## NATURAL QUANTIFICATION IN OPTIMALITY THEORY

by

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# ABSTRACT OF THE DISSERTATION 

## Natural Quantification in Optimality Theory

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This investigation of Natural Language quantification posits interpretation as the optimization of the description of language. An account of the optimization of quantificational description is given within the optimality framework of Prince and Smolensky (1993) for English. A strict dominance hierarchy of constraints on the goodness of a quantified description is proposed. Firstly, a quantificational description is an expression of epistemo-linguo correspondence. Secondly, a description is not bound to any particular set of individuals, but depicts an empirical trend concerning all possible members of the subject. Next, a description is not probabilistic, but certain. Then, a description is actual for incorporating all relevant knowledge that may bound it.

The constraint hierarchy isolates the optimal quantificational description which is accurately predicted to be the interpretation. Explicit arguments are provided for the ranking of these rules. General aspects of Natural Language quantification are addressed such as Universals, Generalization, Probability, Modality, and Expectation. An additional constraint to be Public is proposed to account for additional phenomena such as Partitivity, Proportionality vs. Cardinality (Milsark, 1974), and readings associated with the article "the". Under satisfaction of this constraint, a description is based on widespread knowledge/belief.

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### 1.0 Introduction

By this account of the interpretation of Natural Language (NL) sentences, the term quantificational interpretation will be used simultaneously to indicate a quantificational description of a sentence and its parts, and also to indicate the calculation of such a description. The calculation of an interpretation under this approach involves the identification of the best, or optimal, quantificational description that can be made of the sentence. Therefore it will be said that a salient interpretation of a sentence is a description of it, and that the mechanism of interpretation yields the best of such descriptions.

The mechanism of interpretation developed here is constructed within Optimality Theory (OT) first proposed as a general framework of optimization for theories of NL grammar and their associated rule systems (Prince and Smolensky, 1993). OT introduces partial and total orderings of violable constraints as a means to support the acclaimed conjecture of Universal Grammar (UG) (Prince and Smolensky, 1997; McCarthy and Prince, 1993). Optimality Theory is a framework for building theories within the Cognitive Science of Language under violable constraint optimization (Grimshaw, Legendre, and Vikner 2001; Grimshaw 1997; McCarthy 2002; Prince 2002; Prince and Tesar 1999; Samek-Lodovici and Prince 1999; Smolensky 1995, 1996; Tesar and Smolensky 2000; and Tesar 1995).

### 1.1 Natural Language Sentence under Description as a Formula

Interpretation is presented here as the selection of the best description. This account supposes that a quantificational description is an expression of equality between two quantities called i.) the Linguistic Model $\left(\Omega^{\delta}\right)$, and ii.) the Epistemic Ratio $\left(\Omega^{\varepsilon}\right)$.

Accordingly, quantificational interpretation is made of the sentence as an expression of an Epistemo-Linguo correspondence ${ }^{1}$.
(1) Epistemo-Linguo Correspondence: $\Omega^{\delta}=\Omega^{\varepsilon}$

Both quantities are taken to represent the level of participation in some property $\varphi$, as measured among a set of individuals $S$. That is, both quantities $\left(\Omega^{\delta}, \Omega^{\varepsilon}\right)$ represent the participation level, taken as the individual members of $S$ which are members of $\varphi$, measured against the total membership of $S$.
(2) Participation Level: $|S \cap \varphi| \div|S|$

However, $\Omega^{\delta}$ and $\Omega^{\varepsilon}$ differ by purpose. The linguistic model $\Omega^{\delta}$ is an overt statement of the level of participation, and the initial value of $\Omega^{\delta}$ will be presented as a NL determiner (Det). $\Omega^{\delta}$ serves as a model of the epistemic ratio $\Omega^{\varepsilon}{ }^{2}$ In general, the value of $\Omega^{\delta}$ may vary along the open unit interval [0..1] where, at the extremes, $\Omega^{\delta}=0$ represents no participation in $\varphi$ among the members of $S$, and $\Omega^{\delta}=1$ depicts total member participation.

By contrast, the epistemic ratio $\Omega^{\varepsilon}$ will represent knowledge/belief of the participation level in $\varphi$ among $S$ according to the experience of the interpreter. The epistemic ratio $\Omega^{\varepsilon}$ is then taken to reflect interpreter-belief of participation-level in a property $\varphi$ among $S$ according to hearsay and/or else observational experience.

[^0]Allowing that the interpreter has knowledge/belief regarding the membership of the set $|S|=s$, and also knowledge regarding individual participation ${ }^{3}(p)$ in some property, then the epistemic ratio $\Omega^{\varepsilon}$ will stand as the level of participation in the property according to the interpreter's experience.
(3) $\Omega^{\varepsilon}=p / s$

Under epistemo-linguo correspondence, a substitution (4ii) is possible given the definition under (3), and the assumption that $\Omega^{\delta}$ is a model of $\Omega^{\varepsilon}$.
(4) i. $\Omega^{\delta}=\Omega^{\varepsilon} \quad$ assume
ii. $\quad \Omega^{\delta}=p / s \quad$ by substitution
iii. $\quad \Omega^{\delta}{ }_{S}=p \quad$ by reformulation

The equality under (4ii) may be reformulated quite simply as (4iii) and so, (4iii) implies (4i) and conversely. Then, the current proposal of quantified interpretation may be expressed compactly as follows.

If a NL sentence can be accurately described as the expression under (5) below, then the description may be taken as an expression of epistemo-linguo correspondence and therefore may stand as a quantificational interpretation of the sentence.
(5) $\Omega^{\delta}{ }_{s}=p$

To what extent is it possible to describe a NL sentence in terms of the form in (5)? Beginning with a standard syntactic structure, the general form of quantificational description will be motivated as follows.
(6) i. $S \rightarrow[\mathrm{NP}][\mathrm{VP}]$
ii. $\mathrm{NP} \rightarrow \mathrm{DET} \mathrm{N}$

[^1]iii. $S \rightarrow\left[{ }^{\mathrm{NP}} \mathrm{DET} \mathrm{N}\right][\mathrm{VP}]$

In (6) above are simple phrase structure rules by which a sentential category (S) may be analyzed into the basic constituents of NP and VP. Furthermore, the NP may be analyzed into a standard decomposition of determiner (Det) and subject noun (N). A quantificational interpretation of the sentence, namely a description in the form of (5), will be possible in so far as we can map the description onto the syntactic structure of the sentence that is commonly assumed.
i. Quantificational Description:
$\left[\begin{array}{ll}\Omega^{\delta} & s\end{array}\right]=[p]$
ii. Syntactic Structure:
[Det N] [VP]

In order for this to be the case, we must be able to i.) associate the NL determiner (Det) with the linguistic model ( $\Omega^{\delta}$ ), ii.) associate the subject noun ( N ) with knowledge of the cardinality ( $s$ ) of the subject set of individuals, and iii.) associate the verb phrase (VP) with knowledge of individual participation ( $p$ ).

For the purposes of quantificational interpretation, let the quantificational denotation of an expression reflect cardinality, if the semantic denotation of the expression will be a set of individuals. Otherwise, let the quantificational denotation of Det be a set of possible values or, the denotation interval of the model $\Omega^{\delta}$. Then letting the semantic denotation of the subject noun N be a set of individuals $S$, the quantificational denotation will be the absolute value, or cardinality $(s=|S|)$ according to the interpreter.
(8) $\quad$ i. $S=\|\mathbb{N}\|$

$$
\text { ii. } s=|||\mathrm{N}||
$$

Furthermore, let the quantificational denotation of the verb phrase ( $\|\|\mathrm{VP}\| \mid)$ be the cardinality of the semantic denotation of the lexical verb phrase (\|VP\|). Given that the standard first-order denotation of the lexical verb phrase is taken to be a set of individuals, the quantificational denotation of VP will be the cardinality of the set. This set will also be called the individual participants $P$, and the cardinality will be $p=|P|$.
(9) i. $\quad \Omega^{\delta}=\|\mid \operatorname{Det}\| \|$
ii. $\quad s=\||N|\|$
iii. $\quad p=\| \| \mathrm{VP} \|$

The quantificational denotation of the VP is suggested to be the interpreter's knowledge of individual participation ( $p$ ). Participants are members of the subject denotation who bear the property expressed by the predicate. Therefore, at some level, the VP must represent that intersection between the subject denotation and the predicate denotation. A fact exploited in Barwise and Cooper (1981), demonstrates this relation. In the following under (10), the second occurrence of the term "dogs" is optional although this occurrence has no further bearing on the meaning of the sentence.
a. [np All [dogs]] [vp are [xp good]].
b. [ NP All [dogs]] [vp are [xp good (dogs) $)]$.

The VP in (10a), can be taken as "be good dog", as opposed to just "be good". The intersection of "good" and "dog" are the individual members of "dog" that participate in "good", and this much is implied by (10a). ${ }^{4}$

[^2]In summary, quantificational interpretation is possible where a special description of the sentence is possible. Primarily, the description must be as an expression of epistemo-linguo correspondence as (11).
(11) $\Omega^{\delta} \times s=p$

Since the ultimate value of the model $\left(\Omega^{\delta}\right)$ is taken to occur within the unit interval [0..1], this value when combined with the cardinality of the subject denotation, must yield the cardinality of some subset of the subject denotation. This cardinality must correspond to quantificational knowledge of individual participation $p$ if (11) is an accurate description of the sentence.

### 1.2 A Problem of Generalization in Natural Language Interpretation

Upon being forced to guess... the reader may assert the odds in this world, of any dog being a participant in say, the behavior of barking. Whatever the response may be, let this assertion represent the reader's probabilistic opinion ( $\omega$ ) of a dog's participation in barking. ${ }^{5}$ As a probability, the value of $\omega$ must lie somewhere on the unit interval between and including zero and one; $\omega \in[0.11$.

Presumably, a reasonable guess maker would not commit themselves to the certainty of a dog's participation in barking $\omega \neq 1$, for as should be clear, the following sentence is never really true.
(12) All dogs bark.

In fact, (12) is always false, for there is never a time when (12) may be uttered truthfully over the set of individuals that may in earnest be called a dog (i.e. the set of possible dogs). Minimally, the truth of (12) requires the following:

[^3](13) i. There have been no non-barking dogs observed.
ii. There will be no non-barking dogs observed.

If (13) must be satisfied at a minimum, then there is never a time when (12) may be uttered truthfully, if simply because of the ineluctable accidents and surprises of nature.

Demonstrably, there is no mundane observable property (e.g. visible to the naked eye, or audible to the unassisted ear etc.), which serves as a criterion property for being a dog (i.e. a property that is itself sufficient unto dog-ness). As a matter of critical importance, if there were even a necessary mundane observable property, then all dogs would have it perforce. Then, statements like (12) could be spoken in truth for any such property. But as the reader may verify, there is no mundane observable property $\phi$, for which a universal statement such as (12) can be made truthfully regarding the set of possible dogs, $\Delta^{\diamond}{ }^{6}$
(14) a. \#All dogs bark.
b. \#All dogs have whiskers.
c. \#All dogs have a tail.
d. \#All dogs swim.
e. \#All dogs dig-holes.
f. \#All dogs chase-cats.
g. \#All dogs like bones.

To show that a property $\phi$ is not a necessary property for being a dog, simply consider any individual $\delta$ that may be earnestly called "dog" (e.g. Fidoe Dog), and pose the following question under (15Q):

[^4](15) Given: $\phi($ Fido $) \wedge$ Fido $\in$ Dog

Q: Does negating the property $\phi$ disqualify Fido from being a dog?
A: No.
Importantly, Fido will not forfeit "dogness", for the lack of any mundane observable property. Counter-factual statements like the following confirm that such properties do not interact with the status of Fido's dogness.
(16) a. \#If Fido did not bark, he would not be a dog.
b. \#If Fido did not have a tail, he would not be a dog.
c. \#If Fido did not dig-holes, he would not be a dog.

Unlike Dogs, there are categories of things having at least some necessary properties, i.e. having four sides is part of the formal definition of being a square etc. This time however, the necessity of a property $\psi$ unto class membership is instead confirmed by counterfactual judgments.
(17) a. If it didn't have four sides, it wouldn't be a square.
b. If it wasn't big, it wouldn't be a mountain.
c. If it wasn't round, it wouldn't be a Euclidean circle.

While having four sides is not alone a criterion property for being a square etc., the expressions under (18) can always be spoken in truth by the necessity of $\psi$.
(18) a. All squares have four sides.
b. All mountains are big.
c. All Euclidean circles are round.

The property of having-four-sides $(\psi)$ is shown to be necessary of any individual $\sigma$ that may in earnest be called a square.
(19) Given: $\psi(\sigma) \wedge \sigma \in$ Square

Q: Does negating the property $\psi$, disqualify $\sigma$ from being a square?
A: Yes.
Strikingly, there are no such non-defeasible properties for dogs among the mundane observable and therefore, there is no mundane observable property that can be said to be universal among dogs. ${ }^{7}$ As a direct consequence, the set of possible dogs contains individual dogs lacking each of the properties that are commonly attributed to dogs in general. This constitutes a problem of generalization in NL interpretation that arises from quantification over the non-necessary possible.

### 1.2.1 Quantification over the Non-Necessary Possible

Assume (20).
(20) There is no mundane observable property $\psi$, that is necessary to hold of an individual $\delta$, so that $\delta$ may be a member of a set $\Delta$.

If (20), then every mundane observable property $\phi$, that may hold of a $\delta: \delta \in \Delta$, may also be singly negated without changing the status of a $\delta$ 's being a member of $\Delta$. Then, (21) is allowed for every such property $\phi$ that may hold of a $\delta: \delta \in \Delta$.
(21) $(\delta \in \Delta) \wedge \neg \phi(\delta)$,

Then, no universal statement such as (22) will be true on the set of possible members of $\Delta$, for any such property $\phi$ that may hold of a member of $\Delta$.
(22) $\forall \delta:(\delta \in \Delta) \rightarrow \phi(\delta)$

[^5]The set of dogs $\Delta$ will be regarded as an empirical set of individuals. An empirical set will be one such that for each ordinary observable property that holds among the members of the set, there exists a non-participant in the set of possible members. Now, a set such that no mundane observable property is necessary for membership will qualify as being empirical. Specifically, where no property demonstrated among the actual members is necessary, the set will always have possible members that are non-participants regarding every such property.

An interpretation of Empirical Generalization may result from sentences in which universal quantification is attempted on the possible members of an empirical set. For example, the set of possible dogs includes dogs which do not participate in each ordinary observable property found among the actual set of dogs. As a consequence, nondisjunctive universal statements of such properties that are intended to range over the possible members of an empirical set of individuals will always strictly fail.
(23) a. All dogs bark.
b. Every cat meows.
c. Every car rusts.
d. All birthday cakes have candles.
e. Every baby cries.

In (23) we have universal statements ranging over the possible members of some empirical sets (dogs, cats, cars, and birthday-cakes, etc.). Strictly speaking, each of the statements under (23) is false, because no individual may be excluded from an empirical set solely for the lack of a basic observable property. Taking (23d), if we posit a thing which may in earnest be called a birthday cake, then a lack of candles does not change its
status as such. Then, a birthday cake without candles is possible, and (23d) is strictly false if it ranges over possible birthday cakes. Because there is no basic observable property that is necessary unto set membership, then all such properties are optional if allowed. Then members lacking the optional property are possible and so, no universal quantification involving the property is truthful on the possible set. If one requirement of Natural Quantification is that it hold over the possible members of an empirical set, then universal natural quantification is always strictly false.

### 1.3 Quantificational Meaning and the Interpreter's Probabilistic Opinion

The speaker/interpreter's probabilistic opinion of participation ( $\omega$ ), may refer to the chance of any member ( $\delta$ ) of a set ( $\Delta$ ) participating in some property $(\phi)$, according to the opinion bearer's experience.

Let the magnitude $p$ represent that part of the set $\Delta$ which has demonstrated the property $\phi$. This will be understood as the amount of individual participation in $\phi$ among $\delta \in \Delta$. Furthermore, let the magnitude $s$ represent the cardinality of the set $\Delta, s=|\Delta|$. Then the ratio $\left({ }^{p} / s\right)$ may stand for the chance of participation in $\phi$, as considered at a particular time among the members of $\Delta$. Finally, let the speaker/interpreter's probabilistic opinion of participation ( $\omega$ ) be equal to this ratio $\left({ }^{p} / s\right)$, understood as the magnitude of set participants that have been observed, over the total magnitude of set members. Then, given a set $\Delta$ of individuals $\delta$ and some property $\phi$, we have:
(24) i. Individual Participants in $\phi,(P): \quad P=\{\delta: \delta \in \Delta \wedge \phi(\delta)\}$
ii. Cardinality of $\Delta(s): \quad s=|\Delta|$
iii. Cardinality of Participants $(p): \quad p=|\mathbb{P}|$
iv. Probabilistic Opinion $(\omega): \quad \omega={ }^{p} / s$

### 1.3.1 Effects of Probabilistic Opinion on Quantified Interpretation

The gross value of $\omega$ controls the quantificational meaning of natural language artifacts.
a. Not every cat sleeps standing-up.
b. Not every horse sleeps standing-up.

The sentences under (25) differ according to their quantificational reading. Where opinion is less than chance, the sentence about cats (25a) means most cats do not. Where opinion is greater than chance, the sentence about horses (25b) means most horses do. Both sentences may be spoken while viewing horses and cats sleeping on all-fours.

The problem revealed by such examples is that the overt linguistic determination is identical in both cases (i.e. not every), although the quantificational meaning varies significantly between (25a) and (25b). As a result, it becomes difficult to treat the linguistic determination "not every" as having a simple and static meaning. Instead it would appear that the meaning of this quantificational element corresponds to the opinion $(\omega)$ as it relates to chance $(\omega<1 / 2, \omega>1 / 2)$. This hypothesis is further supported by the effect on the interpretation, of making deliberate modifications to the gross interval of chance in which $\omega$ resides.
a. Not every cat sleeps curled-up in a ball.
b. Not every horse sleeps curled-up in a ball.

Here, the artifacts have exchanged their quantificational readings to correspond with the interval of chance that includes $\omega$. The quantificational reading of (26a) is that there are negative exceptions to universal participation. In contrast, the meaning of (26b) is that the
behavior is uncommon among horses, and this horse is an exception (looking at such a strange horse).

A very salient (if not the most salient) reading of (27a) is that barking is a universal behavior among dogs. However, if not assumed to be false, the best reading of (27b) is that there are only a limited number of dogs that do math. ${ }^{8}$
(27) a. Dogs bark.
b. Dogs do math.

Again, it is clear that the examples under (27) are identical regarding their linguistic determination. In these cases under (27), the Bare-Plural subjects (See Diesing, 1992) are exactly the same, i.e. "Dogs". The effect is also clear with the linguistic determiner $a$, called the indefinite article (Heim, 1982; Diesing, 1992; de Swart, 1996).
(28) a. A dog barks.
b. A dog does math.

In (28a), a quantificational reading is strongly available whereby dogs are asserted to universally participate in barking. This universal reading is not available to the artifact listed in (28b). Instead, it has only a bounded reading where it is possible to ask, "How many?" This difference is made clear according to the acceptability of the continuations in (29). Again, the availability of the universal reading corresponds to the interval of chance that contains $\omega$.
a. A dog barks. That's what dogs do.
b. A dog does math. \#That's what dogs do.

