previous studies (Heim 1985, Carlson 1987, Beck 2000, Barker 2007, Brasoveanu 2011), is that adjectives like \textit{butong} bear an anaphoric index pointing to an object for comparison. With this assumption, the external reading can be modeled as involving cross-sentential anaphora (Brasoveanu 2011).

(93) a. Zilu kan-le Lunyu\textsubscript{y}.
   Zilu read-ASP the.Analects
   ‘Zilu read the Analects.’

b. \( \exists x \land x = zl \land \exists y \land y = ly \land \text{read} y x \)

(94) a. Ziyou kan-le butong\textsubscript{y} y\textsuperscript{k}-de shu\textsubscript{y}\textsuperscript{k}.
   Ziyou read-ASP different-MOD book
   ‘Ziyou read books different from the Analects.’

b. \( \exists x' \land x' = zy \land \exists y^{k} \land y^{k} = \text{bk-kind-disjoint-from} y \land \text{read} y^{k} x \)

(95) \[ \text{bk-kind-disjoint-from} y := \cap z. \text{book} z \land z \neq y \]

Suppose (94-a) is interpreted following (93-a), which introduces a salient person \textit{Zilu} and a salient book \textit{Lunyu}, as shown in the interpretation in (93-b). The noun phrase \textit{butong-de shu} introduces a kind term. However, instead of picking out \textit{bk-kind}, it picks out a slightly smaller object, namely,
a kind-level object that does not include the value assigned to \( y \) as a part. This is notated as \textit{bk-kind-disjoint-from-} \( y \) in the interpretation sketched in (94-b). The formation of this kind is shown in (95). Since \( y \) is a variable, the predicate subject to nominalization may vary depending on the value assigned to \( y \). As a consequence, the output of nominalization also may deliver different kinds for different values assigned to \( y \). This is how kind formation can take advantage of the sentence-external reading of \textit{butong} to generate different kinds based on the variable present on the adjective.

Based on cross-linguistic generalizations, Brasoveanu (2011) argues that the analysis for the sentence-external reading of adjectives like \textit{different} should be extended to a particular version of \textit{different} with a sentence-internal reading, known as ‘singular \textit{different}’ (see also Carlson 1987, Beck 2000 for the distinction). The precise classification of \textit{different} is not important to the discussion here. What is important is that the internal reading of adjectives like English \textit{different} and Mandarin \textit{butong} can be decomposed into a series of external readings with an appropriate semantics for distributive quantification. Take (96) as an example. The decomposition allows books read by Zilu to be determined based on books read by Ziyou and vice versa, inside the scope of distributive quantification.

(96) Zilu he Ziyou ge kan-le butong-de shu.
    Zilu and Ziyou GE read-ASP different-MOD book
    ‘Zilu and Ziyou each read different books.’

The contribution of the sentence-internal reading of \textit{butong}, with an enriched semantics of distributive quantification, patterns much more closely with the contribution of a bound pronoun. The present semantics of distributivity does not handle this kind of decomposition. However, Brasoveanu (2011) and Bumford (2015) have both proposed semantics of distributive quantification that can handle the decomposition.

---

\(^{24}\)Mandarin also makes a distinction between a singular \textit{butong} and a number-neutral \textit{butong}. The former is only compatible with overtly marked atomic distributivity (e.g., when a \textit{meige} NP ‘every NP’ or \textit{ge} is present). The latter does not require atomic distributivity.
5.3.5 *Respective* distributivity

In this section, I offer a formulation of *respective* distributivity in DPILM based on the analysis of Gawron and Kehler (2004). Chaves (2012) and Kubota and Levine (2016) have offered alternative analyses of the same set of phenomenon. However, since modeling the *respective* interpretation is not the main goal of this chapter, I pick Gawron and Kehler (2004)'s proposal because it can be easily transformed into a plural logic without the assumptions of events (used in Chaves (2012)) or multisets (used in Kubota and Levine (2016)).

At the heart of Gawron and Kehler (2004)'s proposal is the *respective* distributive operator in (97) (to avoid notational confusions, I have swapped their $g$ (group) for $x$, which is variable for both atomic individuals and plural individuals in the present work):

\[(97) \quad \text{RESP}_f := \lambda P \lambda x. \bigcup_{1 \leq i \leq |f|} [f(P)(i)](f(x)(i)) \quad \text{(Gawron and Kehler 2004:14)}\]

According to Gawron and Kehler (2004), $f$ is a pragmatically available sequencing function. With the help of this function, the *respective* distributive operator takes a property sum $P$, a plural individual $x$, and returns a proposition sum. The proposition sum is the collection of all propositions that is obtained by applying one property from the property sum to one individual in the plural individual. The application is guided by the sequencing function $f$. $f$ breaks down a plurality (a plural individual or a property sum) into sub-pluralities (typically atoms) and labels each sub-plurality with a bigger integer starting from 1. The set of integers used for labeling the sub-pluralities is the cardinality of $f$, i.e., $|f|$. The integers serve as a probe for $f$ to find a particular sub-plurality.

Function Application of one plurality to another plurality is guided by the integers assigned to the two pluralities, such that $f(P)(1)$ is applied to $f(x)(1)$ and $f(P)(2)$ is applied to $f(x)(2)$, so on and so forth. $f$ is additionally required to satisfy the following requirements, according to Gawron and Kehler (2004) (pp.173–174).

\[(98) \quad \text{Requirements on a sequencing function}\]

\[\begin{align*}
\text{a.} & \quad \text{Same cardinality: all pluralities that serve as arguments to } \text{RESP}_f \text{ must have the same cardinality.} \\
\text{b.} & \quad \text{Proper subgroups: for each } x \text{ and } i, f(x)(i) \text{ picks out a proper subpart of } x.
\end{align*}\]
c. Exhaustivity: summing up all the sub-pluralities generated by \( f \) on \( x \) returns \( x \), i.e.,
\[
\bigcup_{1 \leq i \leq |f|} f(x)(i) = x
\]

These requirements are intended to explain the restricted distribution of *respective* distributivity. The same cardinality requirement predicts that a typical cumulative interpretation that lacks information about one-to-one correspondence between the sub-pluralities in two pluralities does not give rise to a *respective* interpretation:\(^{25}\)

\[(99) \quad \text{Five hundred companies used six hundred computers, (*respectively).}\]

The requirement on proper subgroups makes sure that a plurality and a singleton do not license respective distributivity:

\[(100) \quad \text{John and Mary saw Peter, (*respectively).}\]

Lastly, the exhaustivity requirement rules out cases in which \( f \) does not pick out all the parts in a plurality. For example, if there is an \( f \) that, for all integers, only picks out *John* from the plurality *John and Mary* in (101), and the property *jogged* from the property sum *jogged and swam*, then this \( f \) is not usable for *respective* distributivity because it fails exhaustivity. If such a function were used, (101) would be true as long as it is true that John jogged.

\[(101) \quad \text{John and Mary jogged and swam, respectively.}\]

Given the definition in (97) and the requirements in (98), (102) can be evaluated as in Table 5.1.\(^{26}\)

---

\(^{25}\)Determining the cardinality of a plurality is not as straightforward as just finding out all the atoms in the plurality. Gawron and Kehler (2004) discuss pluralities involving duplicate parts, such as the coordinated VP in (i-a) below. If pluralities are treated as sets or sums, then the coordinated VPs in (i-a) and (i-b) have the same cardinality, namely, 2. However, the fact that (i-a) is acceptable while (i-b) is not suggests that some way is needed to model pluralities with duplicate parts. In this study, I assume, along the lines of Gawron and Kehler (2004), that duplicates can be represented, but remain open as to how to model them. One possibility is discussed in Kubota and Levine (2016), who model pluralities using multisets, i.e., sets that allow for duplicate occurrences of identical elements.

\[(i) \quad \text{a. Sue, Karen, and Bob jog, drive, and jog respectively.}\]
\[\text{b. #Sue, Karen, and Bob jog and drive respectively.}\]

\(^{26}\)Given that a sequencing function is pragmatically determined, (102) in principle allows a different \( f \) that pairs Zilu with dancing and Ziyu with singing. I believe this is true, as (102) is not a contradiction.
(102) Zilu he Ziyou fenbie chang-le ge he tiao-le wu. Zilu and Ziyou respectively sing-ASP song and dance-ASP dance ‘Zilu and Ziyou sang and danced, respectively’

<table>
<thead>
<tr>
<th>Individual sum</th>
<th>Property sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(zl \oplus zy)(i)$</td>
<td>$f(\text{sing} \oplus \text{dance})(i)$</td>
</tr>
<tr>
<td>1</td>
<td>$zl$</td>
</tr>
<tr>
<td>2</td>
<td>$zy$</td>
</tr>
</tbody>
</table>

Table 5.1: Respective distributivity as in Gawron & Kehler (2004)

There are close connections between Gawron and Kehler (2004)’s analysis of respective distributivity and distributivity in a plural logic (DPIL/DPILM/PCDRT).27 The sequencing function $f$ plays a similar role as a set of assignments in three respects. First, in Gawron and Kehler (2004), $f$ splits up a plurality, whereas in a plural logic, a set of assignments splits up a plurality. Second, $f$ establishes a correspondence relation between parts in two pluralities, whereas the same job is tasked to a set of assignments in a plural logic. Third, the correspondence relation established by $f$ allows an evaluation to proceed pair by pair, giving rise to distributivity. In a plural logic, since a set of assignments is used to store a correspondence relation, distributivity is achieved by splitting up the set of assignments. In addition, both frameworks return a plurality as the result of (respective) distributivity. In Gawron and Kehler (2004), the result is a proposition sum (i.e., a list of propositions), whereas in a plural logic, the result is a set of assignments, which stores information that can verify a proposition sum. Based on these similarities, Gawron and Kehler (2004)’s insights can be straightforwardly translated into the plural logic used in this dissertation.

To ease into the discussion, let us first consider a sentence with two coordinate noun phrases under a simple cumulative interpretation.

(i) Zilu he Ziyou fenbie chang-le ge he tiao-le wu. Zhunquedeshuo, Zilu tiao-de wu, Ziyou Zilu and Ziyou respectively sing-ASP song and dance-ASP dance Precisely Zilu dance-DE dance Ziyou chang-de ge. sing-DE song ‘Zilu and Ziyou sang and danced, respectively. More precisely, Zilu danced and Ziyou sang.’

27This is not surprising because respective distributivity is also closely related to ordinary distributivity, as pointed out in Gawron and Kehler (2004).
As suggested in Section 2.3 of Chapter 2, this sentence receives the interpretation in (104). The outcome is a collective/cumulative interpretation, which does not encode any dependency between the two variables $x$ and $y$.

(104) $\text{max}^x (x = zl \oplus zy) \land \text{max}^y (y = zx \oplus zs) \land \text{saw} \, y \, x$

A respective interpretation differs minimally from a collective/cumulative interpretation in the addition of a respective distributive operator. The respective distributive operator has flexible arity. In (103), I assume that it is adjoined to the transitive verb and takes the verb as one of its arguments. It takes two other arguments, in the form of two d-refs.\footnote{It also takes a sequencing function as an argument, but I have suppressed the representation of the function.} One d-ref stores the plurality contributed by the coordinated subject noun phrase and the other stores the plurality contributed by the coordinated object noun phrase. These two d-refs will be used to construct a pair of new d-refs subject to distributive evaluation, as shown in (105).\footnote{When a coordinated VP, such as jogged and swam, is involved, the respective distributive operator then only has two arguments, i.e., a pair of d-refs. One d-ref stores a plural individual and the other stores a property sum.}

(105) $\text{max}^x (x = zl \oplus zy) \land \text{max}^y (y = zx \oplus zs) \land \text{Resp}^{x', y'}_{x, y} (\text{saw} \, y' \, x')$

The definition of $\text{Resp}^{x', y'}_{x, y} (\phi)$ is given in (106). It can be divided into two parts: a pair-wise variable introduction, notated as $\text{Resp}^{x', y'}_{x, y}$ and defined in (106-a), and a distributive evaluation of $\phi$, defined in (106-b).\footnote{In sentences with more than two coordinated noun phrases that exhibit respective distributivity, such as (i), pair-wise variable introduction will need to be generalized to tuple-wise variable introduction.}

(i) John and Mary wanted to give a book and a pen to Sue and Jane, respectively.
b. For all $a \in H'(x'). H'|_{x'=a} \models \phi \models H|_{x'=a}$