[^6]To summarize, the quantificational reading of otherwise identical quantificational expressions is clearly co-variant with the interval of chance that contains the probabilistic opinion of the interpreter. In the cases where $\omega$ is greater than chance, the quantificational reading is of majority, or else of universality. However, where $\omega$ is equal to a value less than chance, the quantificational reading is either of minority, or of bounded occurrences.

### 1.3.2 Epistemic Ratio ( $\omega$ ) and Epistemic Interval ( $\boldsymbol{\Omega}^{\boldsymbol{\varepsilon}}$ )

Because the value of $\omega$ may not be consciously available, let the epistemic interval $\left(\Omega^{\varepsilon}\right)$ be the smallest interval which can be said, with confidence, to contain the opinion $(\omega) .{ }^{9}$ Then, as it is defined, the value of $\omega$ is limited to the unit interval between and including zero and one. Therefore, the epistemic interval will always be a subset of the unit interval.
(30) Some Relevant Values/Sub-Intervals of the Unit Interval


For the purposes of exposition, several basic sub-intervals of the unit interval are identified, which carry unique implications for the value of participation ( $p$ ).
(31) Basic Epistemic Intervals

|  | Name | $\omega={ }^{p} / s$ | Participation $(p ; s \neq 0)$ |
| :---: | :---: | :---: | :---: |
| 1. | Certainty | $\omega=1$ | $p=s$ |
| 2. | $(-)$ Exception | $\omega \in(1 / 2.1)$ | $p>1 / 2 s$ |
| 3. | $(+)$ Exception | $\omega \in(0.1 / 2)$ | $p<1 / 2 s$ |
| 4. | Denial | $\omega=0$ | $p=0$ |
| 5. | Non Denial | $\omega \in(0.1]$ | $p \neq 0, p>0$ |

[^7]For simplicity, only gross epistemic intervals have been presented. The two singleton intervals [1] and [0], will often be referenced by their single member values.

The singleton intervals of certainty and denial contain the unique values of $\langle 1,0\rangle$, and therefore entail that participation among set members is known to be total $(p=s)$, or null ( $p=0$ ) respectively.

The intervals of negative ( - ) and positive ( + ) exception contain the possible values of $\omega$ where a minority of exceptions exist regarding certainty of participation $(p>1 / 2 s)$, and a minority of exceptions exist regarding denial of participation $(p<1 / 2 s)$ respectively. ${ }^{10}$ Finally, the epistemic interval of non-denial contains the possible values of $\omega$ such that participation is not zero. Because the implication is that some participation in $\phi$ is assumed to be the case, the interval of non-denial will also be referred to as the interval of allowed possibility, (0..1]. The interval of allowed possibility will be considered as the default epistemic interval in cases where the interpreter lacks specific knowledge, but does not reject the possibility. ${ }^{11}$

The epistemic intervals of negative and positive exception, are also those intervals which contain the possible values of $\omega$, such that the rule of Logical Generalization may not apply. This is because, within an interval of $\left(\begin{array}{c} \\ \sim\end{array}\right)$ exception, $\Phi$ does not hold arbitrarily.

[^8](i) Integumented megasporangia float, Echinoidia sink.

Given that the interpreter has no expectation regarding the properties of megasporangia and Echinoidia, an initial epistemic interval of possibility is assumed unless the interpreter rejects the possibility for other reasons.
(32) Logical Generalization: From a well-formed formula $\Phi$ containing $x$, infer $\forall x \Phi$ where $\Phi$ holds for any arbitrary individual $x$.

Furthermore, the rule of logical generalization may not apply concerning any mundane observable property $\phi$ in the domain of possible dogs, because $\Omega^{\varepsilon}$ will always be a positive or negative interval of exception.

As was demonstrated by examples $(25,26)$ above, the quantificational reading of a sentence may vary according to the epistemic interval that contains $\omega$. In (33) below, the epistemic interval is claimed by the author to be that of negative exception.
(33) Not every fish can stay under-water.

This means that the author is aware of something that may be called a fish that cannot stay under-water, although the majority of fish do participate in this ability. Therefore, an individual fish constitutes a negative exception to the certainty of a fish being able to stay under water. By contrast, the epistemic interval of (34) is that of positive exception, if spoken when looking at such a special cat.
(34) Not every cat can stay under water.

Example (34) means that a cat which can stay under-water is a positive exception to denial of participation among cats. Again, the difficulty is that many sentences are quantificationally identical on the surface, although the quantificational reading of the sentence corresponds to the epistemic interval of the interpreter.

### 1.4 Natural Language Determination

Determiners of The English Language have an intuitive quantificational meaning when considered in isolation. The quantificational denotation of the determiner in
isolation will be called its denotation interval. The following set of denotation intervals is given for the basic determiners of English.
(35) The Basic Determiner System of English ${ }^{12}$

|  | Determiner Expression | Denotation Value/Interval |
| :---: | :---: | :---: |
| 1. | all, every | 1 |
| 2. | most | $(1 / 2.1)$ |
| 3. | few | $(0 . .1 / 2)$ |
| 4. | no, none | 0 |
| 5. | a, bare-plural | $(0 . .1]$ |
| 6. | not all, not every | $[0.1)$ |

The interval assignments given in (35) reflect the quantificational meaning of the determiner in isolation. However, as demonstrated, the value of the determiner does not always match the quantificational reading of the sentence, which appears to vary with $\omega$.
(36) a. Not all fish can stay under-water.
b. Not all cats can stay under water.

As should be clear from the data in (36), the determiner meaning does not correspond exactly to the quantificational reading of a sentence which contains the determiner. What should be appreciated is that (36a) is a quantificational claim regarding fish. Likewise, (36b) is an entirely different quantificational claim regarding cats. Mysteriously, the two quantificational readings are taken from identical linguistic expressions of quantification. As a result, the denotation interval $\left(\Omega^{\delta}\right)$ of the determiner in isolation must be theoretically distinguished from the ultimate quantificational meaning of the sentence ( $\Omega$ ).

[^9]
### 1.5 Quantificational Meaning and Epistemo-Lingual Correspondence

Let the quantificational meaning ( $\Omega$ ) of a sentence arise as the value of the Epistemo-Linguo correspondence between the epistemic interval ( $\Omega^{\varepsilon}$ ), and the Linguistic Model $\left(\Omega^{\delta}\right)$. That is, let the quantificational meaning of the sentence be the value upon which $\Omega^{\varepsilon}$ and $\Omega^{\delta}$ can be made to agree, if that is possible.

$$
\begin{equation*}
\Omega \leftrightarrow\left(\Omega^{\varepsilon}=\Omega^{\delta}\right) \tag{37}
\end{equation*}
$$

Therefore, if the epistemic interval and linguistic model can be made to agree, a quantificational interpretation of truth will occur for the sentence in question. Otherwise, the sentence will be false $\left(\Omega^{\varepsilon} \neq \Omega^{\delta}\right)$.

### 1.5.1 Correspondence under Semantic Operations

It is only under limited circumstances (Chapter 2) that correspondence obtains initially, without making changes to the model, nor making changes to the epistemic interval. Therefore, if correspondence will hold, changes must be made to either the model $\Omega^{\delta}$, the epistemic interval $\Omega^{\varepsilon}$, or both. Secondly, changes in the subject cardinality $s$ may enable correspondence. The first type of change will apply to intervals under operations of Enhancement. Changes to the model will be model enhancement ( $\mathrm{M}^{+}$), and changes to the epistemic interval will be knowledge enhancement ( $\mathrm{K}^{+}$).

The second type of change affects the subject cardinality $s$, through careful selection of the sentential subject. Such selection may place an upward bound on the cardinality of the subject set. A selection of the subject that places an upward bound on the set's cardinality will be called Trivialization (TRV). The operation of trivialization controls $\Omega^{\varepsilon}$ inversely according to the value of $s$, and may therefore be used to drive epistemo-linguo correspondence. Quantification over bounded subjects constitutes trivia.

### 1.5.2 Constraints on Operations

Allowable instances of knowledge enhancement $\left(\mathrm{K}^{+}\right)$will be limited to the removal of $(/ /$ ) exceptions. Then, the current delimitation of exceptionality, prohibits the removal of anything but a minority of counter-examples. Consider the following (38).
(38) All dogs fly.

Now, assume that a couple of special dogs do actually fly. Then at least we have, $\Omega^{\varepsilon}=$ $\left(0 . .^{1 / 2}\right)$. But, there is no interval of exceptionality that represents the non-flying dogs as negative counter-examples to (38). Certainly, the flying minority do not constitute positive exceptionality to (38), but instead to (39).
(39) No dogs fly.

With respect to (38), the set of non-flying dogs is not relevant to the operation of knowledge enhancement as it is defined. Therefore, knowledge enhancement is considered to be unavailable as an operation regarding non-flying dogs in the interpretation of (38). In such cases, the epistemic interval regarding flying-dogs ( $\Omega^{\varepsilon}=$ $\left.\left(0 . .^{1 / 2}\right)\right)$ would not agree with the model $\left(\Omega^{\delta}=[1]\right)$, and cannot be brought into correspondence with $\Omega^{\varepsilon}$ via $\mathrm{K}^{+}$. It is for this reason that the theory predicts the example (38) to be patently false, while (39) on the other hand, is a reasonable generalization. The allowable instances of model enhancement $\left(\mathrm{M}^{+}\right)$, will be limited to those changes of any denotation interval $\Omega^{\delta}$, to one of its sub-intervals $\left\{\Omega^{\delta+}: \Omega^{\delta+} \subseteq \Omega^{\delta}\right\}$. Therefore, enhancement has no effect on models of certainty and denial.

Therefore, both proposed enhancement operations of "interval change" are constrained in their application. Knowledge enhancement ( $\mathrm{K}^{+}$) may only apply to remove $(+/)$ exceptions to a property-pair (i.e. $\operatorname{dog} \wedge$ bark). Due to the interval delimitations, this
constraint implies that the enhanced intervals will be constrained to either certainty, or denial, and that only a minority of exceptions may ever be removed under this operation.
(40) Determiner Denotations and Possible Enhancements

|  | Determiner | Denotation | Enhancement |
| :---: | :---: | :---: | :---: |
| 1. | all, every | 1 | N/A |
| 2. | most | $(1 / 2.1)$ | $\Omega^{\delta+}: \Omega^{\delta+} \subseteq(1 / 2 . .1)$ |
| 3. | few | $(0.1 / 2)$ | $\Omega^{\delta:}: \Omega^{\delta \dagger} \subseteq(0.1 / 2)$ |
| 4. | no, none | 0 | N/A |
| 5. | a, bare-plural | $(0 . .1]$ | $\Omega^{\delta+}: \Omega^{\delta+} \subseteq(0 . .1]$ |
| 6. | not all, not every | $[0 . .1)$ | $\Omega^{\delta+}: \Omega^{\delta+} \subseteq[0 . .1)$ |

On the other hand, model enhancement ( $\mathrm{M}^{+}$) may only apply to bring a denotation interval $\left(\Omega^{\delta}\right)$ to one of its sub-intervals. The general constraints on possible enhancement operations can be stated as follows under (41).
(41) Constraints on Operations

|  | Operation | Initial Interval | Enhancements $(+)$ |
| :---: | :---: | :---: | :---: |
| 1. | Knowledge Enhancement $\left(\mathrm{K}^{+}\right)$ | $\Omega^{\varepsilon}$ | $\Omega^{\varepsilon \dagger}: \Omega^{\mathrm{\varepsilon}} \in\{[1],[0]\}$ |
| 2. | Model Enhancement $\left(\mathrm{M}^{+}\right)$ | $\Omega^{\delta}$ | $\Omega^{\delta+}: \Omega^{\delta+} \subseteq \Omega^{\delta}$ |

Let it also be acknowledged that the types and number of intervals that will ever emerge as optimal descriptions, and therefore as salient interpretations, are those which find agreement under correspondence. Therefore, constraints placed upon possible epistemic intervals will limit the kind and number of possible interpretations under this account. For instance, although the constraint on model enhancement allows enhancements of the intervals $\Omega^{\delta}=(0.1]$ and $\Omega^{\delta}=[0.1)$ to any sub-interval, only those sub-intervals of $\Omega^{\delta}$ which correspond to an epistemic interval $\left(\Omega^{\varepsilon}\right)$ will ever arise as a true quantificational interpretation $(\Omega)$. Therefore, the enhanced intervals for these models are effectively constrained to sub-intervals of the intervals $(1 / 2 . .1),\left(0 . .^{1 / 2}\right),[1]$, and $[0]$, if we
confine the epistemic interval to these intervals, and sub-intervals thereof. This state of affairs would constitute an instance of Harmonic Bounding in OT, regarding many otherwise viable candidates that always fall short of optimality. ${ }^{13}$

Another way to satisfy correspondence is to select a special set of individuals standing as the subject of the sentence. For this purpose, the third semantic operation of trivialization is proposed (TRV). Trivialization refers to the selection of a special bounded set $\delta!$, or special number of individuals $s!=|\delta!|$. This selection may satisfy the model of participation $\left(\omega=p / s!\in \Omega^{\delta}\right)$. Among the bounded sets of relevant individuals $\&$, only some sets $\delta$ !, will produce the desired ratio $\omega$. The reader should recognize that the value of $s$ ! has a direct impact on the value of $\omega$, and that the two values are inversely related for fixed values of $p$. Therefore, the desired value of $\omega$, may be achieved through manipulations of the cardinality $\xi!=\mid\{!\mid$ under TRV.

### 1.6 Optimization of Quantified Interpretation

The current theory of quantified interpretation is developed within the framework of Optimality Theory (Prince and Smolensky, 1993), where the specific OT implementation is the topic of Chapter two. Optimality theory is a framework for developing specific theories of cognitive rule systems, such as NL grammar, whereby the observed linguistic phenomena is held to be the best output according to a domain specific optimization function of the theorist's assembly. Under the hood, the optimization function is a hierarchy of rules (constraints) some of which may be broken by the winning candidate for output if a higher ranking rule is satisfied as a result.

[^10]Optimality theory is primarily concerned with such constraint interaction as a means to explain the very complex patterns of linguistic and other cognitive phenomena, which may frustrate standard fixed-rule systems with "hard" unviolable rules (cf. Hard vs. Soft Constraints in OT).

In the large sense, optimization simply means to seek the most desirable outcome according to some criteria of goodness. Here, the goodness of a quantified interpretation is measured in terms of the satisfaction of the following possible interpretation states.
(30) Desired States of a Quantificational Description

True: $\quad$ The candidate description is true iff there is epistemo-linguo correspondence ( $\Omega^{\varepsilon}=\Omega^{\delta}$ ).

Natural: The candidate description has a natural significance iff the subjectset $S$ is not trivialized (i.e. not bounded by interpreter choice).

Certain: The candidate description is certain iff $\Omega$ is not probabilistic (i.e. $\Omega=[1], \Omega=[0])$.

Actual: The candidate description is actual iff it is not the result of enhancement operations.

Firstly, is the description is true under correspondence? Secondly, is the quantification of natural significance, or is it trivia? The Natural status of a description may be assessed according to the relevance of bounding questions (§ 1.8). Next, is the value certain, (i.e. $\Omega=[1], \Omega=[0])$, or is it probabilistic, $\Omega \subseteq(0 . .1)$ ? Lastly, is the quantification actual, and therefore not the result of any semantic operations of enhancement $\left(\mathrm{K}^{+}, \mathrm{M}^{+}\right)$?

Under enhancement ( $\mathrm{K}^{+}, \mathrm{M}^{+}$), the intervals ( $\Omega^{\varepsilon}, \Omega^{\delta}$ ) can be brought into correspondence for the satisfaction of truth. However, under enhancement the
quantification fails to be actual because certain knowledge is disregarded ( $\mathrm{K}^{+}$), or else the speaker's model is altered $\left(\mathrm{M}^{+}\right)$.

State: | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: |
| $\checkmark$ | $\checkmark$ |  | $*$ |

Because the trivialized set is bounded, correspondence is only achieved in isolated space-time regions. Under trivialization, correspondence can be made at any desired quantificational value, however this depends entirely upon the proper selection of the subject. The trivialized subject that satisfies correspondence ( $\$!$ ) is a member of the set of bounded sub-sets of the subject, which place $\omega$ within the participation model.
(32) Trivialization (TRV): $\mathcal{S}!\in\left\{\mathcal{S}: \omega \in \Omega^{\delta}\right\}$

The careful selection of $\mathcal{F} \neq \varnothing$, such that $S$ of sufficient variety, will produce any desired ratio $\left(\omega={ }^{p} / s\right)$. Those bounded sets of individuals $(\xi)$ that place $\omega \in \Omega^{\delta}$, guarantee that $\Omega^{\varepsilon}$ $\subseteq \Omega^{\delta}{ }^{14}$ Unlike enhancement, the operation of trivialization does not violate actuality because the quantification is quite actual despite being local.

State: | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: |
| $\checkmark$ | $*$ |  | $\checkmark$ |

The states under (34) can be separated into two groups according to their resistance to change by semantic operation (34ii, 34iv), or else according to the satisfaction of a desired value (34i, 34iii).
(34) Interpretation States
i. True: $\quad \Omega^{\varepsilon}=\Omega^{\delta}$

[^11]ii. Natural: Not Trivialized
iii. Certain: $\Omega=[1], \Omega=[0]$
iv. Actual: Not Enhanced

In addition to the satisfaction of truth, the operations of trivialization and enhancement may also satisfy the desired state of certainty. Because the operation always removes a minority of counter-cases, $\mathrm{K}^{+}$will only bring $\Omega^{\varepsilon}$ to either positive certainty ( $\Omega^{\varepsilon+}=[1]$ ), or negative certainty $\left(\Omega^{\varepsilon+}=[0]\right)$. Furthermore, since trivialization may bring correspondence at any desired value that is allowed, then both bounded certainty $\left(\Omega^{\mathfrak{z}}=[1]\right)$ and bounded denial $\left(\Omega^{\varepsilon}=[0]\right)$ are available under trivialization.

The goodness of the quantified description increases with the satisfaction of the states under (34). This implies that the use of semantic operations will be optimally minimal, as semantic operations represent a cost to the system. Because the use of an operation violates a desired state by definition, semantic operations and the goal of maximal state satisfaction must conflict in a simple sense. However, if the states are ranked according to dominance as in OT, then the benefit to one state that occurs by the use of the operation may outweigh the operation's cost to a lesser desired state. In that case, a minor state violation is licensed by the satisfaction of a more significant state. This is referred to as constraint interaction within the framework of Optimality Theory.

### 1.7 Correspondence in the Limit

To some readers, the operation of $\mathrm{K}^{+}$may appear to be an ad hoc solution to the problem of universal truth despite exceptionality. Specifically, the reader may wonder what licenses the interpreter (or the theorist), to overlook a minority of exceptions for a
belief of universal truth? Since the exceptionality is the problem, does the removal of the exceptional cases under $\mathrm{K}^{+}$amount to a mere, solution by stipulation?

The epistemic interval $\left(\Omega^{\varepsilon}\right)$ has been defined as the smallest interval that can be said, with confidence, to contain the interpreter's probabilistic opinion ( $\omega$ ). Moreover in (11), $\omega$ is defined as the observed frequency of participation $\left(\omega={ }^{p / s}\right)$ as taken on a set $\Delta$. In general, the observed frequency of any event will not be very different from the actual probability of the event, when considered over the long run of experience.