The pair-wise variable introduction rule introduces two new d-refs $x'$ and $y'$ based on two extant variables $x$ and $y$ and a sequencing function $f$ as defined in Gawron and Kehler (2004). $x$ and $y$ are independently introduced with use of two coordinated noun phrases and by default do not stand in any dependence relation. However, the variables $x'$ and $y'$ are not introduced by the default variable introduction rule, i.e., $\exists x' \land \exists y'$. In particular, (omitting the subscripted d-refs for the moment) with $\text{Resp}^{x', y'}$ each assignment $g$ in the input $G$ simultaneously updates, in a pair-wise fashion, the $x'$ slot and the $y'$ slot using the pairing information provided by a sequencing function $f$. Transferring the requirements on $f$ from Gawron and Kehler (2004)’s framework to the present one ensures that $x$ and $y$ are both plural and their parts stand in a correspondence relation. For this reason, $x'$ and $y'$ end up standing in a dependence relation, a necessary ingredient for satisfying the monotonicity constraint of $ge$. By assuming that this special variable introduction rule is available only when there is a salient sequencing function inferable from the context has the effect of allowing a respective interpretation and the associated pair-wise variable introduction rule only when all the requirements on the sequencing function are met.\(^{31}\)

The distributive evaluation of $\phi$ is facilitated by the good old distributive operator $\delta_{x'}$ we have been using throughout the dissertation.\(^{32}\) It splits up the evaluation along the $x'$-dimension and checks that in each sub-info-state storing one $x'$-value, evaluation of $\phi$ leads to at least one output info-state. A graphical illustration of the interpretation of (105) is given in Figure 5.4.

\(^{31}\)Chaves (2012) notes that a pair of definite plurals can give rise to a respective interpretation if they contain a matching numeral, as shown below (data cited from Chaves 2012: (7a)):

(i) The three best students received the three best scores respectively.

Similar sentences in Mandarin can also support $ge$:

(ii) Zhe-san-ge xuesheng ge kan-le na-san-ben shu.

\begin{quote}
this-three-cl student GE see-ASP that-three-cl book
\end{quote}

‘These three students read those three books, respectively.’

This shows that respective distributivity is not confined to coordinations. I leave the actual factors governing the emergence of respective distributivity for future research.

\(^{32}\) $x'$ is the variable storing the atomic sub-pluralities extracted from the first mentioned coordinated phrase. Since $x'$ and $y'$ are introduced in a pair-wise manner and their values stand in a one-to-one correspondence, swapping $x'$ with $y'$ does not cause any problem, provided there is a way to model duplication.
As one can see, after evaluating (105), any info-state in the output will store two variables that exhibit dependence. It is for this reason that $ge$ is licensed by respective distributivity. As shown more concretely in (107), $ge$ can use the variables $x'$ and $y'$ introduced by pair-wise variable introduction to satisfy the monotonicity constraint.

\[
\text{max}^x(x = zx \oplus zs) \land \text{max}^y(y = zx \oplus zs) \land \text{Resp}^{x', y'}(\text{saw} y x') \land \text{dm}_{x', y'}
\]

In short, evaluating respective distributivity gives rise to info-states that encode variable dependence, which $ge$ can use to satisfy the monotonicity constraint. By treating $ge$ as a marker contributing a monotonicity constraint, the difficult question of how $ge$ gets to contribute two distinct types of distributive operators is resolved: it does not contribute these distributive operators; it merely requires distributivity and is compatible with more than one type of distributivity.

Our discussion of $ge$ allows us to see how a monotonicity constraint that allows both dependent individuals and dependent measurements conditions the distribution of a distributive marker like
Mandarin \emph{ge}. In what follows, I discuss another direction in which binominal \emph{each}’s monotonicity constraint can be generalized. More specifically, I take up the event differentiation condition observed for determiner \textit{each} (Vendler (1962), Tunstall 1998, Brasoveanu and Dotlačil 2015) and show that it can be understood as a monotonicity constraint involving events and their thematic dimensions.

5.4 Previous studies on Mandarin \textit{ge}

The licensing requirements of \textit{ge} have been reported in a wide variety of syntax and semantics studies, including Kung (1993), Lin (1998b), Soh (2005), Lee et al. (2009a), Tsai (2009), Li and Law (2016). They have been used to argue for analyses that treat \textit{ge} as a pairing function (Lin 1998b, Lee et al. 2009a), as a respective distributive operator (Tsai 2009), and as an anti-cumulativity operator (Li and Law 2016). In the next three subsections I review these analyses and compare them with the present work.

5.4.1 \textit{Ge} as a pairing function

Both Lin (1998b) and Lee et al. (2009a) suggest that \textit{ge} contributes a pairing function, which pairs values drawn from two sets. Since these two studies do not consider compositionality, it is not clear how the two sets are derived. However, it is natural to assume that the first set is contributed by the Key and the second set can be contributed by a distributively evaluated licensing expression in the Share.\footnote{For Lin (1998b) only a counting quantifier is eligible to provide the second set of values. However, Lee et al. (2009a) extend the range of eligible expressions to include those that contain bound pronouns and coordinated phrases.} Since the pairing function essentially pairs values in the Key with corresponding values in the Share respecting the dependency established by distributivity, some way of accessing a quantificational dependency is necessary for their studies. Once these additional assumptions are in place, the pairing function can be seen as a correlate of the monotonicity constraint in the sense that they both serve to constrain the output of distributivity.

Despite the similarities, there is an essential difference between the pairing function and the monotonicity constraint. Although Lin (1998b) and Lee et al. (2009a) do not give definitions of the pairing function contributed by \textit{ge}, from their descriptions it is clear that the pairing function either requires total co-variation (for Lin 1998b) or at least partial co-variation (Lee et al. 2009a).
As such, the pairing function in Lin (1998b) and Lee et al. (2009a) essentially requires value dependence and belong in the same class as the evaluation plurality requirement in Henderson (2014) and Kuhn (2017). While value dependence can adequately capture the fact that ge requires co-variation when the second set consists of individuals (i.e., when the dependent variable stores individuals), it does not handle ge’s sensitivity to the distinction between intensive measure phrases and extensive measure phrases. As I have argued earlier, this type of distinction can only be adequately captured if the mereological structure of two sets of is taken into consideration. In short, the monotonicity constraint has a wider empirical coverage than the pairing function posited in Lin (1998b) and Lee et al. (2009a).

5.4.2 Ge as a respective distributive operator

Tsai (2009) argues that ge is a special distributive operator (called a summation operator), which has a semantics based on RESP_f, the distributive operator that gives rise to respective distributivity in Gawron and Kehler (2004). Tsai (2009) also assumes that ge bears a differentiation condition, which requires the VP following ge to contribute a proper plurality. I illustrate Tsai’s analysis with (108). As RESP_f, ge takes two arguments, as shown in (109). The two arguments are a plural NP and a plural VP, modeled as an individual sum and a property sum, respectively, following Gawron and Kehler 2004’s analysis. RESP_f pairs each individual in the plural NP with a property in the property sum, and obtains a proposition sum.

(108) Zilu he Ziyou ge chang-le ge he tiao-le wu.
Zilu and Ziyou GE sing-ASP song and jump-ASP dance
‘Zilu and Ziyou sang a song and performed a dance, respectively.’

(109) RESP_f (sing ⊔ dance) (Zilu ⊕ Ziyou)

There are important similarities in Tsai’s analysis in terms of respective distributivity and the monotonicity constraint analysis put forward in this study. For one thing, they both predict ge’s compatibility with respective distributivity. In addition, both analyses predict that ge requires two pluralities.

In this sense, the differentiation condition can be reduced to the proper subpart requirement of the sequencing function.
For Tsai, this is necessary for satisfying the proper subpart requirement of the sequencing function as well as the differentiation condition. For the present study, this is necessary for satisfying the monotonicity constraint, which requires order preservation.

However, Tsai’s analysis differs from the present analysis in how the plurality is obtained. According to Tsai, the plurality is obtained before distributive quantification, as an inherent feature of the VP subject to respective distributivity. In fact, Tsai argues that differentiation condition presupposes that ge must combine with a VP denoting a property sum. In the present study, the plurality is obtained after distributive quantification. This timing difference makes distinct predictions on what types of VP are compatible with ge.

For the present analysis, even if there is no coordination in a VP, as long as there is an eligible expression that may co-vary with distributivity, a plurality can be obtained after distributive quantification to help satisfy the monotonicity constraint.

Tsai (2009), on the other hand, predicts that only VPs that inherently denote a property sum can readily combine with ge. As a consequence, when a VP does not provide any coordination, as in the case of (110), Tsai (2009:154) has to assume that it can be shifted into a property sum with help of pragmatics.

\[(110)\quad \text{Zilu he Ziyou ge du-le yi-ben shu}\]
\[
\text{Zilu and Zilu GE read-ASP one-CL book}\]
\[
\text{‘Zilu and Ziyou each read a book.’}\]

The precise relationship between pragmatics and this shift in meaning is not clearly given in Tsai (2009). However it is resolved, a similar pragmatics also has to be extended to account for bound pronouns and sentence-internal adjectives, which also license ge. The present study does not need to posit such a pragmatic mechanism. Counting quantifiers, pronouns, and sentence-internal adjectives pattern together because when they are interpreted in the scope of distributivity, they can help give rise to a plurality, which in term supports ge. In short, the plurality can be said to be derived semantically in the present analysis, rather than pragmatically, as suggested in Tsai.
5.5 Remaining issues

In this section, I discuss two issues not directly addressed in the present study on Mandarin ge and their implications for the analysis. The first issue, discussed in Section 5.5.1, pertains to the locality conditions of ge and the second issue, discussed in 5.5.2 concerns an additional licensing condition of ge, namely, wh-questions with a pair-list interpretation.

5.5.1 Locality conditions of ge

Ge exhibits interesting locality conditions both in relation to the Key and the licensing expression in the Share. These locality conditions have not been taken up in this chapter and will be reserved for future research. To begin with, observe that ge cannot be embedded in a syntactic island or be separated from the Key by a clausal boundary (the island/embedded clause is enclosed in a pair of square brackets):

(111) *Meige ren dou tingshuo-le [Zilu ge jiejue-le yi-ge nanti de xiaoshi]
    Every person DOU heard-ASP Zilu GE solve-ASP one-CL puzzle de news
    ‘Every boy heard the news that Zilu solved one puzzle each.’

(112) *Meige ren dou tingshuo [Zilu ge jiejue-le yi-ge nanti.]
    every person DOU heard Zilu GE solve-ASP one-CL puzzle
    ‘*Every boy heard that Zilu solved one puzzle each.’

Some may suspect that the unacceptability is due to the fact that ge linearly follows dou, which is taken to contribute a distributive operator. In fact, when both ge and dou are inside the same clause, they may co-occur in any order. For example, ge follows dou in (113) (repeated from (1)) but precedes it in (114). 35

(113) Tamen dou ge mai-le yi-ben shu.
    they DOU GE buy-ASP one-CL book
    ‘They each bought a book.’

(114) Tamen ge dou mai-le yi-ben shu.
    they GE DOU buy-ASP one-CL book
    ‘They each bought a book.’

35 The order dou > ge is preferred in this particular example but ge > dou is also a commonly attested order (Lee et al. 2009b).
For this reason, I take the unacceptability of (111) and (112) to indicate that ge must be in the same scope domain as the distributive operator for it to be interpreted outside the scope of the distributive operator. (111) and (112) are ruled out because ge will have to undergo QR beyond the island in the former and beyond the clausal boundary in the latter to be in the same scope domain as the distributive operator. The locality condition of ge in relation to the Key is reminiscent of similar locality conditions of binominal each. For example, the following sentence (repeated from (31) in Chapter 3), shows that binominal each also cannot be separated from the Key by a clausal boundary:

(115) ??The linguists thought two theories each were refuted. (Simon Charlow, p.c.)

In Chapter 3, locality conditions of this kind have been used to defend the view that binominal each undergoes QR to take split scope over distributivity. The fact that ge shows a similar locality condition suggests that it, too, may need to undergo QR to be interpreted outside the scope of distributivity. When it is embedded inside an island or a clause, this movement leads to unacceptability, as shown in Figure 5.5, which is the structural representation of (111)/(112).

![Figure 5.5: Banned QR of ge from a syntactic island or an embedded clause](image)

Besides showing locality conditions in relation to the Key, ge also exhibits locality conditions in relation to the licensing expression in the Share. In particular, ge cannot be separated from its licensor by a clausal boundary or a syntactic island, as exemplified by the following sentences:
(116) *Meige laoshi ge tingshuo/renwei [Zilu xie-le yi-pian wenzhang].
every teacher GE think Zilu write-ASP one-CL article
‘Every teacher heard/thinks that Zilu wrote an article.’

(117) *Meige laoshi ge tingshuo/renwei [Zilu kan-le ziji xian-de shu].
every teacher GE heard/think Zilu read-ASP self write-MOD book
‘Every teacher heard/thinks that Zilu read a book he or she wrote.’