For the purposes of demonstration, assume that $\Omega^{\varepsilon}$ and $\Omega^{\delta}$ correspond $\left(\Omega^{\varepsilon}=\Omega^{\delta}\right)$. Furthermore, let it be assumed that the denotation interval $\left(\Omega^{\delta}\right)$ contains the actual probability. Then, the shared interval is the minimal interval that can be said to contain both the observed frequency and the actual probability of participation. Over the long-run of experience, $\Omega^{\varepsilon}$ and $\Omega^{\delta}$ are expected, being minimal, to converge upon the singleton interval containing the actual probability.
(49) Optimization of $\Omega^{\varepsilon}$ Under $\mathrm{K}^{+}$


Alternatively, base-line exceptionality constitutes a perhaps very small portion of the total membership in the limit of experience. If so, then at most a minority of exceptions may be excused now, in anticipation of an ultimate future correspondence. By this reasoning, enhancement under $\mathrm{K}^{+}$is licensed for the limited removal of countercases. In the diagram under (49), this is depicted as the optimization of $\Omega^{\varepsilon}$.

In absolute terms there may be many counter-examples, but over the long run, the ratio of participants to total relevant individuals, should come to approximate the actual probability of participation. Then, if $\Omega^{\varepsilon}$ and $\Omega^{\delta}$ are minimal, the intervals will correspond after the removal of positive or negative exceptionality. This is based on a belief that the current exceptionality will at some point be a negligible portion of the total membership. Therefore, the interpretation that requires the optimization of $\left(\Omega^{\varepsilon}\right)$, is licensed by a fundamental expectation of correspondence in the limit of experience.

An important constraint on the optimization of $\left(\Omega^{\varepsilon}\right)$ is the probabilistic compatibility with $\left(\Omega^{\delta}\right)$.
(50) No Expectation with Opposing Likelihood


If the likelihoods expressed by $\Omega^{\delta}$ and $\Omega^{\varepsilon}$ are in opposition across chance, then $\Omega^{\delta}$ and $\Omega^{\varepsilon}$ can be said to be probabilistically incompatible. If incompatible, then there is no expectation of correspondence, even over the long-run. This condition is enforced as a hard constraint on the operation of enhancement $\left(\mathrm{K}^{+}\right)$, which is defined so that it may only apply to remove negative or positive exceptions to certainty and denial. This can explain why interpretations of generalization are typically not available, when not supported by an expectation of correspondence in the limit. If not taken to be false, the interpretations under (51) are never generalizations.
(51) a. Every cat sleeps standing-up.
b. Cats swim under-water.
c. A dog climbs trees.

### 1.8 Unboundedness of Generalization

Unboundedness is taken to be a salient component of generalization. If a generalization is made despite known counter-evidence, a necessary characteristic of the interpretation will be interpretational unboundedness of number, place, and time.
(52) a. All dogs bark.
b. Every cat meows.
c. No cat sleeps standing-up.
d. Dogs bark.
e. A dog barks.
f. Babies cry.
g. Cars rust.

On these readings, the interpretation of number, place, and time is completely unbounded. Take for example the first and most salient reading of the following sentence.
(53) Dogs bark.

The most natural reading of (53), does not allow bounding questions such as,
(54) i. How many?
ii. When?
iii. Where?

Any contentful response by the speaker of (53), to the questions in (54) imposes a bound on the possible interpretations of the sentence. Answers to such questions will bound the set of possibilities in terms of the number of individuals involved, the location, and time
of situation/event(s). Instead, the only appropriate response to such questions as found under (54) should naturally be (55).
(55) It doesn't matter how many, when, or where, because what I'm saying is, bark is what dogs do. Dogs bark!

Therefore, the foremost natural interpretation of (53) is entirely unbounded regarding matters of number, place, and time. A grammatical notion of unboundedness regarding generics is already present in the literature, and can be found in Declerk (1986).

The bounded status of an interpretation will be assessed through the use of bounding questions as in (54). If the questions are irrelevant to a particular interpretation, then the interpretation will not be considered as bounded.

### 2.0 Natural Quantification in Optimality Theory

The Optimality Theory of Prince and Smolensky (2002, 1999, 1993), explains that the grammaticality of a linguistic form depends on its relative goodness in the satisfaction of a rule hierarchy, as compared to that of the alternative competing structural possibilities. The set of structural variations that are possible on an input form yields a space of candidates for optimality. The incremental satisfaction/violation of the rule hierarchy by any of these forms determines its potential status among competitors as being the optimal candidate. By its unique accord with the rule system, the winning candidate is grammatically optimal, and therefore distinguished from sub-optimal forms which are never considered for output. Therefore, the optimality theoretic (OT) grammar computes a function from input form $\left(I_{0}\right)$, to a unique, grammatically optimal output $\left(I^{\circ}\right)$. The computation implies the generation of a candidate space that is winnowed by rule violation to isolate the observed grammatical form. Here, the optimal candidate for output stands as the best quantificational description of the sentence according to the rule hierarchy, and therefore is the quantified interpretation $\left(I^{\circ}\right)$.

Central concepts of the OT framework are the Constraint Hierarchy $(\mathrm{CH})$ and the Candidate Space (CS). The constraint hierarchy is a type of rule system in which the set of constraints on the possible output is ordered by dominance. This means that some constraints in the system are more dominant than others, and so their influence on the possible output is felt more acutely relative to other less dominant rules. Such dominance relationships among the constraints are represented by a hierarchical ranking. The constraints are also violable, meaning that any candidate, including the optimal form, may stand in violation of one or more rules. Violations of the more lowly ranked
constraints may prove to be optimal so long as the satisfaction of a more highly ranked constraint is achieved as a result. Such interplay among the constraints is known in optimality theory as constraint interaction.

### 2.1 Constraint Hierarchy

The current OT account assumes the five violable constraints listed under (1), where the quantificational constraints (i-iv) are ranked as they are presented.

## (1) Constraints on Quantified Interpretation

i. Interpretation is True
ii. Interpretation is Natural
iii. Interpretation is Certain
iv. Interpretation is Actual
v. Interpretation is Public ${ }^{15}$

Under certain conditions, the optimal candidate will satisfy the entire constraint set yielding a perfect true, natural, certain, actual, and public interpretation. However as will be shown, such perfect conditions are rarely encountered, and the satisfaction of the constraint to be true, for instance, often requires the violation of less dominant constraints depending upon the initial state of the system $\left(I_{0}\right)$.
(2) Constraint Ranking

True >> Natural >> Certain >> Actual : Public
This Chapter deals with the motivation of the constraint system in (2). A detailed ranking argument will be made for each pair-wise constraint ranking as the optimality theoretic tableaux are considered in the sections below. Ranking arguments will either stand on violation patterns, or else by inference according to the transitivity of the ordering
${ }^{15}$ § 3.3.4
relation. For example, if the ranking arguments under (3i) and (3ii) can be made according to some violation pattern, then the ranking argument under (3iii) can be made by the transitivity of strict dominance.
(3) Ranking Argument by Transitivity of Strict Dominance
i. $\mathrm{X} \gg \mathrm{Y}$
ii. $Y \gg Z$
iii. $X \gg Z$

Constraint satisfaction/violation is evaluated according to the simple criteria that were developed in the first chapter. The constraint to be True will be violated iff the intervals $\Omega^{\delta}$ and $\Omega^{\varepsilon}$ are not equal $\left({ }^{*} \Omega^{\delta} \neq \Omega^{\varepsilon}\right)$, and satisfied iff the intervals are equal $\left(\Omega^{\delta}=\Omega^{\varepsilon}\right)$. Next, the constraint that the description be Natural is satisfied iff it is not the result of any specific choice, or number, of individuals. If the description results from the specific choice of individuals, or choice of number, then the quantification is considered to be trivia. Trivialization will also occur if space and time are simultaneously constrained (§ 3.3.3). The constraint to be Natural will therefore be called a threshold constraint that is violated only once, if either the subject is bounded (s), or if both space and time are simultaneously constrained $(t, t)$. Next, the constraint to be Certain is violated just in case the quantified interpretation is probabilistic $(\Omega \notin\{1,0\})$. Next, the constraint to be Actual is violated by any departure from the facts through enhancement operations. Therefore, the constraint to be Actual will be violated for any application of $\mathrm{K}^{+}$or $\mathrm{M}^{+}$. Finally, the constraint to be Public is violated by any Privatization under the action of PRV, which marks the subject as speaker-defined, and therefore non-public (\$ 3.3.4).
（4）
Constraint Mapping and Ranking

| Constraint |  | Condition | Evaluation |
| :---: | :---: | :---: | :---: |
| 1． | True | $\Omega^{\delta}=\Omega^{\varepsilon}$ | $* \Omega^{\delta} \neq \Omega^{\varepsilon}$ |
| 2. | Natural | Not Trivial | $* \mathrm{TRV}$ |
| 3． | Certain | $\Omega \in\{1,0\}$ | $* \Omega \notin\{1,0\}$ |
| 4． | Actual | Not Enhanced | $* \mathrm{~K}^{+}, * \mathrm{M}^{+}$ |
| 5． | Public | Not Private | $* \mathrm{PRV}$ |

Arguments for particular rankings between constraint pairs will be made during the following analysis．The following is a summary of all ranking arguments as they pertain to the constraint hierarchy given above in（2）．Numbered tableaux are listed beside particular pair－wise rankings which together support the total ranking of CH ．

## （5）Summary of Ranking Arguments

| Argument | Ranking | Tableau | CH |
| :---: | :---: | :---: | :---: |
| RARG1 | True 》 Natural | T4，T5 | True＞）Natural |
| RARG2 | Natural＞＞Actual | T9，T11 | True ） 7 Natural Natural＞＞Actual |
| RARG3 | True 》）Actual | By <br> Transitivity | True 》）Natural 》＞Actual |
| RARG4 | Certain＞＞Actual | T12，T13 | True $\gg$ Natural＞＞Actual Certain＞）Actual |
| RARG5 | Natural＞＞Certain | T18 | True ）＞Natural 》＞Certain 7＞Actual |

## 2．2 Markedness and Faithfulness Constraints

The canonical OT distinction between Markedness constraints and Faithfulness constraints is relevant here．Markedness constraints are those constraints that are violated when a candidate form fails to occupy a certain specified state．Because it does not occupy the specified state，the candidate is said to be marked．For example，the constraint to be True is violated once by any candidate interpretation where $\Omega^{\delta} \neq \Omega^{\varepsilon}$ ，and the constraint to be Certain will be violated once by any candidate such that $\Omega \notin\{1,0\}$ ．

These constraints are the Markedness constraints of the system, for violations imply a candidate which is marked by divergence from the specified states of Truth and Certainty.
(6) Markedness Constraints On Interpretation
i. Be True
ii. Be Certain

Faithfulness constraints are those constraints violated when the output diverges from the initial state of the system $\left(I_{0}\right)$. Any operation of Enhancement, Trivialization, or Privatization may cause a departure from the initial state of the system. Therefore, faithfulness constraints are those constraints that are violated in any candidate that represents a change from the initial unmodified interpretation $\left(I_{0}\right) .{ }^{16}$

## (7) Faithfulness Constraints on Interpretation

i. Natural $=$ Not Trivia $\left({ }^{*}\right.$ TRV $)$
ii. Actual $=\operatorname{Not}$ Enhanced $\left({ }^{*} \mathrm{~K}^{+},{ }^{*} \mathrm{M}^{+}\right)$
iii. Public $=$ Not Private $($ *PRV $)$

Therefore, the totality of five constraints is partitioned into two sub-sets which are described as markedness and faithfulness constraints respectively.

### 2.3 Candidate Space of Quantified Interpretation

Following standard OT practice, it is assumed here that a function exists (GEN) which generates a space of "candidates for optimality". The candidate space results from considering all structural possibilities that are allowed by the structural theory. This is

[^12]accomplished by considering the outcome of all structural operations that may apply to the input form. Therefore in the current context, the available structural operations; Enhancement ( $\mathrm{K}^{+}, \mathrm{M}^{+}$), Trivialization (TRV), and Privatization (PRV), collectively and in combinations define the space of possible output candidates, according to their specific effects.

The initial candidate ( $I_{0}$ ) is defined as the unadulterated quantified description of the bare input artifact. Therefore, the initial candidate reflects the initial state of the system given the input. Otherwise, candidates will be annotated according to the operations that define them. Below under (8), we have the initial candidate in addition to four basic descriptions that are annotated according to the defining operations.
(8) Basic Interpretations by System Action
i. $I_{0}$
ii. $I_{\mathrm{K}+}$
iii. $I_{\mathrm{M}+}$
iv. $I_{\text {TRV }}$
v. $I_{\text {PRV }}$

Additionally, multiple operations may apply to create complex descriptions.
(9) Some Complex Candidates
i. $I_{\mathrm{K}+\mathrm{M}+}$
ii. $I_{\text {TRV }+\mathrm{M}+}$
iii. $I_{\text {PRV-TRV }}$
iv. $I_{\mathrm{PRV}-\mathrm{K}+\mathrm{M}+}$

If the constraint of Truth may be satisfied by some candidate, then any combination of individual operations that does not satisfy Truth will be sub-optimal. Therefore, many possible candidates that arise as multiple (or recursive) applications of system actions will always be sub-optimal. This is because all system actions obtain at the cost of constraint violation. Therefore, all system actions are superfluous and self-defeating if not instrumental in the satisfaction of more highly ranked constraints. As a result, the set of interesting candidates turns out to be quite small.
(10) Candidate Space by Faithfulness Violation


If we consider only single applications of any one operation, then the space of possible structural candidates is reduced to the space of semi-faithful interpretation shown in (10). ${ }^{17}$ The solid arrows connect the candidates that are possible without the operation of PRV, which may not be available given certain linguistic inputs. The space of semi-

[^13]faithful interpretation represents those candidate types that diverge from the initial state of the system ( $\left(I_{0}\right)$, with increasing faithfulness violations. The least faithful candidates are those where three operations have applied. If any of these violations happen to result in the satisfaction of higher constraints such as Truth and Certainty, then the candidate may be improved.

According to the ranking, the faithfulness violations caused by Enhancement, TRV, and PRV differ by severity. Then, starting at $I_{0}$, the most faithful candidate in the space of semi-faithful interpretation must be found which best satisfies the markedness constraints of Truth and Certainty. This will be the optimum, and is theorized to be the favored interpretation of the input artifact.

### 2.4 Hard Constraints

The OT function of GEN produces the structural variations that are allowed by the structural theory. Then, the structural theory itself imposes constraints on this set of candidates. Structural modifications are made according to the defined semantic operations. Firstly, knowledge enhancement ( $\mathrm{K}^{+}$) may only remove a minority of accidental exceptions (§ 4.4). Therefore, intervals that are enhanced beyond a minority will not be produced by GEN. Regarding $\mathrm{M}^{+}$, all model enhancements will bring the input model to a sub-interval. Furthermore, Privatization by the interpreter is only allowed where the input is overtly marked as known (i.e. the) or else where there are embedded constraints (§ 3.3.4) on the model (i.e. numerals and some), or else where space-time location is specified. These various inputs indicate specific speakerknowledge.
(11) Hard Constraints upon GEN
i. $\mathrm{K}^{+}$removes only a minority of unreasoned exceptions (accidents).
ii. $\mathrm{M}^{+}$only constrains a model to a sub-interval. Not for embedded models.
iii. PRV is only relevant given an embedded model at input ( $I_{0}$ ).

### 2.5 Analysis

The tableau is separated into three major regions from left to right i.) The Candidate Space, ii.) $\Omega^{\delta}-\Omega^{\varepsilon}$ Resolution Details and iii.) The Constraint Hierarchy. The symbol "*" indicates violation, and "*!" indicates terminal violation. A terminal violation means that the candidate is decidedly non-optimal, and is thereafter shaded.
(12) Sample Tableau ( $A^{\prime}$ ): All cats meow.
i. iii. iii.

| Tableau (A) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\mathbf{\Omega}^{\varepsilon} \mathbf{S}\right]$ | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{I}_{o}$ | $[1]$ Cats | $(1 / 2.1)$ Cats | $*!$ |  | $*$ |  |
| $\boldsymbol{I}_{K}^{+}$ | $[1]$ Cats | $\left(1 / .^{+}[1]\right.$ Cats |  |  |  | $*$ |
| $I_{T R V}$ | $[1]$ Cats | $[1]$ Gats |  | $*!$ |  |  |

All candidates are possible interpretations of the artifact. Candidates are identified according to the relevant defining operations as described above. The initial candidate ( $I_{o}$ ) is pure input, reflects no actions of any kind, and therefore is entirely faithful as it represents the initial conditions. The initial value of $\Omega^{\varepsilon}$ approximates the interpreter's $\omega$. The optimal interpretation is identified by the familiar OT optimal symbol (ब). In the example given above, the model of participation is $\Omega^{\delta}=[1]$, and the opinion is of likelihood, $\Omega^{\varepsilon}=(1 / 2 . .1)$.

Here, the contest is between candidates $\boldsymbol{I}_{K}{ }^{+}$and $I_{T R V}$. The constraint ranking decides between Enhancement and Trivialization. The optimal candidate is $\boldsymbol{I}_{K}{ }^{+}$, leading to
a Public, Non-Actual, Natural Truth, which is the interpretation-type of empirical generalization.

### 2.5.1 Quantified Genericity

If epistemic anomaly constitutes a break between what is said and what is known, then linguistic models of certainty are always anomalous on an empirical set.

Remembering that an empirical set of individuals is one where possible non-participants exist for every property that is demonstrated among the actual members, universal statements such as those under (13) will never find Natural epistemic support. Since counter-examples are allowed, they must be considered in the set of possible members. Therefore, if the following sentences are understood modally, then they must be false.
(13) a. All cats climb-trees.
b. Every cat climbs-trees.
c. No cats swim under-water.

Each sentence under (13) will be considered as input to the optimization function, where candidate descriptions are generated and evaluated according to CH .
(14) $\mathrm{T} 1^{\prime}$ Inputs
i. Artifact:
ii. Participation Model: $\quad \Omega^{\delta}=[1]$
iii. Epistemic Interval:
iv. Subject Class:
$\Omega^{\varepsilon}=(1 / 2 . .1)$
$S=$ Cats

| Tableau (1) | $\left[\Omega^{\delta} \mathbf{S}\right]$ | $\left[\Omega^{\varepsilon} S\right]$ | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{o}$ | $[1]$ Cats | $(1 / 2.1)$ Cats | $*!$ |  | $*$ |  |
| $I_{K}^{+}$ | $[1]$ Cats | $(1 / 2 .[1]$ Cats |  |  |  | $*$ |
| $I_{T R V}$ | $[1]$ Gats | $[1]$ Gats |  | $*!$ |  |  |

As can be seen in Tableau (1) above, the initial candidate ( $I_{0}$ ) does not satisfy Truth because while the model is certain $\left(\Omega^{\delta}=[1]\right)$, knowledge only supports simple majority $\left(\Omega^{\varepsilon}=(1 / 2 . .1)\right)$. However, $\mathrm{M}^{+}$has no effect on a model of certainty because there is no room for enhancement. Because $\mathrm{M}^{+}$does not apply, there are no candidates generated that involve the operation. In contrast, knowledge enhancement ( $\mathrm{K}^{+}$) may remove a minority of counter examples (i.e. those cats that do not climb-trees). As can be seen in candidate $\left(I_{\mathrm{K}}{ }^{+}\right)$, both constraints of Truth and Certainty are satisfied under the violation of Actuality. However, any trivialized candidate ( $I_{\mathrm{TRV}}$ ) will be sub-optimal because $\mathrm{K}^{+}$is less costly than TRV. TRV requires the selection of a special set of cats and therefore violates Natural significance. The tableau indicates that the optimal candidate is $I_{K}{ }^{+}$, and predicts that the interpretation of the input artifact is a True, Natural, Certain, Non-Actual quantification. The non-actual status of the interpretation accounts for its generic nature. Further examples demonstrate similar behavior, except that Tableau (3) shows the dynamic under a model of denial $\left(\Omega^{\delta}=[0]\right)$ and an epistemic interval of simple minority $\left(\left(\Omega^{\varepsilon}=(1 / 2.1)\right)\right)$.

Tableau (2) Every cat climbs-trees.

| Tableau (2) | [ $\Omega^{\delta} \mathrm{S}$ ] | $\left[\Omega^{\varepsilon} \mathrm{S}\right]$ | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{o}$ | [1] Cats | (1/2.1) Cats | *! |  | - 4 |  |
| G $\boldsymbol{I}_{K}{ }^{+}$ | [1] Cats | (\%.. ${ }^{+}$[1] Cats |  |  |  | * |
| $I_{\text {TRV }}$ | [1] Gats | [1] Gats |  | *! | + | S |

Tableau (3) No cats swim under-water.

| Tableau (3) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\Omega^{2} \mathrm{~S}\right]$ | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{I}_{0}$ | $[0]$ Cats | $\left(0 . .^{1 / 2) \text { Cats }}\right.$ | $*!$ |  |  |  |
| $\boldsymbol{I}_{K}{ }^{+}$ | $[0]$ Cats | $\left.[0]^{+}, . / 2\right)$ Cats |  |  |  | $*$ |
| $I_{T R V}$ | $[0]$ Cats | $[0]$ Cats |  | $*!$ |  |  |

In the previous tableau, it was the case that the denotation interval of the determiner and the epistemic interval were probabilistically compatible. This means that the intervals occupied the same side of chance. However, in the set of tableaux (T4-T7) the intervals are incompatible if not trivialized. This means that the sentence is incompatible with what the interpreter takes to be generally known about cats.

The next two tableaux (T4, T5) are distinguished by input models of Certainty and Denial respectively. In both cases however, the intervals $\left(\Omega^{\delta}, \Omega^{\ell}\right)$ are in conflict across chance. In (T4), we have ( $\Omega^{\delta}>1 / 2 ; \Omega^{\varepsilon}<1 / 2$ ), while in (T5) the conflict is reversed $\left(\Omega^{\delta}<1 / 2 ; \Omega^{\varepsilon}>1 / 2\right.$ ). Tableaux (T4, T5) provide a ranking argument for, True )> Natural.

The two examples ( $\mathrm{T} 4, \mathrm{~T} 5$ ) are similar in that trivialization is optimal, due to the fact that $\mathrm{K}^{+}$does not satisfy Truth. It can be verified by considering the artifacts, that the only truthful interpretation of the following two examples involves the choice of a special bounded group of cats (Gats).
(15) a. Every cat swims under-water.
b. No cat meows.

Therefore, the quantification fails to have a natural significance concerning cats in these cases. The operation of knowledge enhancement $\left(\mathrm{K}^{+}\right)$cannot satisfy Truth here because the swimming cats are the exceptional ones. Therefore, if the operation does apply, it must drive knowledge to negative certainty $\left(\Omega^{\varepsilon}=[0]\right)$ considering the input epistemic interval $\left(\Omega^{\varepsilon}=(0 ., 1 / 2)\right)$. Therefore, we have $\left(\Omega^{\delta} \neq \Omega^{\varepsilon}\right)$ under $K^{+}$.
(16) T4 Inputs
$\begin{array}{ll}\text { i. Artifact: } & \text { Every cat swims under-water. } \\ \text { ii. Participation Model: } & \Omega^{\delta}=[1]\end{array}$
$\begin{array}{ll}\text { iii. Epistemic Interval: } & \Omega^{\varepsilon}=\left(0 .{ }^{1} / 2\right) \\ \text { iv. Subject Class: } & S=\text { Cats }\end{array}$
Tableau (4) Every cat swims under-water.

| Tableau (4) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | [ $\left.\Omega^{\text {e }} \mathrm{S}\right]$ | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | [1] Cats | (0..1/2) Cats | *! |  | * |  |
| $\mathrm{IK}_{K}^{+}$ | [1] Cats | [0] $\left.{ }^{+}, 1 / 2\right)$ Cats | *! | + | * | - |
| $I_{T R V}$ | [1] Gats | [1] Gats |  | * |  |  |

The ranking argument (RARG1) for the constraints True >> Natural is bought out by an annotated comparative tableau (T4') which allows the data tableau to serve as a comparative tableau (Arguing Optimality, Prince 2002). In a comparative tableau, the optimum row ( $I_{T R V}$ ) is spotlighted as the first candidate. Then, winner-loser marks (W-L) are assigned to the sub-optima. $A$ "W" is placed in every cell in which the desired optimum is a winner, an "L" is placed in every cell in which the desired optimum is a loser, while every cell that does not distinguish the optimum from sub-optima is unchanged.

Tableau (4') Annotated Comparative Tableau

| Tableau (4) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\Omega^{\mathbf{E}} \mathrm{S}\right]$ | True | Natural | Certaim | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ( $I_{T R V}$ | [1] Gats | [1] Cats |  | * |  |  |
| $I_{0}$ | [1] Cats | (0..1/2) Cats | *!W | L | *W |  |
| $\mathrm{IK}_{K}^{+}$ | [1] Cats | $\left.[0]^{+} .2 / 2\right)$ Cats | *!W | 1 | * ${ }^{\text {d }}$ | *W |

With constraints in domination order, a successful optimum $\left(I_{T R V}\right)$ is marked by the occurrence of "W" in the leftmost non-blank cell in the row (Prince, 2002). In (T4"), this means that (ITRV) correctly beats the other candidates at the highest constraint that distinguishes them (Grimshaw 1997), and therefore establishes the ranking between the constraints True and Natural. Furthermore, the appearance of both "W" and "L" on the
same rows indicates a constraint conflict (Prince, 2002). If the two constraints under consideration were ranked alternatively, then $\left(I_{T R V}\right)$, although being the desired optimum, would no longer be optimal according to the ranking. Similar remarks can be made for tableau (5).
(17) T5 Inputs

| i. Artifact: | No cat meows./No cats meow. |
| :--- | :--- |
| ii. Participation Model: | $\Omega^{\delta}=[0]$ |
| iii. Epistemic Interval: | $\Omega^{\varepsilon}=(1 / 2 . .1)$ |
| iv. Subject Class: | $S=$ Cats |

Tableau (5) No cat meows./No cats meow.

| Tableau (5) | $\left[\Omega^{\delta} \mathbf{S}\right]$ | $\left[\Omega^{\varepsilon} \mathbf{S}\right]$ | True | Natural | Certain | Actual |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{I}_{o}$ | $[0]$ Cats | $(1 / 2.1)$ Cats | $*!$ |  | $*$ |  |
| $\boldsymbol{I}_{K}^{+}$ | $[0]$ Cats | $1 / 2 .+[1]$ Cats | $*!$ |  | $*$ | $*$ |
| $I_{T R V}$ | $[0]$ Gats | $[0]$ Cats |  | $*$ |  |  |

Despite the probabilistic conflict of T4 and T5, an interpretation of Truth at Natural significance may be achieved under privatization (PRV). Strictly, PRV may apply only where overt speaker-knowledge has been indicated at input.

Tableau (6) Every cat around here swims under-water.

| Tableau (6) | $\left[\Omega^{\delta} S\right]$ | $\left[\Omega^{\mathrm{E}} \mathbf{S}\right]$ | True | Ntr | Crtn | Actl | Public |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{o}$ | $[1]$ Cats | $(0.1 / 2)$ Cats | $*!$ |  | $*$ |  |  |
| $I_{K}^{+}$ | $[1]$ Cats | $\left.[0]^{+}, 1 / 2\right)$ Cats | $*!$ |  | $*$ | $*$ |  |
| $I_{T R V}$ | $[1]$ Cats | $[1]$ Cats |  | $*!$ |  |  |  |
| $I_{P R V}$ | $[1]$ Cats | $[1]\langle$ Cats $\rangle$ |  |  |  |  | $*$ |
| $I_{P R V-K}{ }^{+}$ | $[1]$ Cats | $\left(C^{+}[1]\langle\right.$ Cats $\rangle$ |  |  |  | $*!$ | $*$ |
| $I_{P R V-T R V}$ | $[1]$ Cats | $[1]\langle$ Cats $\rangle$ |  | $*!$ |  |  | $*$ |

Although the inputs are the same, it can be seen that the violation patterns in T6, T7 differ from the previous examples. Since privatization is allowed, a truthful interpretation is possible without trivializing the quantification.

Tableau (7) No cat around here meows./No cats around here meow.

| Tableau (7) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | [ $\Omega^{\varepsilon} \mathrm{S}$ ] | True | Ntr | Crtm | Actl | Public |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{1}$ | [0] Cats | (1/2.1) Cats | *! |  | * | \% |  |
| $I_{K}^{+}$ | [0] Cats | (1/2.. $\left.{ }^{+} 1\right]$ Cats | *! | \% | \% | $\stackrel{4}{4}$ |  |
| $I_{\text {TRV }}$ | [0] Cats | [0] Gats |  | *! | + |  |  |
| $I_{\text {cr }}$ | [0] Cats | [0]〈Cats〉 |  |  |  |  | * |
| $\mathbb{I}_{\text {PRV-K }}{ }^{+}$ | [0] Cats | $\left.[0]^{+}, x\right)\left\langle\right.$ Cats ${ }^{\text {a }}$ |  |  |  | *! | * |
| $I_{P R V-T R V}$ | [0] Cats | [0] <Cats) |  | *! |  |  | * |

It is clear that the examples under (19) are not bounded to any special number of cats. In fact, the interpretation does not constrain the subject to any number of cats, although the subject is clearly not the generally known (i.e. public) class of cats.
a. Every cat around here swims under-water.
b. No cat around here meows.

In these cases (T6, T7), speaker-knowledge is expressed in the input artifact as spacetime location, around here, where time defaults to the current.

### 2.5.2 Probabilistic Drift

Probabilistic Drift occurs in those cases where models of probability are too weak for knowledge, and are therefore enhanced up to a more restricted interval for the satisfaction of Truth.
(21) a. Most people understand the stock market. (Agree)
ii. Most people kind-of understand the stock market. (Strongly Agree)
b. Few people play the harp. (Agree)
ii. Few people really play the harp well. (Strongly Agree)

There is a marked difference among the pairs of examples under (21), where (21a) and (21b) are less agreeable than (21aii) and (21bii). According to the intuitive meanings of the predicates below in (22), it can be said with confidence that there are fewer participants in the set denoted by ( $p$ ) than there are denoted by ( $p$ ). There are fewer individuals expected to participate in really playing the harp well, than there are expected to participate in just playing the harp, under some appropriate construal. Therefore the participation-values for the following two examples under (22), differ as in (23).
(22) a. Few people ( ${ }^{p}$ play the harp).
b. Few people ( ${ }^{p^{\prime}}$ really play the harp well).
(23) $p^{\prime}<p$

Directly, this means that if $s=s^{\prime}$ (i.e. neither subject is trivial here because the quantification is Natural) then we have the following inequality, where less-than ( $<$ ) means more restricted.
(24) $\Omega^{\varepsilon \prime}<\Omega^{\varepsilon}$

If both sentences (22a) and (22b) are true, then both $\Omega^{\varepsilon \prime}$ and $\Omega^{\varepsilon}$ are on the interval of minority $\left(0 . .^{1 / 2}\right)$. However, since $\Omega^{\varepsilon \prime}$ is strictly less than $\Omega^{\varepsilon}$, then $\Omega^{\varepsilon}$ is more restricted than the model of simple minority. This is because even if $\Omega^{\varepsilon}$ is identical with the basic denotation of $f$ ew as $\left(0 . .^{1 / 2}\right)$, the value of $\Omega^{\varepsilon /}$ is nevertheless constrained to a sub-interval of $\Omega^{\varepsilon}$ according to clear judgments concerning (22). The conclusion is that the truth of (22b) obtains on a more tightly constrained interval than that of the simple minority model.

Tableau T8 is one of the rare examples where we have optimality at the initial candidate ( $I_{0}$ ). If we allow the epistemic interval to be simple majority, then the model is initially satisfied, and all further manipulations are sub-optimal.
(25) T8 Inputs

| i. Artifact: | Most people understand the stock market. |
| :--- | :--- |
| ii. Participation Model: | $\Omega^{\delta}=(1 / 2 . .1)$ |
| iii. Epistemic Interval: | $\Omega^{\varepsilon}=(1 / 2 . .1)$ |
| iv. Subject Class: | $S=$ People |

Tableau (8) Most people understand the stock market.

| Tableau (8) | $\left[\Omega^{\delta} \mathbf{S}\right]$ | $\left[\Omega^{\mathrm{e}} \mathbf{S}\right]$ | True | Ntrl | Crtn | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{I}_{o}$ | $(1 / 2.1)$ People | $(1 / 2.1)$ People |  |  | $*$ |  |
| $\boldsymbol{I}_{M}{ }^{+}$ | $(1 / 2 .(x . .1)$ People | $(1 / 2.1)$ People | $*!$ |  | $*$ | $*$ |
| $\boldsymbol{I}_{K}^{+}$ | $(1 / 2.1)$ People | $\left(1 / . .{ }^{+}[1]\right.$ People | $*!$ |  | $*$ | $*$ |
| $\boldsymbol{I}_{M}{ }^{+}-K^{+}$ | $\left(1 / 2 .{ }^{+}(x .1)\right.$ People | $\left(1 / 2 . .{ }^{+}[1]\right.$ People | $*!$ |  | $*$ | ${ }^{* *}$ |
| $\boldsymbol{I}_{T R V}$ | $(1 / 2.1)$ People | $(1 / 2.1)$ People |  | $*!$ | $*$ |  |
| $\boldsymbol{I}_{T R V-M^{+}}$ | $\left(1 / 2 .{ }^{+}(x .1)\right.$ People | $(1 / 2.1)$ People |  | $*!$ | $*$ | $*$ |

Here in T8 a notation is used in candidate $\left(\mathrm{I}_{\mathrm{M}}{ }^{+}\right)$to represent the set of candidates resulting from all possible model enhancements on the majority model ( $1 / 2{ }^{+}\left(x_{.} .1\right)$. Letting $x$ be any real number between chance and certainty, the notation ( $/ 2 . .^{+}\left(x_{0.1}\right)$ represents an infinite set of candidates that are never considered for output, but may be evaluated against the initial candidate $\left(I_{0}\right)$. The entire set represented by $\left(I_{\mathrm{M}}{ }^{+}\right)$is sub-optimal because no enhancement $\left(\mathrm{M}^{+}, \mathrm{K}^{+}\right)$is necessary for the satisfaction of Truth. Furthermore, TRV is sub-optimal for the same reason. Now, the case of drift is considered in $T 9$ below. Because of the more restrictive epistemic interval, the probabilistic opinion must concern a larger portion of participating individuals than in T8. Therefore, the interpretation of the
input model qua denotation interval must suffer model enhancement $\left(\mathrm{M}^{+}\right)$for the satisfaction of Truth. This constitutes an instance of model drift.

Considering the input of a minority model $\left(\Omega^{\delta}=\left(0 . .^{1 / 2}\right)\right.$ ), we have a similar drift in T10 and T11 depending of the epistemic interval. T10 demonstrates initial probabilistic Truth. As above, all non-initial candidates incur some additional violation.
(26) T9 Inputs

| i. Artifact: | Most people kind-of understand the stock market. |
| :--- | :--- |
| ii. Participation Model: | $\Omega^{\delta}=(1 / 2 . .1)$ |
| iii. Epistemic Interval: | $\Omega^{\varepsilon}=(9 / 10 . .1)$ |
| iv. Subject Class: | $S=$ People |

Tableau (9) Most people kind-of understand the stock market.

| Tableau (9) | [ $\Omega^{\delta} \mathrm{S}$ ] | [ $\Omega^{\varepsilon} \mathrm{S}$ ] | True | Ntrl | Crtn | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | (1/2..1) People | (9/10..1) People | *! |  | - |  |
| $\cdots{ }_{M}{ }^{+}$ | ( $/ 2 . .{ }^{+}(1 / 10.0$ 1) People | ( $9 / 10.1$ ) People |  |  | * | * |
| $I_{K}{ }^{+}$ | (1/2..1) People | (\% ${ }_{\text {e, }}{ }^{+}$[1] People | *! |  | ${ }^{2}$ | * |
| $\boldsymbol{I}_{M}^{+}-{ }_{K}^{+}$ | (1/2. ${ }^{+}(x .1)$ P People | ( $9_{60 .}+{ }^{+}$[1] People | *! | - | \% | ** |
| $I_{\text {TRV }}$ | (1/2..1) People | (1/2..1) Peeple |  | *! | $\stackrel{1}{4}$ | \% |
| $\mathrm{I}_{\text {TRV-M }}{ }^{+}$ | (1/2.. ${ }^{+}\left(x_{0} .1\right)$ People | (1/2..1) People | *! | * | * | * |

Knowledge enhancement under $\mathrm{K}^{+}$will never satisfy Truth in this example because $\mathrm{M}^{+}$ will never produce a model of certainty as it is defined. Then, $\mathrm{K}^{+}$combined with any model enhancement will be sub-optimal as in the set of candidates depicted by candidate $I_{M}{ }^{+}-K^{+}$. Finally, TRV is sub-optimal as the operation of $M^{+}$is preferred. This is verified by the unbounded status of the subject in the favored interpretation of (26i).