(118) *Meige laoshi ge sheng-le taolun yi-ge shuxue nanti de wenzhang.
every teacher GE review-ASP one-CL discuss math puzzle MOD article
‘Every teacher reviewed articles that discussed a math puzzle.’

(119) *Meige laoshi ge sheng-le taolun ziji-de lilun de wenzhang.
every teacher GE review-ASP discuss self-POSS theory-MOD article
‘Every teacher reviewed articles that discussed their theories.’

In (116) and (117), the licensing expressions are a counting quantifier and a bound pronoun embedded in a finite clause, respectively. In (118) and (119), the licensing expressions are a counting quantifier and a bound pronoun embedded in a complex NP island. The discovery of data like (118) – (119) requires some qualification to my previous claim that ge does not need to be adjacent to its licensor in the Share: if ge is only related to its licensor via dynamic binding, there is no reason why locality conditions of this form should be observed as dynamic binding is insensitive to island boundaries or clausal boundaries.36

The qualification, I suggest, points to the direction that ge is not just related to its licensor by dynamic binding, but also by a syntactic mechanism. In particular, ge may originate from a position that forms a constituent with its licensor. It then undergoes syntactic movement to be in a pre-verbal position. Although previous syntactic studies have not considered the possibility that ge undergoes movement from within its licensor, they have demonstrated that ge exhibits some flexibility in its

36Typical cases of donkey anaphora involve an indefinite introduced in a complex NP island:

(i) Every farmer who owns a donkey beats it.

Cases involving cross-sentential binding shows that dynamic binding is insensitive to clausal boundaries:

(ii) A man came in. He sat down.

Charlow (2014) argues that dynamic binding and exceptional scope should be understood as two sides of the same coin. If Charlow’s thesis is right, then the ability of an indefinite to take exceptional scope out of a certain domain (say, a syntactic island) should entail its ability to dynamically bind a d-ref outside that domain.
surface position as long as it adjoins to a VP/V’ category (Lin 1998b) or a vP/VP category (Soh 2005).

5.5.2 Licensing by pair-list questions

Besides counting quantifiers, extensive measure phrases, noun phrases with bound pronouns, respective distributivity and adjectives with a sentence-internal reading, there is another condition in which ge is licensed, namely, in wh-questions with a pair-list interpretation (Lin 1998b, Lee et al. 2009a).

Consider (120-a), in which distributivity is marked with *dou*. This question admits a pair-list answer, such as (120-b), as well as a single answer, such as (120-c).37

(120) a. Zhe-xie haizi dou kan-le shenme shu?
   this-CL.PL child DOU read-ASP what book
   ‘What book did these children each read?’

   b. Zilu kan-le Emma, Ziyou kan-le Jane Eyre, Mali kan-le Pride and
   Zilu read-ASP Emma  Ziyou read-ASP Jane Eyre  Mary read-ASP Pride and
   Prejudice
   ‘Zilu read Emma, Ziyou Jane Eyre, and Mary Pride and Prejudice.’  
   Pair-list

   c. Tamen kan-le Emma.
   they  read-ASP Emma
   ‘They read Emma.’  
   Single

However, once the distributive marker becomes *ge*, as in (121-a), only the pair-list answer (121-b) is acceptable. The single answer (121-c) is infelicitous.

(121) a. Zhe-xie haizi ge kan-le shenme shu?
   this-CL.PL child GE read-ASP what book
   ‘What book did these children each read?’

   b. Zilu kan-le Emma, Ziyou kan-le Jane Eyre, Mali kan-le Pride and
   Zilu read-ASP Emma  Ziyou read-ASP Jane Eyre  Mary read-ASP Pride and
   Prejudice.
   ‘Zilu read Emma, Ziyou Jane Eyre, and Mary Pride and Prejudice.’  
   Pair-list

37 A single answer is also known as an ‘individual answer’ (e.g., Engdahl 1986, Groenendijk and Stokhof 1984, Krifka 2001b).
Binominal *each* can also be licensed by *wh*-expressions, as shown in (122) (Safir and Stowell 1988). However, they must be *how many*-NPs, which have a measurement component due to the presence of *many* (Hackl 2000, Schwarzschild 2006, Kennedy 2015, a.o.).

(122) How many books each did the girls read?

(123) How many girls each did the men see? (Safir and Stowell 1988:(5a))

In fact, it is the measurement in (122) that licenses binominal *each*, rather than the pair-list interpretation. This is because a felicitous answer to this question may be a pair-list answer (e.g., five, three, and six) or a single answer (e.g., five). Using a *wh*-phrase that lacks a measurement component does not license binominal *each*, as shown in (124).

(124) *What books/which book each did the girls read?*

To capture the licensing effect of a *wh*-question with a pair-list reading, the current dynamic semantics will have to be extended to model questions and their pair-list interpretations. There are multiple directions for the extension, depending on one’s choice of a question semantics and one’s assumptions about the pair-list phenomenon. However DPLLM is extended to handle questions with a pair-list interpretation, it is important to set apart *wh*-questions with a distributive quantifier from multiple constituent *wh*-questions (cf. Groenendijk and Stokhof 1989). As noted in many studies (e.g., Higginbotham and May 1981, Dayal 1996), multiple constituent *wh*-questions in English can also give rise to pair-list interpretations. Similar observations have been made on multiple constituent *wh*-questions in Mandarin (Liao and Wang 2007). However, multiple constituent *wh*-questions cannot license *ge*, regardless of the number marking on *wh*-expressions:

---

38For example, a question may be modeled as a function (Hausser and Zaefferer 1979, Xiang 2016), a structured meaning (von Stechow and Zimmerman 1984, von Stechow 1991, Krifka 2001a), a set of propositions (i.e., a set of sets of possible worlds, Hamblin 1973, Karttunen 1977), or a set of mutually incompatible propositions (i.e., a set of partitions of possible worlds, Groenendijk and Stokhof 1984, 1989). Questions expecting a pair-list answer may be modeled as involving Skolem functions (Engdahl 1986, Chierchia 1993, Dayal 1996), quantification into speech acts (Karttunen 1977, Krifka 2001b), or a family of questions (Groenendijk and Stokhof 1989).
(125) *Shei ge kan-le shenme shu?
   who GE read-ASP what book
   'Who read what?'

(126) *Na-xie ren ge kan-le na-xie shu?
   which-CL.PL people GE read-ASP which-CL.PL book
   'Which people read which books?'

The unique role of the pair-list interpretation of a wh-question involving a distributive quantifier is reserved for future research.