Tableau (9') Annotated Comparative Tableau

| Tableau (9) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\Omega^{\varepsilon} \mathrm{S}\right]$ | True | Ntrl | Crtm | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\cdots{ }_{M}^{+}$ | (/2.. ${ }^{+}(1 / 10.0$ 1) People | ( $9 / 10.1$ ) People |  |  | * | * |
| $I_{o}$ | (1/2..1) People | ( $9 / 10.0 .1$ ) People | *!W |  | $\underline{L}$ | 1 |
| $\mathrm{I}_{K}{ }^{+}$ | (1/2..1) People | (\%.. ${ }^{+}$[1] People | *! W |  | 7 | * |
| $\bar{I}_{M}^{+}-{ }^{+}{ }^{+}$ | (\%/2. ${ }^{+}\left(x_{0} .1\right)$ People | ( 7 to. ${ }^{\text {T }}$ [1] People | *! W |  | * | **W |
| $I_{\text {TRV }}$ | (1/2..1) People | (1/2..1) People |  | *! W | * | 1 |
| $I_{T R V-M}{ }^{+}$ | (1/2.. ${ }^{+}\left(x_{0} .1\right)$ People | (1/2..1) People | *!W | *W | * | * |

Among others, a ranking argument (RARG2) for the constraints Natural 》Actual is provided by ( $\mathrm{T} 9^{\prime}$ ). A constraint conflict $\mathrm{W}-\mathrm{L}$ is visible from the candidate $I_{T R V}$. With the current ranking, the conflict favors the desired optimum $\left(I_{M}{ }^{+}\right)$as the quantified interpretation. A similar demonstration for the pair-wise ranking can be made for (T11) below.
(27) T10 Inputs
i. Artifact: Few people play the harp.
ii. Participation Model: $\quad \Omega^{\delta}=(0 . .1 / 2)$
iii. Epistemic Interval: $\quad \Omega^{\varepsilon}=(0 . .1 / 2)$
iv. Subject Class: $\quad S=$ People

Tableau (10) Few people play the harp. (Agree)

| Tableau (10) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\Omega^{\varepsilon} S\right]$ | True | Ntrl | Crtn | ActI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| To $I_{0}$ | (0..1/2) People | (0..1/2) People |  |  | * |  |
| $\underline{M M}^{+}$ | (0.x) ${ }^{+}$. $1 / 2$ People | (0.1/2) People | *! |  | * | * |
| $\mathrm{I}_{K}^{+}$ | (0..1/2) People | $[0]^{+}$. $/ 2$ P People | *! |  | 4 | 7 |
| $\mathrm{IM}^{+}-{ }^{+}{ }^{+}$ | (0..x) ${ }^{+}$. $1 / 2$ ) People | $[0]^{+}$, , /2) People | *! |  | 3 | 8* |
| $I_{T R V}$ | (0..1/2) People | (0..1/2) People |  | *! | \% | - |
| $I_{T R V-M}{ }^{+}$ | (0.0x) ${ }^{+}$. $1 / 2$ ) People | (0.1/2) Peeple |  | *! | * | 8 |

Also in T11, strong agreement with $\Omega^{\varepsilon}$ is achieved under the operation of $\mathrm{M}^{+}$.
i. Artifact:
ii. Participation Model: $\quad \Omega^{\delta}=(0 . .1 / 2)$
iii. Epistemic Interval:
iv. Subject Class:

Few people really play the harp well.
$\Omega^{\varepsilon}=\left(0 .{ }^{1} / 10\right)$
$S=$ People

Tableau (11) Few people really play the harp well. (Strongly Agree)

| Tableau (11) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | [ $\Omega^{\text {e }}$ S] | True | Ntrl | Crin | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{o}$ | (0..1/2) People | (0.. $1 / 10$ ) People | *! | S | * |  |
| $\mathrm{I}^{+}{ }^{+}$ | (0.0 $\left.{ }^{1 / 10}\right)^{+} . .^{1 / 2}$ ) People | (0.. $1 / 10$ ) People |  |  | * | * |
| $\mathrm{I}_{K}^{+}$ | (0..1/2) People | [0] ${ }^{+}$, /, \%) People | *! |  | * | * |
| $\boldsymbol{I}_{M}^{+}{ }^{+}{ }^{+}$ | (0.x) ${ }^{+}$. $1 / 2$ ) People | $\left.[0]^{+} . .1 / 0\right)^{+1}$ People | *! |  | * | ** |
| $I_{\text {TRV }}$ | (0..1/2) People | (0..1/2) People |  | *! | + |  |
| $I_{\text {TRV-M }}{ }^{+}$ | (0.x) $\left.{ }^{+} .1 / 2\right)$ Peeple | (0..1/2) People |  | *! | \$ | * |

### 2.5.3 Bare Genericity

Bare Genericity refers to those generic interpretations which can arise from artifacts where no overt universal expressions are present in the linguistic artifact. ${ }^{18}$ The following under (29) are typical examples of Bare Genericity.
a. A dog barks. That's what dogs do.
b. Dogs bark. That's what dogs do.

The phenomenon is explained here under the combined effects of $\mathrm{M}^{+}$and $\mathrm{K}^{+}$. First, because the input model is mere possibility $\left(\Omega^{\delta}=(0 . .1]\right), \mathrm{M}^{+}$must apply in order to yield a truthful interpretation at certainty $\left(\Omega^{\delta}=\Omega^{\varepsilon}=1\right)$. However, because the epistemic

[^14]interval is never certain with respect to an empirical set such as Dogs, $\mathrm{K}^{+}$must also apply. Plurality distinctions are assumed to be irrelevant on an unbounded set, but will figure into a bounded candidate under trivialization in T14, T15.

As explained, single enhancements will not satisfy Truth as in candidates $I_{M}{ }^{+}$and $I_{K}{ }^{+}$. Then because of the ranking between Natural and Actual, no number of violations ${ }^{19}$ to Actuality will overcome the desired satisfaction of Natural quantification.
(30) T12 Inputs

| i. Artifact: | A dog barks. |
| :--- | :--- |
| ii. Participation Model: | $\Omega^{\delta}=(0 . .1]$ |
| iii. Epistemic Interval: | $\Omega^{\varepsilon}=(1 / 2 . .1)$ |
| iv. Subject Class: | $S=$ Dogs |

Tableau (12) A dog barks.

| Tableau (12) | [ $\Omega^{\delta} \mathrm{S}$ ] | $\left[\Omega^{\text {e }}\right.$ S] | True | Ntrl | Crtn | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | (0..1] Dogs | (1/2.1) Dogs | *! |  | * |  |
| $I_{M}{ }^{+}$ | (0. ${ }^{+}(1 / 2.01)$ Dogs | (1/2..1) Dogs |  |  | *! | * |
| $\mathrm{I}^{+}{ }^{+}$ | (0..1] Dogs | (1/2. ${ }^{+}$[1] Dogs | *! | , | $\stackrel{*}{*}$ | * |
| $\cdots \quad \boldsymbol{I}^{+}{ }^{+}-{ }^{+}$ | (0. ${ }^{+}$[1] Dogs | (\%.. [1] Dogs |  |  |  | ** |
| $\mathrm{I}_{\text {TRV }}$ | (0.1] Pegs | [1] Pogs | *! | * | - | - |
| $I_{T R V-M}{ }^{+}$ | (0.. ${ }^{+}$[1] Degs | [1] Dogs |  | *! |  | * |
| $\mathbb{I}_{T R V}$ | (0..1] Degs | (x..1] Pegs |  | *! | \$ |  |
| $I_{\text {TRV-M }}{ }^{+}$ | $0.1_{+}^{+}$(x.ol] Degs | (x..1] Degs |  | *! | * | * |

As a result all candidates involving TRV will be sub-optimal. The favored interpretation is predicted to be a generalization of considerable enhancement.
(31) a. An apple a day keeps the doctor away.

[^15]b. A bird in the hand is worth two in the bush.

This case (T12) provides the ranking argument (RARG4) for the constraint ranking Certain >> Actual. Considering the comparative tableau (T12), the candidate $\left(I_{M}{ }^{+}\right)$shows the crucial constraint conflict.

Tableau (12') Annotated Comparative Tableau

| Tableau (12) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\Omega^{\varepsilon} S\right]$ | True | Ntrl | Crim | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{6} I_{M}^{+}{ }_{-K}{ }^{+}$ | 10. ${ }^{+}$[1] Dogs | (1/2. ${ }^{+}$[1] Dogs |  |  |  | ** |
| $I_{0}$ | (0..1] Dogs | (1/2., 1) Dogs | *! W | + | *W | 1 |
| $\boldsymbol{I}_{M}{ }^{+}$ | (0. ${ }^{+}(1 / 2.1 .1)$ Dogs | (1/2..1) Dogs |  |  | *!W | ${ }^{\text {L }}$ |
| $I_{K}^{+}$ | (0..1] Dogs | (1/2. ${ }^{+}$[1] Dogs | *!W |  | \% | ${ }^{\text {\% }}$ |
| $I_{\text {TRV }}$ | (0.1] Dogg | [1] Degg | *!W | *W | *W | L |
| $I_{\text {TRV-M }}{ }^{+}$ | (0.+ [1] Degs | [1] Degs |  | *! W |  | +L |
| $I_{T R V}$ | (0..1] Dogs | (x.1] Degs |  | *!W | *W | L |
| $\mathrm{I}_{\text {TRV-M }}{ }^{+}$ | (0.) ${ }^{+}$(x.01] Degs | (x..1] Degs |  | *! W | *W | *L |

An identical violation pattern is found in (T13) with the bare-plural model of possibility.
(32) T13 Inputs
i. Artifact: Dogs bark.
ii. Participation Model: $\quad \Omega^{\delta}=(0 . .1]$
iii. Epistemic Interval: $\quad \Omega^{\varepsilon}=(1 / 2 . .1)$
iv. Subject Class: $\quad S=$ Dogs

Tableau (13) Dogs bark.

| Tableau (13) | $\left[\Omega^{\delta} \mathrm{S}\right]$ | $\left[\Omega^{\varepsilon} S\right]$ | True | Ntrl | Crtn | Actil |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | (0..1] Dogs | (1/2..1) Dogs | *! | - | \% |  |
| $\mathrm{IM}^{+}$ | (0.) ${ }^{+}(1 / 2.01)$ Dogs | (1/2..1) Dogs |  |  | *! | * |
| $\mathrm{I}^{+}$ | (0..1] Dogs | (\%. ${ }^{+}$[1] Dogs | *! | - | * | * |
| $\mathrm{H}^{+}{ }^{+}{ }^{+}$ | (0.. ${ }^{+}$[1] Dogs | ( $1 / 2 . .+$ [1] Dogs |  |  |  | ** |
| $\mathrm{I}_{T R \mathrm{~V}}$ | (0.11] Degs | [1] Dogs | *! | \% | ${ }^{4}$ |  |
| $\mathrm{ITRV}-\mathrm{M}^{+}$ | (0). ${ }^{+}$[1] Dogs | [1] Degs |  | *! |  | * |
| $H_{T R V}$ | (0.1] Degs | (0.11] Degs |  | *! | * |  |
| $I_{\text {TRV-M }}{ }^{+}$ | 0.. ${ }^{+}$(x.o. 1 ] Degs | (x..1] Dogs |  | *! | * | * |

(33) a. Red skies at night, sailor's delight.
b. Birds of a feather flock together.

Next we consider the optimization function under probabilistic conflict at input. In the following tableaux (T14, T15), the epistemic interval is minority. However as it is defined, the operation $\mathrm{M}^{+}$will never constrain a model of Possibility towards Denial. ${ }^{20}$ Therefore, dual enhancement $\left(\mathrm{M}^{+}, \mathrm{K}^{+}\right)$will not bring Truth at denial $\left(\Omega^{\delta}=\Omega^{\varepsilon}=[0]\right)$ in this case. Therefore, Truth will only be satisfied under TRV to a specific cat. The supporting effort of Model enhancement to certainty is required.
(34) T14 Inputs

| i. Artifact: | A cat swims under-water. |
| :--- | :--- |
| ii. Participation Model: | $\Omega^{\delta}=(0 . .1]$ |
| iii. Epistemic Interval: | $\Omega^{\varepsilon}=\left(0 . .^{1 / 2}\right)$ |
| iv. Subject Class: | $S=$ Cats |

Tableau (14) A cat swims under-water.

| Tableau (14) | [ $\Omega^{\delta} \mathrm{S}$ ] | [ $\left.\Omega^{\varepsilon} \mathrm{S}\right]$ | True | Ntrl | Crtm | Actll |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | (0.1] Cats | (0..1/2) Cats | *! |  | * |  |
| $\mathbb{I M}^{+}$ | (0) ${ }^{+}(x .01]$ Cats | (0..1/2) Cats | *! | , | * | + |
| $\mathrm{I}_{K}^{+}$ | (0.1] Cats | $\left.[0]^{+} .1 / 2\right)$ Cats | *! |  | * | 4 |
| $\mathrm{IM}^{+}-{ }^{+}{ }^{+}$ | 10. ${ }^{+}(x .01]$ Cats | $\left.[0]^{+} .1 / 2\right)$ Cats | *! |  | * | ** |
| $I_{\text {TRV }}$ | (0.1] Cats | [1] Cats | *! | * | 4 |  |
| G $\mathrm{I}_{\text {TRV-M }}{ }^{+}$ | (0.. ${ }^{+}$[1] Cats | [1] Gats |  | * |  | * |
| $I_{T R V}^{\prime}$ | (0.1] Cats | (x.1] Gats | *! | \% | \% |  |
| $I_{\text {TRV-M }}{ }^{+}$ | $10 .{ }^{+}(\mathrm{x} .11]$ Gats | (x.1] Gats |  | * | *! | * |

In $T 14$, the requirement of the singular is met by trivialization to a specific cat. Then $\mathrm{M}^{+}$ may only satisfy Truth at certainty. The candidates $I_{T R V}^{\prime}$, and $I_{T R V-M}^{\prime}{ }^{+}$are two of many

[^16]possible sub-optimal variations of $I_{T R V}$, and $I_{T R V-M}{ }^{+}$. In these candidates, it is assumed that $x \neq 0$. This will result in a violation of Truth for $I_{T R V}^{\prime}$. However, Truth avoids violation under $\mathrm{M}^{+}$in candidate $I_{T R V-M}^{\prime}$. The predicted interpretation is the claim that a single cat swims under water which may be paraphrased as under (35).
(35) There is a cat that swims under-water.

If the interpreter allows the possibility, then trivialization to some special set of cats that swim under-water will satisfy truth in T15. Otherwise the interpretation is falsity $\left(\Omega^{\delta} \neq\right.$ $\Omega^{\varepsilon}$ ).
(36) T15 Inputs
i. Artifact:
Cats swim under-water.
ii. Participation Model: $\quad \Omega^{\delta}=(0 . .1]$
iii. Epistemic Interval:
$\Omega^{\varepsilon}=\left(0 . .^{1 / 2}\right)$
iv. Subject Class:
$S=$ Cats

Tableau (15) Cats swim under-water.

| Tableau (15) | [ $\Omega^{\delta} \mathrm{S}$ ] | $\left[\Omega^{\varepsilon} \mathrm{S}\right.$ ] | True | Ntrl | Crtn | Actl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | (0.1] Cats | (0..1/2) Cats | *! |  | * | - |
| $\mathrm{IM}^{+}$ | $10 .{ }^{+}(x .01]$ Cats | (0.1/2) Cats | *! |  | * | * |
| $\mathbb{I}_{K}^{+}$ | (0.1] Cats | $[0]^{+}$. $/ 3 /$ ) Cats | *! | \% | * | * |
| $\mathrm{IM}^{+}{ }^{+}{ }^{+}$ | (0.. ${ }^{+}\left(x_{0.1} 1\right]$ Cats | $[0]^{+}$, /2, Cats | *! | , | * | * |
| $\mathrm{I}_{\text {TRV }}$ | (0.1] Gats | [1] Gats | *! | 4 | * | 4 |
| $\square \mathrm{I}_{\text {TRV- } M}{ }^{+}$ | (0.. [1] Cats | [1] Gats |  | * |  | * |
| $I_{\text {TRV }}^{\prime}$ | (0.1] Gats | (x.1] Cats | *! | * | * |  |
| $\mathrm{I}_{\text {TRV-M }}{ }^{+}$ | (0.) ${ }^{+}$(Xo.1] Gats | (x..1] Gats |  | * | *! | \% |

Now the optimal candidate is a trivialized plural at certainty. The interpretation is paraphrased by the following under (37), where no partitivity is granted among the chosen group of cats. That is, there are cats that swim under-water, and they all do so.
(37) There are cats that swim under-water.

This is in contrast to the final sub-optimal candidate where only some portion of the group are participants. The topic of proportional readings and Partitvity is taken-up in Chapter three.

### 3.0 Representations

### 3.1 Situation Notation

Here, representational knowledge is taken to be of a partial nature, and therefore the representational form chosen for the purposes of this account will be a situation. Relative to a naturalistic-representational debate sparked by Barwise and Perry (1983), the term situation in the current theory simply means: a partial set-theoretic information structure. Ontological matters regarding situations will not be considered in great detail. Contra B\&P (1983), situations or situation-types do not contain truth-values as they are found to be useful here. However, the Situation Theory offers (at least) a representational perspective that seems to be essential. Therefore, the following set-theoretic constructs of the situation theory will be assumed with minor alterations to $\mathrm{B} \& \mathrm{P}$ (1983) as mentioned.

- Situation: Set of individuals and properties taken over some space-time interval.
- Sub-situation: A situation $s^{\prime}$, is a sub-situation of $s$ iff $s^{\prime} \subseteq s$.
- Situation-type: A situation $s$ is of type $s^{\prime}$, iff $s^{\prime} \subseteq s$.

The lattice-theoretic and algebraic properties of sets of situations are well-documented, and can be found in the literature on situation theory (Barwise and Etchemendy, 1997). Given the following individuals, properties, and space-time regions/intervals:

| Set of Individuals ( $i$ ): | $\mathrm{N}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ |
| :--- | :--- |
| Set of Properties: | $\mathrm{P}=\{\operatorname{dog}(0, \mathrm{do}(0$, bark(0, wag-tail() $\}$ |
| Space-time Indexes $(l, t):$ | $@ l$ space-region : $t_{o}-$ t time interval |

We may construct situations $s, s^{\prime}$ as a set (possibly empty) containing an individual, necessarily dressed in-property, taken over some region of space-time.
(1) Two Situations: $s^{\prime}, s$

$$
\begin{array}{rlrl}
s^{\prime}:= & @ l, l_{0}-t_{o}-t\{ & s:= & @ l, t_{o}-t\{ \\
& & \mathrm{a}, \mathrm{~b} \\
& \operatorname{dog}(x) & & \operatorname{dog}(x) \\
& x=\mathrm{a} & & \operatorname{dog}(y), \text { wag-tail }(y), \operatorname{do}(y) \\
& & & x=\mathrm{a}, y=\mathrm{b}
\end{array}
$$

A situation $s^{\prime}$, is a sub-situation of $s$ iff $s^{\prime}$ is contained by $s$. As can be confirmed, we have $s^{\prime} \subseteq^{S} s$ under (1) above.

### 3.1.1 Situation Types

A situation $s$ is of type $s^{\prime}$ iff $s^{\prime}$ is a sub-situation of $s$. The idea is that a situation is of every type of sub-situation it contains. Therefore, a situation in which there is a dog and a cat is of the cat type and also of the dog type. A situation type $\uparrow s^{\alpha}$ is the set of situations of type $s^{\alpha}$ namely, the set of super-sets of $s^{\alpha}$.
(2) Situation Type: $\uparrow s^{\alpha}=\left\{s: s^{\alpha} \subseteq^{s} s\right\}$

In situation theory, entailment is taken as a relation between situation types, where the implication,
(3) $\operatorname{kiss}(a, b) \rightarrow \operatorname{touch}(a, b)$
may be represented as the following condition on situations.
(4) $s^{\text {kiss }} \subseteq s^{\text {touch }}$

This condition requires that the set of kissing situations (kissing-type) is a subset of touching situations (touching type). Equivalently, it is said that every kissing situation contains a touching situation as a sub-situation, which corresponds naturally to the intuition that touching is a part of kissing. Or, in terms of the relation of involvement,
kissing involves touching (Barwise and Perry, 1983). Therefore in our current model, barking and tail-wagging are types of doing ( $s^{\text {bark }} \subseteq s^{\mathrm{do}}$, and $s^{\text {wag-tail }} \subseteq s^{\mathrm{do}}$ ).

### 3.1.2 Space-time

At first, an intuitive picture of space-time is presented for purposes of clarity, and with the hope of facilitating the argumentation which follows. Imagine looking down from a hot-air balloon onto a large, fenced yard containing three dogs $\{a, b, c\}$. Let's say this is dog-world $\left(S^{\omega}\right)$, or the maximal situation when extended in time.
(5) A Situation in Dog-World
$s:=@ l, t_{o-t}$

Let the situation $s$, be some dollop of space-time, or time interval $\left(t_{o}-t\right)$ at space region ( $)$, containing three dogs $\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$. Let's say that in this particular situation, dogs ( $\mathrm{b} \& \mathrm{c}$ ) were running ( b was faster), and dog (a) was just barking. In $s$, each dog/dog-path constitutes a sub-situation, $\left(s_{1}, s_{2}, s_{3}\right)$.
(6) Lattice of Sub-situations


Furthermore, a situation, say $s_{2}$, may be taken at an infinite number of sub-intervals ( $t_{o}^{\prime}-$ $t$ ), that constitute sub-situations $s^{\prime}$ of $s_{2}$ such that, $s^{\prime} \subseteq^{\mathrm{S}} s_{2}$.
(7) A Sub-situation of $s_{2}$ Taken Over $\left(t_{0}^{\prime}-t\right)$


However, considering that individuals are extended in time, a minimal form of situation $s^{0}$ will be used, which will be called a situation of observation.
(8) Situation of Observation $s^{0}$


A situation of observation $s^{\circ}$, may be understood as a sample of an individual ( $i=b$ ), that is careening through space-time, over an interval that is no smaller than some minimal duration (to $-t$ !). The following example situation is the unfortunate course of a bug. In truth, the entire room is extended in time, and moving through space. However, since our bug is accelerating with respect to the room, we may consider the room as being fixed in space relative to the bug.
(9) A Sample Situation


### 3.2 A Semantics of Participation and Observation

Let an Observation ( $o$ ) be a symbolic representation of visual experience. For now, an observation is required to contain the representation of some minimal perceptible distinction. Furthermore, it will be assumed that observations are keyed to a certain particular object that they represent, called the focus. Any set of observations ( $O$ ) may be grouped by focus into partitions ( $\pi$ ) or, exhaustive non-overlapping subsets. Each partition will then be considered as a way to represent the individual that serves as the focus.

Let there be a set of observations that may be called "dog" observations in the most intuitive of senses possible (c.f. Quine's, Word and Object). Then, let each of these observations belong to only one of possibly many equivalence classes (partitions on the set of dog observations) that correspond to individual dogs. These equivalence classes will be called individual partitions. The set of such individual partitions will be called the Subject ( $S$ ). Accordingly, $S$ is a set of sets of observations. The cardinality of the set ( $s=$ $|S|$ ) is equal to the number of partitions on $S$, or the number of individual dogs that are discerned among the set of observations.

The generalized union of a set of sets, results from taking the union of all its members.

$$
\begin{equation*}
\bigcup_{F}=\{x: \text { for some } A, A \in F \wedge x \in A\} \tag{10}
\end{equation*}
$$

The individual partitioning of a set of observations $O$, yields a set $S$ of non-overlapping subsets of $O$ that jointly exhaust all members of $O$.
(11) i. $\cup_{S}=0$
ii. if two individual partitions $A$ and $B$ are both in $S$ and $A \neq B$, then $A \cap B=\varnothing$.

### 3.2.1 Participation

As an observation may represent objects, an observation may also represent certain properties. If a set of observations $O$ can be grouped by property, then the intersection of a property $\phi$ as a set of observations, and an individual partition $\pi$ may be considered. Therefore an observation $o$, member of an individual partition $o \in \pi$, may also be a member of a property $o \in \phi$. In such a case, it will be said that the individual that is the focus of $\pi$ bears the property $\phi$ according to observational knowledge. It will be said that such knowledge constitutes evidence of participation of the individual $\pi$ in $\phi$. In other words, $\pi$ participates, or is a participant in $\phi$.

Those individual partitions $\pi$, that share an observation with a property $\phi$, are said to be the active partitions $(A P)$ in $\phi$. The physical value of participation $(p)$ may be defined as the cardinality of the active set.
(12) $p=|A P|=3$

As above, the subject cardinality is the number of partitions on the class of observations.

$$
\begin{equation*}
f=|S|=5 \tag{13}
\end{equation*}
$$

Finally, the Probabilistic Opinion of Participation ( $\omega$ ) is defined as the ratio of
participants to partitions.
(14) $\omega={ }^{p} / s=3 / 5=.6$

### 3.2.2 The Natural Language Determiner as Participation Model ( $\Omega^{\delta}$ )

A determiner is defined as denoting fixed proper sub-interval of the unit interval, $\Omega^{\delta} \subseteq[0.1]$. This initial meaning of the determiner has been called the denotation interval. Quantificational description of the sentence must be taken from some subinterval of the items in the following list, where it is understood that the quantificational meaning of the sentence $(\Omega)$ is constrained to within some basic denotation interval ( $\Omega^{\delta}$ ).
(15) Functional Interpretation of NL Sentence at $\Omega$

| i. Certain | $\Omega \subseteq[0]$ or,$\Omega \subseteq[1]$ |
| :--- | :--- |
| ii. Probable | $\Omega \subseteq(1 / 2 . .1)$ or $\Omega \subseteq\left(0 . .^{1 / 2}\right)$ |
| iii. Possible | $\Omega \subseteq(0 . .1]$ |
| iv. False | $\Omega^{\delta} \neq \Omega^{\varepsilon} \quad$ (no function) |

In the following, a presentation of the denotations of ten linguistic determiners of English will be made in terms of such denotation intervals.
(16) English Determiners Under Consideration

1. all 6.a
2. every 7. bare-plural
3. most 8. numeral
4. few 9. some
5. no 10. the

It is possible to derive a basic vocabulary of constraints on $\Omega$, which serve as models of participation. First, a correspondence is observed with the classic set-theoretic determiner meanings (Barwise and Cooper, 1981).
(17) Correspondence With Set-theoretic Constraints
i. $\left[\left[\mathrm{DET}^{\text {all/every }}\right]\right]: \quad \lambda A \lambda B \cdot \mathrm{~A} \subseteq B \quad \leftrightarrow \quad p=s$
ii. $\left[\left[\mathrm{DET}^{\text {no/none }}\right]\right]: \quad \lambda A \lambda B \cdot|A \cap B|=0 \quad \leftrightarrow \quad p=0$
ii. $\left[\left[\mathrm{DET}^{\text {some/a }}\right]\right]: \quad \lambda A \lambda B \cdot|A \cap B| \neq 0 \quad \leftrightarrow \quad p \geq 1$

By simple manipulation of the representation, this correspondence can be expressed in terms of the proportion $p / s$.
(18) Correspondence Between Constraints On $p, s$ and ${ }^{p} / s$

$$
\begin{array}{lll}
p=s & \leftrightarrow & p / s=1 \\
p=0 & \leftrightarrow & p / s=0 \\
p \geq 1 & \leftrightarrow & p / s>0
\end{array}
$$

Also the following may be included,

$$
\begin{array}{llll}
\text { (19) } & p>1 / 2 S & \leftrightarrow & p / s>1 / 2 \\
& p<1 / 2 s & \leftrightarrow & p / s<1 / 2
\end{array}
$$

Interestingly, constraints on just three locations of $\Omega$, i.e. $[1,0,1 / 2]$, will suffice to account for the quantificational aspects of the natural language determiners under consideration. Therefore, the following set of basic participation models is proposed.
(20) Basic Participation Models

| Certain | Probable | Possible |
| :---: | :---: | :---: |
| $\Omega \in[1]$ | $\Omega \in(0.1 / 2)$ | $\Omega \in(0.1]^{\text {sl }}$ |
| $\Omega \in[0]$ | $\Omega \in(1 / 2.1)$ | $\Omega \in(0.1]^{\text {plr }}$ |

A plurality distinction is made regarding the model of possibility. This distinction will be treated as follows under (21), as a constraint on bounded values of participation. Briefly, the thought is that because the distinction of plurality only pertains to bounded values, the constraint of plurality should be conditional on bounding.
(21) i. Sgl: $p \rightarrow(\beta=1)$
ii. Plr: $\beta \rightarrow(\beta>1)$

In addition, two partial constraints on $\Omega$ are needed to account for numerals, some, and the. Embedded constraints do not constrain the range of values that $\Omega$ may take. Instead embedded constraints are imposed directly on either participation (p) or the subject cardinality ( $s$ ). Such constraints are called embedded, because they are imposed directly on the internal parameters $p$, and $s$ of $\Omega$.

## (22) Embedded Constraints on $\Omega$

|  | Determimer | Constraint | Description |
| :---: | :---: | :---: | :---: |
| i. | Numeral | $\Omega(\beta \geq i, s)$ | Participation $>$ minimal value $i$. |
| ii. | Some | $\Omega(\neq s)$ | Participation is bounded. |
| iii. | The | $\Omega(p,\langle s\rangle)$ | The subject is private. |

The embedded constraint on $\Omega$ imposed by a numeral determiner serves to place a lower bound on participation. Therefore, according to the constraint, there is a bounded value of participation that is greater than some specified minimal value (i). ${ }^{21}$ Importantly, participant bounding does not itself trivialize the subject. The embedded constraint on $\Omega$ imposed by the determiner some simply places an unspecified lower-bound on

[^17]participation. Considering the bounding status of natural vs. trivial significance, we have the following typology of known and unknown, upper and lower bounds.

## (23) Embedded Lower Bounds Upon the Subject Cardinality

i. known lower: numeral
ii. unknown lower: some

It is the idiosyncratic nature of the determiner all that the subject may not be implicitly trivialized. It is for this reason that the determiner all will be called the natural-certain model $\Omega^{\text {all }}=[1]^{\mathrm{N}}$. As a final consideration, it is possible to implement the plurality distinction mentioned above in terms of embedded constraints on $\Omega$. These eight $\Omega$ constraints form a basic vocabulary of restrictions on possible quantificational descriptions that may be made of the sentence.
(24) Determiners As Models of Participation

| Determiner | Model |
| :---: | :---: |
| 1. all | $[1]^{\text {N }}$ |
| 2. every | [1] |
| 3. no | [0] |
| 4. most | (1/2.1) |
| 5. fow | (0.1/2) |
| 6. $a$ | (0..1] $]^{\text {sg }}$ |
| 7. bare-plural | (0.11) ${ }^{p / r}$ |
| 8. numeral | $\Omega(p \geq i, s)$ |
| 9. some | $\Omega(p, s)$ |
| 10. the | $\Omega(p,\langle s\rangle)$ |

Significantly, the proposal reveals that despite these constraints, much of the $\Omega$ relation remains unspecified. The underspecification of $p$ and $s$ discussed in this section, is considered to be a natural language design feature which allows for epistemo-linguo correspondence. Underspecification allows the necessary wiggle-room for semantic
action to aid the purpose of truthful interpretation. The table under (25) shows a lack of specification involving the value of $s$. This allows for the actions of knowledge enhancement ( $\mathrm{K}^{+}$) and TRV. By contrast, it is knowledge of participation ( $p$ ) that is modeled by linguistic determination.
(25) Implications for the Values of $p$ and $s$ Given $\Omega^{\delta}$

| Determiner |  | $p$ | $s$ |
| :---: | :---: | :---: | :---: |
| all, every | $[1]$ | $p=s$ | - |
| no | $[0]$ | $p=0$ | - |
| most | $(1 / 2.1)$ | $(1 / 2 s . s)$ | - |
| few | $(0.1 / 2)$ | $(0 . .1 / 2 s)$ | - |
| $a^{\text {sing }}$ | $(0 . .1]$ | $p \rightarrow(p=1)$ | - |
| bare-plural | $(0 . .1]$ | $p \rightarrow(p>1)$ | - |
| numeral $i$ | $\Omega(\beta \geq i, s)$ | $p \geq i$ | - |
| some | $\Omega(p, s)$ | $p$ | - |
| the | $\Omega(p,\langle s\rangle)$ | - | $\langle s\rangle$ |

### 3.3 Semantic Operations

Three semantic operations have been proposed for the attainment of Truth and Certainty. These are:
(26) i. Knowledge Enhancement ( $\mathrm{K}^{+}$)
ii. Model Enhancement $\left(\mathrm{M}^{+}\right)$
iii.Trivialization (TRV).

Each quantificational operation (i-iii) imposes a restriction on a particular physical set $\varphi$, to one of its proper subsets $\gamma . \mathcal{\mathcal { C }} \varphi$.

$$
\begin{equation*}
O p(\varphi) \in\{\gamma \cdot \gamma \subset \varphi\} \tag{27}
\end{equation*}
$$

A quantificational action cannot be said to have taken place if there has been no change discerned upon the input set $\varphi$.
(28) Quantificational Operations:

1. Knowledge Enhancement $\left(\mathrm{K}^{+}\right)$: Excuses a minority of exceptions from $S$.
2. Trivialization (TRV):

Bounds the subject cardinality (s).
3. Model Enhancement ( $\mathrm{M}^{+}$):

Constrains the model of participation $\left(\Omega^{\delta}\right)$.
The actions of knowledge enhancement ( $\mathrm{K}^{+}$) and trivialization (TRV) are operations defined on the subject $(S)$. The action of model enhancement $\left(\mathrm{M}^{+}\right)$is defined on the denotation interval of the determiner. In the next three sub-sections these operation are described in detail.

### 3.3.1 Knowledge Enhancement $\left(\mathbf{K}^{+}\right)$

The effect of knowledge enhancement is to forgive a minority of accidental exceptions. Under the semantics of participation suggested earlier, positive enhancement is equivalent to removing the inactive partitions from the subject. Negative enhancement is to disregard an exceptional minority of active participants when the claim is of denial.
(29) Positive Enhancement of Knowledge to Certainty of Participation


The only possible effect of knowledge enhancement is to bring the value of $\Omega^{\varepsilon}$ to either that of affirmative certainty $\left(\Omega^{\varepsilon}=1\right)$, or denial $\left(\Omega^{\varepsilon}=0\right)$. This will depend on either the valence of the property, or the valence of the participation model. Negative enhancement
will bring either certainty (30a) or denial (30b) with the excusal of an unexpected minority of active participants.
(30) a. All cats avoid being wet.
b. No cats like being wet.

Finally, there is no knowledge enhancement $\left(\mathrm{K}^{+}\right)$predicted when there may be a reason for the exceptional sub-class. Therefore, (31a) is not acceptable as there is a known subclass of cats without tails (Bobcat, Manx, etc.). This is because non-accidental (perhaps reasoned) sub-classes of exceptions are not considered to be candidates for enhancement. Still, a very important question concerns the difference between (31a) and (31b). The interpretation of (31b) ultimately may be called an empirical generalization, where the intended meaning approaches certainty with the appropriate emphasis added.
(31) a. \#All cats have tails.
b. Cats have tails.
c. C'mon, cats have tails.

In (31a), enhancement cannot bring equality $\left(\Omega^{\delta}=\Omega^{\varepsilon}\right)$ by removing non-participants from the subject. This is because there is a non-accidental sub-class of counter-examples that is of interest, and is therefore significant (\$4.4). Regarding (31b) the solution presented below involves enhancement up-to-the-level that is permitted by collections of significant counter-cases such as Bobcats. In any case, to the extent that (31b) is considered to be true, the interpretation is non-Actual for permitting known counter-examples. ${ }^{22}$

22 (i) \#All dogs are good.
(ii) Dogs are good.
(i) is strongly false. (ii) permits a generic reading. Because of (i)'s falsity, we know that (ii) cannot be enhanced to certainty. This appears to be a case of genericity under enhancement up to reasoned counterexamples.

### 3.3.2 Model Enhancement (M)

The operation $\mathrm{M}^{+}$is proposed as a means to satisfy Truth through constraining the denotation interval of the determiner. The denotation interval is considered to be the standard "meaning" of the natural language determiner. However, the standard determiner meaning may be enhanced under $\mathrm{M}^{+}$, to accommodate a more highly constrained level of knowledge ( $\Omega^{\varepsilon}$ ) when necessary, and when possible.

## (32) Primary Model Enhancements

| Model <br> Type | Initial <br> Denotation Interval | Enhanced <br> Denotation Interval |
| :---: | :---: | :---: |
| Majority | $(1 / 2.1)$ | $(1 / 2 .+(x . .1)$ |
| Minority | $\left(0 . .^{1 / 2)}\right.$ | $\left.(0 . x)^{+}, 1 / 2\right)$ |
| Possibility | $(0 . .1]$ | $\left(0 .{ }^{+}[1]\right.$ |

One basic restriction is placed on the action to strengthen the model $\left(\mathrm{M}^{+}\right)$such that the enhanced denotation remains Faithful to the original denotation interval (See Faithfulness in OT). Intuitively, this means that $\mathrm{M}^{+}$may not be used to enhance the initial interval to an interval that was never part of the original model under determination. Firstly, this forbids enhancement to the denotation interval of another determiner that was not a sub-model of the original denotation. Finally, initial certainty models [1] and [0] that are constrained to one value, are never candidates for $\mathrm{M}^{+}$.

Model Enhancement For few and most

(34) Model Enhancement For Possibility-Models $\left(\Omega^{L}>0\right)$ to $\left(\Omega^{L}=1\right)$


Regarding the first case, it must be asked if the enhanced interval was ever part of the original determiner denotation. For instance, affirmative certainty [1] is part of the possibility model (0..1] and therefore enhancement from possibility to affirmative certainty is legal.
(35) $[1] \subset(0 . .1]$

However, denial is not part of the possibility interval as defined, and therefore $\mathrm{M}^{+}$from possibility to denial is not allowed.
(36) $[0] \not \subset(0.1]$

Moreover, neither affirmative certainty nor denial, are ever part of probability models, and so $\mathrm{M}^{+}$from probability to certainty is never allowed.
(37) i. $[1] \not \subset(1 / 2 . .1)$
ii. $[0] \not \subset\left(0 . .^{1 / 2}\right)$

Therefore, under the general constraint that enhancement be faithful, the following is not allowed.
(38) Possibility to Unlikely Not Allowed Under $\mathrm{M}^{+}$

$$
(0.1] \rightarrow(0.1 / 2) . .1]
$$

In general, an upper bound may be enhanced from closed "]" to open ")". This specific enhancement is in keeping with the overall conception of model constraint because the
open interval removes a possible value of $\Omega$ that was formerly available under the closed interval, i.e. $\{1\}$. Therefore, the theory predicts the following enhancement from possibility to the stronger interval of possibility, (i.e. no knowledge).

$$
\begin{equation*}
(0 . .1] \rightarrow(0 . .1) \tag{39}
\end{equation*}
$$

### 3.3.3 Trivialization (TRV)

Trivialization occurs when the members, or cardinality of the subject is explicitly chosen or fixed by the interpreter. As above, the set of observations may be partitioned into equivalence classes of observations relative to particular individuals. Familiar partitions should be the largest. However, the grain-size of such partitions may also be fine enough to isolate single observations. At this point, these are singleton partitions containing only a single observation. It is also at this point, that the cardinality of the set of observations $|O|$, bounds the cardinality of the set of its own possible partitions (s). Therefore, if the set of observations is itself bounded, then the set of partitions upon the set is also bounded.

Let the subject $S$, be non-trivial iff it represents an unbounded set of possible partitions. This means that the subject will be non-trivial iff there is no bounding of partitions, which in turn requires there to be no bounding of observations.

A trivial subject however, may still be unbounded with respect to observations. This is because the set of observations may increase indefinitely, despite any non-null partitioning. Partition bounding does not imply observation bounding.
(40) Bounding Asymetry


As above, let a subject be trivial iff there is an upper bound on the number of its individual partitions. Specifically, trivialization has been defined as the operation to bound the subject partitions. Therefore, it is important to understand the alternative circumstance that implies trivialization, namely, the bounding of the set of observations.