5.6 Extension: Generalized monotonicity with events

We have paid exclusive attention to Mandarin ge so far in this chapter. In what follows, I outline an extension of generalized monotonicity to include events. The payoff of including events in our logic is an account for the so-called event differentiation condition of English each in the determiner position and an adverbial position.

To see the effect of event differentiation, consider (127) and (128), of which Vendler (1962) notes a difference:

(127) Take every apple.

(128) Take each apple.

He notes that with (127) the speaker doesn’t care whether the apples are being taken one by one or together. However, with (128) the speaker intends for the apple to be taken one by one. This contrast has been taken up further in Beghelli and Stowell (1997) and Tunstall (1998) as evidence that every and each have distinct grammatical properties. Tunstall (1998) posits an event differentiation condition to distinguish between quantification with every and quantification with each.39

39 Every NPs in an object position may receive a non-distributive use, as observed in Schein (1993) and Kratzer (2002). For this reason, the contrast in (127) and (128) does not provide sufficient evidence that it is the distributive reading arising from the use of every and each that have distinct properties. A more convincing contrast is provided below:

(i) a. Every student left.
   b. Each student left.

(i-a) is compatible with all the students leaving at the same time as a group but (i-b) is not. Every NPs do not generally give rise to a non-distributive use in the subject position, as evidenced by (ii) (see also Gil 1995) (with the exception of
condition is provided in (129).

(129) Differentiation Condition (Tunstall 1998:100):

A sentence containing a quantified phrase headed by *each* can only be true of event structures which are totally distributive. Each individual object in the restrictor set of the quantified phrase must be associated with its own subevent, in which the predicate applies to that object, and which can be differentiated in some way from the other subevents.

Although Tunstall’s attempt to experimentally verify the differentiation condition was unsuccessful, Brasoveanu and Dotlačil (2015) were able to experimentally verify the effect of the event differentiation condition.

At a descriptive level, the event differentiation condition is very similar to the variation requirement of Mandarin *ge*. The only difference seems to be the locus of the variation: the variation requirement of *ge* is imposed on the individual values associated with its host, while the event differentiation condition of determiner *each* is imposed on the event values, presumably introduced by a verbal predicate interacting with distributivity.

To visualize the similarity, we can turn Tunstall (1998)’s event differentiation condition into an event differentiation constraint accompanying distributive quantification, as shown in (130-b) and interpreted in (130-c):

(130)  

a. John took each apple.

b. \[\max^x(\text{apple } x) \land \delta_x(\exists e \land \text{take } e \land \text{ag } e = j \land \text{th } e = x) \land \text{ed}_{x,e}\]

c. \[G[H := \top \text{ iff } G = H \text{ and } \forall a, b \in G(x) : G|_{x=a}(e) \neq G|_{x=b}(e)\]

(130-c) says every assignment that assigns a different apple to *x* should assign a different value to *e*. In other words, each apple-taking event is differentiated from the other.

However, at a more technical level, the event differentiation condition is almost always trivially

\[\text{everyone}, \text{which is compatible with collective predicates, as in everyone gathered.).}\]

(ii) *Every man gathered in a dimly-lit alleyway. (Syrett (2019:(7a))

The contrast in (i-a) and (i-b) provides better support for the claim that distributivity with *every* and *each* are different.
satisfied. Suppose there are three apples: apple1, apple2, and apple3, and John took them all at once. Intuitively, the event differentiation condition is violated. However, (130-b) is not violated. This is because for each apple, there is an apple-taking subevent in which John is the agent and the apple is the theme. If we assume the widely held principle, due to Carlson (1998), that events with distinct thematic participants are distinct events, these apple-taking events are all differentiated, by virtue of having different apples as their themes. In short, event differentiation should be trivially satisfied.

To resolve this problem, Tunstall (1998) suggests instead of comparing event values, we compare the thematic dimensions of the relevant events. Correspondingly, this means an enrichment of the event differentiation constraint along the lines in (131). The contribution of the thematic relation \( \theta \), as spelled out in (132), is to relate events with their thematic participants. \( \theta \) here may be resolved to agent, theme, location, temporal trace, or other relevant thematic dimensions.\(^40\)

\[
G \% \text{ed}_x,e(\theta)\% H := T \iff G = H \text{ and } \forall a,b \in G(x) : \theta(\bigoplus G_{x=a}(e)) \neq \theta(\bigoplus G_{x=b}(e))
\]

\[
\theta := \lambda e \lambda x. \theta e = x
\]

We are no longer comparing the values in \( e \), but a thematic dimension of the values in \( e \). If two different events happen to yield the same value along a certain thematic dimension \( \theta \), then the event differentiation constraint in (130-c) is satisfied but the version in (131), which appeals to thematic transformation, is violated. So, if John took all the apples in one fell swoop, (130-c) is trivially satisfied, but (131) is only trivially satisfied if \( \theta \) is set to be \( \text{th} \), i.e., the thematic dimension that happens to be the Key. Although using the thematic version of event differentiation does not completely avoid the possibility that information coming from a Key helps satisfy the differentiation constraint in a trivial manner, it is a significant improvement. To fix the triviality issue with (131), we can posit a variety of reasons to discourage (or ban) the use of the thematic dimension that also identifies the Key. However, it is unclear how to fix (130-c) without some major reconceptualization of events.

The thematic function \( \theta \) in the event differentiation constraint in (131) plays the same role as a measure function \( \mu_{\text{dim}} \) in the monotonic measurement constraint. Of course, we can turn the

\(^{40}\)I have not spelled out how \( \theta \) is obtained. I discuss a few possibilities and their implications in the next subsection.
event differentiation constraint in (131) into a full-blown monotonicity constraint, in the form of a monotonic thematic constraint, as shown in (133-a).\footnote{Here, I’m using a strictly increasing version of monotonicity to make it more in line with Tunstall (1998)’s original event differentiation condition. A weaker version may be adopted if it turns out that the some degree of overlapping is tolerated. For example, if John took each apple is compatible with a scenario in which John took all of the apples but in different batches.}

\begin{equation}
\text{(133) Monotonic thematic constraint}
\end{equation}

\[
G \ll dm_{x,e}(\theta) \gg H := T \iff G = H, \text{ and (a) and (b) below:}
\]

\begin{itemize}
  \item[a.] \( \forall A, A' \in G(x). A \subseteq A' \rightarrow \theta(\bigoplus_{x \in A} G|_{x \in A}(e)) \leq \theta(\bigoplus_{x \in A'} G|_{x \in A'}(e)) \)
  \item[b.] \( \exists A, A' \in G(x). \theta(\bigoplus_{x \in A} G|_{x \in A}(e)) \neq \theta(\bigoplus_{x \in A'} G|_{x \in A'}(e)) \)
\end{itemize}

Informally, the constraint requires a positive correlation between the size of the Key and the size of a thematic dimension of the events in the scope of distributivity. More precisely, the requirement is that the mapping from the size of the Key to a particular thematic dimension of the event plurality has to be non-decreasing and non-constant.

There is a close relationship between the monotonicity constraint of Mandarin \textit{ge}, as shown in (134) and the monotonic thematic constraint of determiner \textit{each}, as shown in (135).

\begin{equation}
\text{(134) } dm_{x,y}
\end{equation}

\begin{equation}
\text{(135) } ed_{x,e}(\theta)
\end{equation}

They both make reference to the Key as the independent variable. In addition, they both take a dependent variable. Although \( y \) stores a set of individuals and \( e \) stores a set of events, applying the thematic function \( \theta \) can turn the events in \( e \) into a set of thematic participants of those events, which can be individuals, times, locations, or other appropriate dimensions of events. If any type of eventuality can contribute \( e \) and if the \( \theta \) is allowed to be pragmatically inferred, as suggested in Tunstall (1998), then the monotonic thematic constraint is an even more flexible constraint than the monotonicity constraint of Mandarin \textit{ge}. Of course, neither of these is an innocuous assumption. More investigation of the distribution of determiner \textit{each} has to be conducted before one can proceed with any of these assumptions.
5.7 Conclusion

In this chapter, I have shown that the monotonic measurement constraint first developed in Chapter 3 for English binominal *each* can be generalized to account for distributive markers that bear partial but important resemblance to the distribution of binominal *each*. The majority of the chapter is devoted to the distributive marker *ge* in Mandarin, which requires licensing and is sensitive to the distinction between intensive and extensive measurement in the Share, just like binominal *each*. However, the expressions that license *ge* is a proper superset of the expressions that license binominal *each*. These properties have motivated a monotonicity constraint for *ge* that is compatible with mappings from individuals to degrees as well as mappings from individuals to individuals. Besides *ge*, I have taken up the event differentiation condition of determiner *each*, showing that it can be understood as a generalized monotonicity constraint satiable by monotonic mappings from individuals to thematic dimensions of events.
6

CONCLUSION

6.1 Summary

In this dissertation, I have investigated distributive markers in English, Cantonese, and Mandarin. What unites these distributive markers is the morphosyntactic and/or interpretive requirements they impose on parts of their Share. The investigation has led to both empirical and theoretical advancement in the research on distributivity and distributivity-related phenomena, as discussed below.

The empirical advancement is two-fold. First, it broadens the range of lexical items that may impose constraints on functional dependencies. Previous studies on dependent indefinites have shown that these constraints can be lexically packaged into indefinites (Farkas 1997, 2002b, Henderson 2014, Kuhn 2017; see also Gil 1988, Balusu 2005, Cable 2014). In this study, I have shown that they can also be packaged into lexical items that are more traditionally categorized as distributive markers. Distributive markers imposing such constraints can be diagnosed by various forms of licensing requirements on the Share, which are not predicted by classical theories of distributivity, such as those put forward in Link (1987, 1983), Roberts (1987), Schwarzschild (1996), Lasersohn (1995), Landman (2000), and Champollion (2010, 2017). I have argued that to properly understand these distributive markers, we must recognize their dual functions in signaling distributivity and imposing constraints on the functional dependencies that arise from such a quantification.

Second, it enriches the variety of constraints on functional dependencies. Previous studies have
already identified a few of the constraints that enforce co-variation (i.e., variable dependence), including the ‘dependency constraint’ identified in Farkas (2002b) for dependent indefinites in Hungarian, the ‘evaluation-level plurality’ constraint identified in Henderson (2014) for dependent indefinites in Kaqchikel. Based on the investigation of Cantonese distributive suffix saai, I have established a constraint that enforces variable independence as opposed to variable dependence. In addition, on the basis of the (distinct) measurement-sensitivity in binominal each, Cantonese saai, and Mandarin ge, I have argued that functional dependencies have a mereological structure that is relevant for the formulation of constraints.

The empirical advancements have motivated the development of DPILM, a variant of dynamic plural logic that not only maintains a tight connection between distributivity and variable dependence, like one of its predecessors DPIL (van den Berg 1996), but also harnesses referential pluralities (for modeling mass nouns and non-cardinality measurement) and sub-sentential compositionality, like its other predecessor PCDRT (Brasoveanu 2007, 2008).

Lastly, the current research has made a methodological contribution. Research on dependent indefinites often characterize dependent indefinites as grammatical elements that need to be licensed (Farkas 1997, 2002b, Yanovich (2005), Henderson 2014, Kuhn 2017). Distributivity is among the most common types of licensors (Yanovich 2005, Henderson 2014). For this reason, a class of dependent indefinites are also called distributive numerals (e.g., Balusu 2005, Cable 2014). An important observation emerging from this dissertation research is that many distributive markers, namely, things that are supposed to license dependent indefinites, are themselves subject to licensing requirements. What licenses them seem to be precisely the expressions that need them to be licensed, such as the dependent (or bound) interpretation of indefinites, counting quantifiers, pronouns, and adjectives with a sentence-internal reading. In other words, the ‘licensor’ has to be licensed by the presence of a licensee.

At first glance, this may seem counterintuitive: what does it mean for a licensor to demand licensing? However, the puzzle goes away once we recognize the configurational nature of licensing—a licensing always involves a licensor and a licensee. The licensing requirement is a requirement on how the two elements should interact to give rise to the desirable licensing environment. If there is sufficient regularity in the pairing, a licensing requirement may in principle fall on either element in the pair. This, I argue, is precisely what we observe with constraints on functional dependencies.
A functional dependency requires distributivity and an expression that is distributively evaluated. A constraint on functional dependencies then may fall on an expression indicating distributive evaluation or on an expression being distributively evaluated. Given the wide range of licensing phenomena in semantics, this methodological note is a call to pay attention not only to the canonical licensees, but also the licensors, which may themselves be subject to licensing requirements.

6.2 Loose ends and outlook

The central goal of this dissertation was to establish the relevance of a variety of constraints on the functional dependencies arising from distributive quantification. To facilitate the investigation, I have made simplifying assumptions about the status of distributivity and the status of the constraints. In this section, I would like to discuss some challenges to the simplifying assumptions and ways to refine the assumptions in future research.