The set of observations is bounded if it is constrained to observations of a fixed location and of a fixed time. As above, the observation can be represented as a situation in the basic set-theoretic sense of Barwise and Perry (1983). Then, the observation is a representation of some region of space-time with possible attending properties. However, since the observation is strictly taken as a cognitive artifact, qua representation, we expect to see some physical constraints on its constitution.

Firstly, it is proposed that an absolute limit be placed on the minimum space-time region that may be called an observation. There is no forever-receding notion of an observation. Secondly, assume that one's observations are minimally contiguous in space-time (non-overlapping). Then, because of the assumed absolute minimal size of an observation, any bounded region of space-time must contain only a finite number of minimal observations ${ }^{23}$ In this sense, a set of observations is bounded if it is constrained to observations of a fixed location and time. Then, according to the implication under (40), constraining the set of observations at both space and time, will serve to place an upper bound upon the number of possible partitions on the subject. Bounding the set of observations by both space and time amounts to the trivialization of the subject.

Although placing both space and time constraints on the set of observations will bound partitions, constraining either space or time, but not both, will not bound

[^18]partitions. This is because as long as there is an open dimension (i.e. space or time), then unnumbered minimal distinct observations are possible. Of course, each observation may have its own partition for the sake of the argument. Therefore, constraining the set of observations to either space or time, but not both, does not amount to trivialization. Then, a subject of this description would not be trivial, because neither the number of partitions, nor the number of observations is upwardly-bounded.

Here, an interpretation involving a non-trivial subject is of natural significance. This means that an interpretation of natural quantification concerns a subject having unbounded possible partitions, and unbounded but perhaps constrained observations. Depending on what constraints on the set of observations may be enforced without trivializing the subject, the following typology of local generalization is possible, where a reading of expectation is based on the interpretation of a natural value.
(41) i. Public Expectation: "Dogs bark."
a. Unbounded Individual Partitions
b. No constraints on Observations.
c. You can expect to see dogs bark.
ii. Venue Expectation: "Fish fly around here."
a. Unbounded Individual Partitions
b. Observations constrained to location.
c. You can expect to see fish fly (around here).
iii. Time-Type: "Dogs howl when the moon is full."
a. Unbounded Individual Partitions
b. Observations constrained to time under description.
c. You can expect to hear dogs howl when the moon is full.
iv. Individual Expectation: "Three dogs fight"
a. Unbounded Individual Partitions
b. Minimum participation level $(\mathfrak{\beta})$ is specified.
c. You can expect to see three dogs fight.

To summarize the typology, all expectations involve subjects that have unbounded partitions and observations. However, there are local expectations (Venue, Time-Type, Individual) in which the set of possible observations has been constrained to a place, a time description, or a minimum number of participants.

Typology of Expectations ( $s=|S|=$ Subject Cardinality)

| Expectation | Partitions <br> Unbounded | Observations <br> Unbounded |
| :---: | :---: | :---: |
| Public | Yes | Yes |
| Venue | Yes | Yes w/ constraint: $(l, t)$ |
| Time-Type | Yes | Yes w/constraint: $(l, t)$ |
| Individual | Yes | Yes w/ constraint: $(\mathcal{P} / s)$ |

These are possible interpretation types that have been derived from partially restricting a set of possible observations in the allowable ways. ${ }^{24}$ An interpretation with a set of observations that is restricted yet natural, will be referred to as a Local Generalization/Expectation.

A special case of quantificational certainty arises under TRV, and will be called trivial certainty. The notion of trivial certainty corresponds intuitively to the distinction made by Milsark (1974) regarding the Proportional vs. Cardinal readings of some English

[^19]sentences, and also to accidental universal quantification (Kratzer, 1982). ${ }^{25}$ Here, Milsark's notion of a cardinal reading will be equated with trivial certainty made under TRV, while the proportional reading will be equated with non-certainty under TRV. In the following examples under (43), we have Milsark's proportional reading (43a) where the subject is greater than twenty individuals, and Milsark's cardinal reading (43b) where the subject is equal to twenty individuals.
(43) a. Twenty people work.
b. Twenty people work here now.

Here, (43a) will be referred to as a case of trivial uncertainty ( $\Omega<1, \S$ ), and (44b) as a case of trivial certainty $(\Omega=1, \oiint)$. The trivial uncertainty of (43a) is indicated by the favored proportional reading that is paraphrased under (44).
(44) Twenty of the people work.

Lastly, trivialization is found with the specification of sufficient tense information regardless of the interpreter's probabilistic intuition. Therefore, the following are examples of a trivial subject as demonstrated by the relevance of bounding questions (when, where, how many). Grammatical tense information bounds the subject by constraining both time and space. This is unlike the time-type expectation, which simply constrains the time of observation to an unbounded class of observations under some description of time as in (41iii).
(45) a. Dogs barked.
b. Cats were swimming under-water.
c. Dogs are barking.

[^20]d. Dogs are climbing-trees and doing math.

Interestingly, tense information does not necessarily trivialize the subject under the appropriate time-type, although the trivial reading is available.
(46) a. Dogs barked (back in the day).
b. Cats were swimming under-water (before the turn of the century).
c. Dogs are barking (these days).
d. Dogs are climbing-trees and doing math (all the time).

### 3.3.4 Privatization (PRV): A Single Non-Quantificational Operation

A Public class of observations may be thought of as being informationally common to all observers, where levels of participation relative to basic properties are considered, by convention, to be shared among adult speakers. Because a Public class of observations is considered to be shared knowledge, all discrepancies between adult observers regarding a Public class should be eventually resolved in some way.

By contrast, there are Private classes of observations. A private class may not reflect regularities that are supported by the Public class, or may reflect regularities that are not supported by the Public class. Examples of a Public class of observations would be the general set of dog observations or, the general set of cat observations.

A private class of observations is a sub-class that may not reflect commonly assumed participation levels. Here, marking the subject as private will be called privatization (PRV). The result of the semantic operation of privatization is to decouple the interpreter's expectations that are associated with widespread (i.e. public) knowledge. Privatized descriptions may be either bounded or unbounded, and may also turn out to be equivalent to a public description of the sentence, except, of course, for the special
distinction of being private. However, the cost incurred by the operation of PRV relates to the abandonment of a public description of the sentence.

An explicit marker of privatization in English, is considered to be the definite article the.
(47) The dogs run-wild around here.

The notion of a private-subject is necessary to explain the fact that while the subject in (47) is unbounded, it is not the public class of dogs. That the class is unbounded is clear when the condition of running-wild only has something to do with the location. Then, there's no telling how many such observations of dogs running wild may be made around here. Therefore, (47) is example of a subject that is non-trivial under privatization.

In other cases, the subject may be taken as private although the artifact is not explicitly marked as such. However, instances of implicit privatization are associated only with the numeral determiners and some. In section 3.2.2 above it was said that implicit privatization (assumption of a private class), is only allowed when an embedded constraint (i.e. a constraint on the value of $p$ for instance) is imposed, or when space-time information is provided. In short, privatization of the description is allowed only when the speaker has indicated specific knowledge in the form of number, place, or time, or else when the artifact is explicitly marked as being speaker-known (See Heim, 1982).

In the following, the primary paraphrase of (48a) is (48b). However, it should be recognized that the subject of people is unbounded in both examples. There is no numeric bound that may be placed upon the subject cardinality given the information made available in the sentences under (48). This was called the proportional reading in Milsark (1974).
(48) a. Twenty people work.
b. Twenty of the people work.

There is an important difference between a private-class ( $\langle\mathrm{S}\rangle$ ) and a trivial class under TRV. Most importantly, a private-class is not inherently bounded.

Near the previous turn of the century, it was believed that the definite article implied uniqueness under the concept of a Russellian definite description. Examples which support this intuition are the familiar definite descriptions.
a. The President
b. The King of France

However, any definition of uniqueness is discredited by the counter-example of Fodor and Sag (1983), where no interpretation of uniqueness is implied nor inferred.
(50) He was the son of poor farmer.

The concept of private class satisfies both usages however. Regarding the Fodor and Sag example, the subject is marked as private, yet it is not necessarily bounded to singularity.
(51) He was the son of a poor farmer. Now he is the student of a religious leader.

Here, the meaning of the indicates a non-public class definition of son and student. Being a member of these classes has implications that are not associated with a general public understanding of son and student.

Furthermore, it is well known that the article the, demonstrates clear properties of certainty and generalization. The following example shows that certainty is implied by the interpretation of (52i).
(52) i. The bicycles are red.
ii, If you were one of the bicycles, you would be red too.

That is, interpretations involving explicit privatization of the subject (the), involve the entire subject as participants, ceteris paribus. Interestingly, this is also true of denials.
(53) i. The bicycles are not red.
ii. If you were one of the bicycles, you would not be red either.

Finally, common usage includes generalization.
(54) a. The Japanese are heavy smokers.
b. The Japanese have the longest life expectancy.
b. The dogs run wild around here.
c. The fish start biting just after it rains.

Both properties of certainty and generalization are explained under the terms of the current system. Private-unbounded classes may be enhanced to satisfy the desired state of Certainty under $\mathrm{K}^{+}$for local expectations. As can be seen, no subject under (54) is bounded. Examples (54a) and (54b) show non-trivial classes constrained by location and time-description respectively. Lastly, the proper names of individuals such as Bill, Mary, and John, are treated as private subjects that are constrained to a single individualpartition of observations.

### 3.4 Partitive Construction

The description of the sentence is made at Certainty under the following constraint on $\Omega$.

$$
\begin{equation*}
\Omega \in\{1,0\} \tag{55}
\end{equation*}
$$

Then, where $\Omega=p \div s$, the description is certain only where $p=s$, for $p>0$.

$$
\begin{equation*}
\Omega=1 \quad \leftrightarrow p=s, \quad \text { where } p>0 \tag{56}
\end{equation*}
$$

This creates the following entailment of trivial certainty under participant bounding.

$$
P \wedge \Omega=1 \rightarrow s
$$

If $p$ is bounded $(p)$, and $\Omega$ is certain $(\Omega=p / s=1)$, then the subject is trivial ( $s)$. Participant bounding under certainty naturally forces a trivialized subject.

The bounding of participation (p) allows an insight into the relative ranking of the constraints to be Natural, and to be Certain. According to the ranking proposed, a probabilistic description of the sentence would be favored over a trivial description of the sentence under participant bounding (p).

An interesting example of this is the puzzle by Jesperson $(1924,1927)$. The puzzle arises from the fact that in $x$-of- $y$ phrases, $x$ must be less than $y$.
(58) a. Ten of the fingers...
b. Two of the parents...
c. Two of the hands...
d. Four of the tires...

The well-familiar data show that despite the interesting readings under (58), there is no real inequality required by the $x$-of- $y$ construction. More correctly stated, the phenomenon of inequality ( $\Omega>1$ ) happens when it is not otherwise prevented by an input model of certainty ( $\Omega^{L}=1$ ).
(59) a. All Ten of the fingers...
b. Both of the parents...
c. Both of the hands...
d. All Four of the tires...

In this construction $x$-of- $y$, embedded constraints on the parameters $(p, s)$ of $\Omega$ are presented as follows, where the linguistic element "of" appears as the punctuation between the arguments of the part-whole relation.
(60) i. $\Omega(p, s)$
ii. All (Ten of the) men...
iii. $\Omega(\beta,\langle\xi\rangle)$ men...
iv. [1] $(10,10)$ men...

As can be seen, (60ii) is a case where $\Omega$ is fully constrained. First, there is the interval constraint of the certainty-model all. Second, participation is bounded ( $\%$ ), and therefore the subject is trivial (s) given the implication under (57). Third, the subject is privatized given the input the. According to the only truthful reading of (60ii), the size of the set denoted by "men" is fixed at ten individuals.

However, when there is no external constraint of certainty on $\Omega$ as above in (60), then $\Omega$ is naturally and intuitively uncertain $(\Omega<1 ; p<s)$.
(61) i. $\Omega(p, s)$
ii. $\Omega$ (Ten of the) men...
iii. $\quad \Omega(\beta,\langle s\rangle)$
iv. $\quad(10 /\langle s\rangle . .1](10,\langle s\rangle)$ men...

The specification of $p$ occurs via an embedded constraint on $\Omega$, where the value of $\Omega$ is not directly constrained, but only constrained by the internal parameter ( $p$ ). Furthermore, the use of embedded constraints numeral, some, and the, enable the consideration of privatized subject $(\langle s\rangle)$.
(62) Numeral Embedded Constraint
i. $\Omega=(p / s)$
ii. $\Omega(p, s)$ where $p=10$

When the input is privatized, then probabilistic opinion is obviated unless the interpreter shares knowledge of the speaker's private subject. In cases of privatization, $\Omega^{\varepsilon}$ is the unit interval, constrained from below according to the specified level of participation ( $p$ ). This is the same to a restricted possibility model.

$$
\begin{equation*}
\left({ }^{10} /\langle\langle \rangle . .1]\right. \tag{63}
\end{equation*}
$$

Without knowledge of the private domain $\langle s\rangle$, there can be no evaluation concerning how much of $\langle s\rangle$ that the parameter $p=10$ represents. Furthermore, it is not often reasonable to assume that the speaker's private class supports the participation levels of the public class.
(64) i. $\Omega \quad(\beta,\langle s\rangle)$
ii. $\left({ }^{10} /\langle s\rangle . .1\right](10,\langle s\rangle)$ men...

With both internal parameters specified by embedded constraints $(\nexists,\langle s\rangle)$, the subject is unbounded, ceteris paribus. This is because as explained above, neither participant bounding $(p)$ nor privatization $(\langle s\rangle)$ alone will trivialize the subject. The notation under (65b) may be used to indicate an unbounded subject under embedded constraints.

$$
\begin{equation*}
\text { a. } s=p / \mathbf{\Omega}^{\varepsilon} \tag{65}
\end{equation*}
$$

b. $s=10 /\left[{ }^{10} / s . .1\right]$

The cardinality of the subject is participation ( $p$ ) divided by some non-zero value ( $s \geq p$ ) falling within the unit interval. However, since $\langle s\rangle$ is private, there is no way to evaluate
the relative magnitude of the specified $p$-level with respect to $s$. If $\langle s\rangle$ is unknown to the interpreter, then the subject is unbounded. The operation of model enhancement has not been defined for embedded models. Therefore, no effect of $\mathrm{M}^{+}$on an embedded model is currently expected.

The crux of the solution is that partitivity is not a syntactic phenomenon, but a constraint interaction between Natural and Certain. In short, the interpretation favors a Natural quantification over a Certain quantification. Firstly, the input marks the subject as private, which simply means that the subject is a set that the speaker has defined.

Now unless there are fewer than twenty men in the world, an epistemic interval of simple majority " $\Omega^{\varepsilon}=(1 / 2.1)$ " can never be met on any generally held class on men. This is because on a public class of men $s, \Omega^{\varepsilon}=10 / s$ will always be very small. Consequently, $10 / s \notin(1 / 2 . .1)$. Therefore Truth can not be met on as Natural-Public quantification, but only under either trivialization (TRV) or privatization (PRV) as shown in T16.

Next, both TRV and $\mathrm{K}^{+}$can make the description satisfy the constraint of Certainty. The satisfaction of Certainty would remove any partitivity $(p<s)$. However, the cost in both cases is an interpreter-choice of the subject cardinality. The candidate $\boldsymbol{I}_{K}{ }^{+}$ satisfies Certainty because all couter-examples are removed. However, the candidate fails to satisfy truth because there must be more than ten men in the public class. Importantly, PRV has not applied in candidate $\mathbb{I}_{K}{ }^{+}$. Also Natural is violated by $\mathbb{I}_{K}{ }^{+}$because the subject must be bounded if it is certain given the constraint placed on $p$. Although in general $\mathrm{K}^{+}$ simply forces $s=p$, because $p=10$ in this case $\mathrm{K}^{+}$forces $s=10$. Straightforwardly, TRV will also violate Natural quantification.

Tableau（16）Ten of the men smoke．

| T（16） | $\Omega^{\delta}(p, S)$ | ［ $\Omega^{\text {e }}$ S］ | Tr | Ntri | Crtm | Acti | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | $\Omega(10$, （Men $\rangle)$ | （1／2．．1）Men | ＊！ |  | ＋${ }^{\text {＋}}$ | 5 |  |
| $\mathrm{I}_{K}{ }^{+}$ | $\Omega(10,\langle$ Men $\rangle)$ | （1／2．${ }^{+}$［1］Men | ＊！ | ＊ | ¢ | ${ }^{4}$ |  |
| $I_{\text {TRV }}$ | $\boldsymbol{\Omega}$（10，〈Men〉） | ［1］Men |  | ＊！ | － |  |  |
| $I_{T R V}$ | $\boldsymbol{\Omega}(10,\langle$ Men $\rangle)$ | $\left[{ }^{10} / /_{s \neq 10\rangle}\right]$ Men |  | ＊！ | \％ |  |  |
| G ${ }^{\text {a }}$ IRV | $\boldsymbol{\Omega}(10,\langle\mathrm{Men}\rangle)$ | $\left.\left[^{10} / s\right\rangle\right]\langle\mathrm{Men}\rangle$ |  |  | ＊ |  | ＊ |
| $I_{\text {PRV－TRV }}$ | $\Omega(10,\langle$ Men $\rangle)$ | ［1］＜Men〉 |  | ＊！ |  |  | \％ |
| $I_{\text {PRV－TRV }}$ | $\Omega(10,\langle$ Men $)$ ） | $\left.{ }^{10} / /_{s \neq 10\rangle}\right]\langle$ Men $\rangle$ |  | ＊！ | ＊ |  | ＊ |
| $I_{P R V-K}{ }^{+}$ | $\Omega(10,\langle$ Men $\rangle)$ | $\int^{10} /\left\langle\bar{c} . .[11]^{+}\langle\mathrm{Men}\rangle\right.$ |  | ＊！ |  | 4 | $\pm$ |

The single non－initial candidate which avoids a Natural violation is simple PRV and is therefore optimal $\left(I_{P R V}\right)$ ．Crucially，this comes at a violation to Certainty，which causes the description of partitivity $(p<s)$ ．The partitivity of T16 is overcome when the input model forces Certainty as in T17．
（66）T17 Input：$\Omega(p=10,\langle\mathrm{Men}\rangle)=1$
Tableau（17）All ten of the men smoke．

| T（17） | $\Omega^{\delta}(p, S)$ | ［ $\mathbf{S}^{\boldsymbol{E}} \mathbf{S}$ ］ | Tr | Ntrl | Crtn | ActI | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | $\Omega(10,\langle$ Men $)=1$ | （1／2．．1）Men | ＊！ |  | 3 |  |  |
| $\mathrm{IK}^{+}$ | $\Omega(10,\langle$ Men $)=1$ | （1／2．．${ }^{+}$［1］Men | ＊！ |  |  | \％ |  |
| $I_{T R V}$ | $\Omega(10$, （Men $)=1$ | ［1］Men | ＊！ | ＊ |  | ， |  |
| $I_{\text {TRV }}$ | $\Omega(10,\langle$ Men $\rangle)=1$ | $\left[{ }^{10} / 8>10\right]$ Men | ＊！ | 3 | ＊ |  |  |
| $\Pi_{P R V}$ | $\boldsymbol{\Omega}(10,\langle$ Men $\rangle)=1$ | $\left[^{10} /\langle s\rangle\right]$（Men〉 | ＊！ |  | 4 |  |  |
| $\square$ ImRV－TRV | $\Omega(10$, （Men $\rangle$ ）$=1$ | ［1］＜Men） |  | ＊ |  |  | ＊ |
| $I_{\text {PRV－TRV }}$ | $\Omega(10,\langle$ Men $\rangle)=1$ | $\left.\left[{ }^{10} / /_{s}>10\right\rangle\right]\left\langle\right.$ Men ${ }^{\text {a }}$ | ＊！ | 4 | ＊ | － | ＊ |
| $\mathrm{I}_{\text {PRV－K }}{ }^{+}$ | $\Omega(10,\langle$ Men $\rangle)=1$ | $\left[^{11} /(s) . .[1]^{+}\langle\mathrm{Men}\rangle\right.$ |  | ＊ |  | ＊！ | ＊ |

Now because Certainty must be satisfied for Truth，straight trivialization under TRV in candidate $I_{P R V-T R V}$ is chosen over $I_{P R V-K}{ }^{+}$．This is because，since $\mathrm{K}^{+}$will violate Natural quantification in this case，the extra violation to Actuality under $\mathrm{K}^{+}$is sub－optimal．

Therefore, the predicted interpretation is not a generalization, but an interpretation of Actual, Trivial, Certainty.