6.2.1 The status of distributivity

Since the constraints studied in this dissertation are constraints that target the dynamic effects of distributive quantification, having distributive quantification is a pre-requisite for the felicitous computation of such constraints. However, I have chosen to be vague about the precise relationship between distributivity and the ‘distributive markers’ that impose constraints on distributivity. In particular, I have modeled distributivity by introducing a possibly covert, VP-level distributive operator that scopes over the entire VP.

There are two concerns stemming from such a distributive operator. The first one pertains to the lexical source of the operator, which may be (i) part of the lexical component of a distributive quantifier (such as every student and each teacher), (ii) contributed by an overt distributive operator, such as English adverbial each, Mandarin dou, and Cantonese cyumbou...dou, or (iii) contributed by a covert distributive operator when there is no other source of distributivity. When a distributive marker such as binominal each, saai or ge is used, one of its roles is to signal the presence of dist, presumably because the constraint is not defined when there is no distributivity. If all that matters is for dist to be present to introduce distributivity, we expect the distributive markers under investigation to be compatible with any form of dist, be it introduced by a covert distributive
operator or coming from of the lexical component of a distributive quantifier. However, this is not universally true. While distributive markers like Cantonese saai and Mandarin ge are compatible with any source of dist, binominal each is only compatible with a covert source of dist.\(^1\) One way to account for the non-uniformity is to analyze binominal each as contributing both dist and the monotonic measurement constraint, while assuming that saai and ge only contribute constraints on distributivity and not dist.\(^2\)

The second concern is related to the position of dist. As mentioned earlier, I have assumed that it is a VP-level operator. As a consequence, it is predicted that dist scopes over the entire VP. However, there is evidence that the scope of dist is delimited by the position of the distributive markers that signal its presence. Take binominal each as an example first. In the following sentences, there seems to be an asymmetry between the italicized indefinites and the hosts of binominal each.

(1) The students shared a story with two classmates each.

(2) The teachers gave a student one book each.

In particular, while the hosts can be easily interpreted inside the distributive scope of the plural subjects, the indefinites, which are inside VP but outside the scope of binominal each, have a much harder time getting a distributive interpretation.\(^3\) This asymmetry is unexpected if there is indeed a covert distributive operator at the VP level.

Cantonese saai is a verbal suffix and cannot attach to a preposition. For this reason, it is harder to test the scope of distributivity when saai only attaches to a lower (indirect) object, as the latter is introduced by a preposition, rather than a verb. However, Mandarin ge may occur in a dative construction to either precede the main verb or precede the preposition (Soh 2005), as shown in (3) and (4).

(3) Na-xie laoshi ge song-le liang-fen liwu gei yi-ge xuesheng. ‘Those teachers each gave two presents to a student.’

1Dependent indefinites in general are compatible with both overt and covert sources of dist.
2In fact, Kuhn (2017) attributes dist to be a lexical component of binominal each.
3I thank Chris Oakden (p.c.) for the judgments.
Interestingly, in (3) the indefinite liang-ben shu ‘two books’ may be interpreted inside the distributive scope of the plural subject; however, in (4) the same indefinite can only receive a collective interpretation relative to the plural subject. In fact, the indefinite, which is plural, has to be interpreted as the Key. What this suggests is that the scope of dist precisely coincides with the position of ge.

More revealingly, although dou and ge may co-occur, they can only do so when they are right next to each other, as in (5). If they are separated by any constituent, the sentence is ungrammatical, as shown in (6). I take this to suggest that the scope of dist must coincide with the surface position of ge.

In short, distributive markers that impose constraints on distributivity seem to also mark the scope position of dist, which determines the scope of distributive quantification. This is even true for distributive markers that are compatible with other overt distributive markers.

6.2.2 The status of output constraints

Constraints on distributivity are modeled as not-at-issue output constraints in this dissertation (see also Kuhn 2017, who model the variation component of dependent indefinites as presuppositions).

The motivation for treating the constraints as a not-at-issue content comes from the lack of a ‘false’ condition when distributivity is evaluated to ‘true’, as shown in the following examples:

(7) The students read one book each.
   a. A ‘true’ scenario: The students each read a book and they did not read the same book.
b. An ‘undefined’ scenario: The students each read a book and they read the same book.

(8) Di-hoksaang taai-saai jat-bun syu.
CL.PL-student read-SAAI one-CL book
a. A ‘true’ scenario: The students each read a book and they read the same book.

b. An ‘undefined’ scenario: The students each read a book and they read different books.

(9) Xuesheng-men ge kan-le yi-ben shu.
student-PL GE read-ASP one-CL book
a. A ‘true’ scenario: The students each read a book and they did not read the same book.

b. An ‘undefined’ scenario: The students each read a book and they read the same book.

Not-at-issue content, such as presuppositions, appositives, and honorifics, typically projects through other sentential operators (see Karttunen and Peters 1979, Heim 1983b, for presuppositions, Potts 2005 and Syrett and Koev 2016 for appositives, and Potts 2005 for honorifics, among many others). However, the projection properties of output constraints have not been systematically investigated. The initial conclusion based on the interaction between negation and the monotonic measurement constraint of binominal each seems to suggest a ‘scope companion’ phenomenon: an output constraint always stays in the same scope domain as distributive quantification and projects as far as distributive quantification does. In other words, when distributive quantification scopes above negation, the constraint does, too. For this reason, we detect projection of the inference associated with the constraint. However, when distributive quantification scopes under negation, the constraint also does, and hence we do not detect any project of the inference associated with the constraint. To what extent scope companion holds for other logical operators and whether it is a general phenomenon for all constraints on distributivity requires further research.

One may wonder whether inferences that piggyback on distributive quantification are not-at-issue in general. This does not seem to be true, as sentence-internal readings of same and different, which can be subsumed under the same rubric of association of distributivity (Brasoveanu 2011, Kuhn 2017), behave like an at-issue content: they are judged as either true or false, depending on the scenario:

(10) The students read the same book.

a. A ‘true’ scenario: The students read the same book.
b. A ‘false’ scenario: The students read different books.

(11) The students read different books.

a. A ‘true’ scenario: The students do not all read the same book.

b. A ‘false’ scenario: The students read the same book.

It is an interesting, and open, question as to why additional inferences that target functional dependencies should exhibit the at-issue vs. not-at-issue split.