## 3.5 "The" Genericity

It is well-known that the definite article "the" has properties of certainty and generalization. Here, this is explained as privatization (PRV) without participant bounding (not $p$ ). The purpose of the in following examples is to indicate that the subject has been speaker-known and is therefore a private-class. However, the private model places no bounding restriction on the interpretation what so ever. Privatization only serves to decouple the interpreter's public knowledge from the interpretation. To show this, we consider the state of Certainty under a private subject. If the interpretation is certain, then we have (67).

$$
\begin{equation*}
\Omega={ }^{p} /\langle\mathbf{S}\rangle=1 \tag{67}
\end{equation*}
$$

Where, very importantly, the following equality exists.

$$
\begin{equation*}
p=\langle\mathbf{S}\rangle \tag{68}
\end{equation*}
$$

However, it cannot be said that the use of the private marker $\langle\mathrm{S}\rangle$ implies bounded subject. In fact, the following subjects are all unbounded from the perspective of this interpreter.
(69) a. The Chinese are heavy smokers.
b. The fish start jumping just after it rains.
c. The dogs run wild, around here.

Examples (69a) and (69b) may be about trivial-private classes (small groups), or else taken at an unbounded cardinality if belief permits. In this case, the interpretations are generic in that they allow counter-examples as universals. Therefore, the operation of
enhancement $\left(\mathrm{K}^{+}\right)$must account for the empirical generalizations under (69) which are tolerant to counter-evidence.

Because the requires the subject to be minimally non-null but imposes no conditions of probability, the initial constraint involves a possibility model. However, as can be seen under the examples in (69), there does not appear to be any ban against plurality by "the".
(70) Quantificational Meaning of the
i. Non-Null Interval of Truth: (0..1]

## ii. No Plurality Constraint

In the following tableau (T18), the subject is marked as plural. Therefore participation must be greater than one ( $p>1$ ). Importantly, the subject is overtly privatized in the artifact.
(71) T18 Inputs: $\Omega(p,\langle$ Men $\rangle) ; p \leq$ Men

Tableau (18) The men are tired.

| T(18) | $\Omega^{\delta}(p, S)$ | [ $\Omega^{\varepsilon} \mathrm{S}$ ] | Tr | Ntrl | Crta | Actl | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | $\Omega(p,\langle$ Men $\rangle)$ | (1/2..1) Men | *! |  | * |  |  |
| $\mathbb{I}_{K}{ }^{+}$ | $\Omega(p,\langle$ Men $\rangle)$ | (1/2, ${ }^{+}$[1] Men | *! | + |  | * |  |
| $I_{T R V}$ | $\Omega(p,\langle\mathrm{Men}\rangle)$ | [1] Men |  | *! | , | S |  |
| $I_{\text {TRV }}$ | $\Omega(p,\langle\mathrm{Men}\rangle)$ | $\left[{ }^{p} /{ }_{s}>p\right]$ Men |  | *! | 3 | 4 |  |
| $I_{P R V}$ | $\Omega(p,\langle\mathrm{Men}\rangle)$ | $\left[{ }^{p} /\langle s\rangle\right]\langle$ Men $\rangle$ |  |  | *! |  |  |
| $I_{\text {PRV-TRV }}$ | $\Omega(p,\langle$ Men $\rangle)$ | [1]〈Men〉 |  | *! | $\stackrel{1}{2}$ |  |  |
| $I_{\text {PRV-TRV }}$ | $\Omega(p,\langle$ Men $\rangle)$ | $\left.\left[{ }^{p} / /_{\text {g }} \times p\right\rangle\right]\langle$ Men $\rangle$ |  | *! | + |  | * |
| (G) $\mathrm{I}_{\text {PRV-K }}{ }^{+}$ | $\Omega(p,\langle$ Men $\rangle)$ |  |  |  |  | * | * |

Above in T18, Truth cannot be satisfied without either TRV or PRV. However, the trivialized candidates ( $I_{T R V}$ ) and ( $\left.I_{P R V-T R V}\right)$ all violate the constraint to be Natural. Then, Certainty is only satisfied under $I_{P R V-K}{ }^{+}$leaving $I_{P R V}$ as sub-optimal.

Tableau (18) Annotated Comparative Tableau

| T(18) | $\Omega^{\delta}(p, S)$ | $\left[\Omega^{\varepsilon} \mathrm{S}\right]$ | Tr | Ntrl | Crtn | Actl | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I $I_{P R V-K}{ }^{+}$ | $\Omega(p,\langle$ Men $\rangle)$ | $1 /\left\langle s, \ldots[1]^{+}\langle\mathrm{Men}\rangle\right.$ |  |  |  | 4 | * |
| $I_{0}$ | $\Omega(p,\langle$ Men $\rangle)$ | ( $1 / 2 . .1$ ) Men | *! W |  | *W | L | $\underline{L}$ |
| $\underline{I}^{+}$ | $\Omega(p$, (Men $\rangle)$ | (1/2.. ${ }^{+11]}$ Men | *!W |  |  | * | I |
| $I_{T R V}$ | $\Omega\left(p\right.$, Men ${ }^{\text {d }}$ ) | [1] Men |  | *!W | , | L | L |
| $I_{\text {TRV }}$ | $\Omega(p,\langle$ Men $\rangle$ ) | $\left[{ }^{p} /{ }_{s}>p\right]$ Men |  | *!W | *W | I | L |
| $I_{P R V}$ | $\Omega(p,\langle$ Men $\rangle)$ | $\left[{ }^{p} /\langle s\rangle\right]\langle$ Men $\rangle$ |  |  | *!W | L | * |
| IPRV-TRV | $\Omega(p,\langle$ Men $\rangle)$ | [1] <Men) |  | *!W |  | 1. | * |
| $I_{P R V-T R V}$ | $\Omega(p,\langle$ Men $\rangle)$ | $\left.\left[{ }^{p} / /_{s}>p\right)\right]\left\langle\right.$ Men ${ }^{\text {a }}$ |  | *!W | *W | L | * |

Here, we have a ranking argument (RARG5) for Natural $\gg$ Actual. The important constraint conflict is demonstrated in candidate $\left(I_{T R V}\right)$, which would have beaten the optimum $\left(I_{P R V-K}{ }^{+}\right)$had the ranking between Natural and Actual been reversed.

It can be seen here as well that the violation of Actual is favored over the violation of Certainty $\left(I_{P R V}\right)$. This accounts for the Certain readings associated with the private marker "the" at the cost of actuality.

An important prediction that is made by the current theory is that the subject is unbounded in the optimal candidate. This corresponds to the fact that the perceived number of men is unconstrained in the favored interpretation of the artifact. Therefore, the system predicts the favored reading to be a generalization over some unbounded set of men. Unlike the T16 and T17, $\mathrm{K}^{+}$does not cause trivialization here. This is because participation ( $p$ ) is not specified at input, and therefore certainty under $\mathrm{K}^{+}$does not force $P=s$ as above. It is for this reason that the predicted interpretation is therefore a privatized generalization.

### 3.6 Proportional vs. Cardinal Readings

In (72a), the reading is uncertain, or Milsarkian-proportional, where twenty participants is less than the subject cardinality $(20<s)$. The most salient reading of $(72 \mathrm{a})$ is close to the paraphrase in (72b).
a. Twenty people work.
b. Only twenty of the people work.

However, the reading of (73) below is certain, or Milsarkian-cardinal, meaning that there is total participation. The subject cardinality is twenty participants ( $20=s$ ).
(73) Twenty people work here now.

Cardinal readings arise where the trivialization of the subject by spatio-temporal constraint forces $\Omega$ Certainty. Here, the explicit use of embedded constraints (i.e. numerals and some) opens up the possibility for privatization of the subject. It is shown in the following Tableaux that constraint interaction accounts for forced-certainty under the specification of space-time (here-now).

Importantly, it is noticed that the $p$-value is bounded by the artifact under a numeral embedded constraint $(\beta \geq 20)$. Since $p$ is bounded, $\Omega$ will be certain only if the subject cardinality is trivialized to equal participation $(s=p)$. In a previous section, it was argued that under the specification of (p), the system seeks uncertainty (c.f. T16). This was explained by the raking of Natural over Certainty, where the constraint to be Natural will not be violated under trivialization for the purposes of Certainty. However, as explained above, the simultaneous specification of space and time independently serves to bound the subject set of observations. As such, the constraint to be Natural is violated by the initial candidate according to the constraining details of the artifact.

Tableau（19）Twenty people work here，now．

| T（19） | $\Omega^{\delta}(p, S)$ | $\left[\Omega^{\varepsilon} S\right]$ | Tr | Ntrl | Crtm | Actil | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | $\Omega$（20，People） | （1／2．．1）People | ＊！ | ＊ | ＊ |  |  |
| $\mathbb{I N}^{+}$ | $\Omega$（20，People） | （1／2．．${ }^{+}$［1］People | ＊！ | ＊ |  |  |  |
| （ $I_{T R V}$ | $\Omega$（20，People） | ［1］Peeple |  | ＊ |  |  |  |
| $I_{T R V}$ | $\Omega$（20，People） | ［ $\left.{ }^{20} / \mathrm{s} \neq 20\right]$ People |  | ＊ | ＊！ |  | ． |
| $I_{P R V}$ | $\Omega$（20，People） | $\left.{ }^{20} /\langle s\rangle\right]$＜People〉 |  | ＊ | ＊！ |  | 4 |
| IPRV－TRV | $\Omega$（20，People） | ［1］＜Reople〉 |  | ＊ |  |  | ＊！ |
| IPRV－TRV | $\Omega$（20，People） | $\left[{ }^{20} / /_{s \neq 20}\right)$［ Peopple〉 |  | ＊ | ＊！ | ， |  |
| $\bar{I}_{\text {PRV－K }}{ }^{+}$ | $\Omega$（20，People） | $\left[{ }^{3 / 3} /\langle \rangle .[1]^{+}\langle\right.$People $\rangle$ |  | ＊ |  | ＊！ | \％ |

The interpretation can only ever be either trivial or non－trivial．Therefore，the constraint of natural quantification may only be violated once．The violation of Natural is strictly a yes，or no affair．Therefore，all allowable candidate interpretations will be trivial given the input in T19．

Because the artifact independently violates Natural at input，then the trivialization of the subject does not present a further cost to the system．Violations that are imposed by the artifact itself are borne by all possible candidates because the artifact itself is a constraint upon the description．Consequently，the system may seek the satisfaction of Certainty at no additional cost．In effect，space－time bounding turns－off the constraint to be Natural with respect to the evaluation of optimality．Therefore，when the artifact constrains both space and time，Certainty becomes a possibility despite participant bounding（p）unlike（T16）above．However，like the preceding examples，Truth only ceases to be a problem for either a trivial or private candidate．

Firstly，as mentioned，（ $I_{K^{+}}$）will not be true because there are more than 20 individuals on the public natural－class of people．However，Certainty under Trivialization
comes with no additional cost to the system because of the constraining details of the input.

The difference between Private and Trivial interpretation is visible when the subject is not initially bounded by space-time (T20). Now, the constraint to be Natural is in full effect, and bans all trivialization for Certainty. Furthermore, since participation is initially bounded (p), enhancement for Certainty under privatization also leads to trivialization of the subject, and therefore is sub-optimal. In conclusion, all efforts to satisfy Certainty either through TRV or Enhancement are sub-optimal, and so the interpretation is uncertain on a natural private subject.

Tableau (20) Twenty people work (hard).

| T(20) | $\Omega^{\delta}(p, S)$ | $\left[\Omega^{\varepsilon} \mathrm{S}\right]$ | Tr | Ntrl | Crtn | Actl | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{0}$ | $\Omega$ (20, People) | (1/2..1) People | *! |  | \% |  |  |
| $I_{M}^{+}$ | $\Omega(20$, People $)$ | (1/2..1) People | *! |  | * |  |  |
| $I_{K}{ }^{+}$ | $\Omega$ (20, People) | (1/2.. $\left.{ }^{+} 11\right]$ People | *! |  |  | * |  |
| $I_{\text {TRV }}$ | $\Omega(20$, People) | [1] People |  | *! |  |  |  |
| $I_{\text {TRV }}$ | $\Omega$ (20, People) | $\left[{ }^{20} / \mathrm{s} \neq 20\right]$ Peeple |  | *! | * | \% |  |
| ( $I_{P R V}$ | $\Omega$ (20, People) | $\left.{ }^{20} /\langle s\rangle\right]\langle$ People〉 |  |  | * |  | * |
| $I_{\text {PRV-TRV }}$ | $\Omega$ (20, People) | [1] <People〉 |  | *! |  |  | * |
| $\mathrm{I}_{\text {PRV-TRV }}$ | $\Omega(20$, People) | $\left.\left.{ }^{20} / /_{s} \neq 20\right\rangle\right]$ (People) |  | *! | * |  | * |
| $I_{P R V-K}{ }^{+}$ | $\Omega(20$, People $)$ | ${ }^{20} /\left\langle\right.$ s) . $[1]^{+}$(People ${ }^{\text {a }}$ |  | *! | + | * | * |

What has been accomplished, is the calculation of the difference between the quantified interpretations of the following two types of examples.
a. Twenty people work.

$$
\begin{equation*}
(p<s) \tag{74}
\end{equation*}
$$

b. Twenty people work here now. $\quad(p=s)$

It has been shown that the specification of both space and time serves to elicit an interpretation of Certainty, while the unspecified artifact (74a) leaves the interpretation
naturally uncertain. It should be recognized that the mere specification of time does not sufficiently constraint the system to produce the effect.
(75) Twenty people work now. ( $p<s$ )

However, the specification of place includes a temporal specification under the context of employment, where the statement is taken to mean, currently.
(76) Twenty people work here. $(p=s)$

Lastly, the final embedded constraint some is shown to bound participation, as does the numeral ( $p$ ). Above, the meaning of the determiner some was defined as the unspecified numeral. The determiner places an unknown positive lower-bound on participation. If $p$ is not bounded, then the required violation to the constraint to be Natural does not necessarily occur under Certainty.
a. Some people work.

$$
\begin{equation*}
(p<s) \tag{77}
\end{equation*}
$$

b. Some people work here now. $\quad(p=s)$

Tableau (21) Some people work here, now.

| T(21) | $\Omega^{\delta}(p, S)$ | [ $\Omega^{\varepsilon} \mathrm{S}$ ] | Tr | Ntrl | Crtn | Actl | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{o}$ | $\Omega$ ( $p$, People) | ( $1 / 2 . .1$ ) People | *! | * | * |  |  |
| $\mathrm{I}_{K}{ }^{+}$ | $\Omega$ ( $\beta$, People) | (1/2.. ${ }^{+}$[1] People | *! | * | \% | S |  |
| G $I_{T R V}$ | $\Omega$ ( $p$, People) | [1] People |  | * |  |  |  |
| $I_{\text {TRV }}$ | $\Omega$ ( $p$, People) | [ ${ }_{\text {/ }} \times \mathrm{p}$ ] $]$ People |  | * | *! |  |  |
| IPRV | $\Omega$ ( $\#$, People) | $\left[{ }^{p / / s s}\right]$ [ People〉 |  | * | *! |  | * |
| $I_{P R V-T R V}$ | $\Omega$ (p, People) | [1] <People〉 |  | * |  |  | *! |
| $I_{\text {PRV-TRV }}$ | $\Omega$ ( $\#$, People) |  |  | * | *! |  | * |
| $\bar{I}_{P R V-K}{ }^{+}$ | $\Omega(p$, People) | [ $/(s) .[1]^{+}$(People) |  | * |  | *! | * |

What is interesting about the determiner some is that although the value of participation is explicitly bounded ( $p$ ), the specific bound is not given.

Tableau（22）Some people work hard，some don＇t．

| T（22） | $\Omega^{\delta}(p, S)$ | ［ $\Omega^{\varepsilon} \mathrm{S}$ ］ | Tr | Ntrl | Crtin | Actl | Pub |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{I}_{0}$ | $\Omega$（ $p$, People） | （1／2．．1）People |  |  | ＊ |  |  |
| $I_{M}{ }^{+}$ | $\Omega$（ $\beta$ ，People） | （ $1 / 2 . .1$ ）People | ＊！ |  | ＊ | ［ | \％ |
| $\mathrm{I}_{K}{ }^{+}$ | $\Omega$（ $p$ ，People） | （1／2．${ }^{+}$［1］People | ＊！ |  |  | 4 |  |
| $I_{T R V}$ | $\Omega$（ $p$ ，People） | ［1］People |  | ＊！ | S |  |  |
| $I_{T R V}$ | $\Omega$（ $p$ ，People） | $\left[/_{s} \neq p\right]$ People |  | ＊！ | ＊ |  |  |
| $I_{\text {PRV }}$ | $\Omega$（ $p$ ，People） | $[P /\langle s\rangle]$＜People〉 |  |  | ＊ |  | ＊！ |
| $I_{P R V-T R V}$ | $\Omega$（ $p$ ，People） | ［1］〈People〉 |  | ＊！ |  | ， | ＊ |
| $\boldsymbol{I}_{\text {PRV－TRV }}$ | $\Omega$（ $\#$ ，People） | $\left[{ }^{P} /{ }_{s \rightarrow p}\right]$（Peeple） |  | ＊！ | ＊ |  | ＋ |
| $I_{P R V-K}{ }^{+}$ | $\Omega(\beta$, People） | $\left[^{+} /\left\langle s\right.\right.$ ）．［1］${ }^{+}$＜People $\rangle$ |  | ＊！ |  | ＊ |  |

Therefore，the statement is compatible with an interpretation on the natural public class of people，and suddenly a statement of uncertainty is available as the initial candidate．In previous examples of participation bounding，this was not possible because explicit numeral values（such as：$p \geq 10, p \geq 20$ etc．）were false on the natural public class as in Tableau（T19）and（T20）．Here，the interpreter has no information regarding a specific bound on participation．Therefore，$p$ can be taken as lower bounded at any value $p<s$ ， and so $\Omega(p$, People $) \in(1 / 2 . .1)$ is possible at $I_{0}$ ．

### 4.0 The Actual Form

A quantificational interpretation will be Actual if the quantification is entirely supported by observations. At a minimum, to be entirely supported by observations means that for each supposed participant, there is at least one observation that supports the claim of individual participation.
(1) A $\psi$-individual is such that $\phi \rightarrow$ a $\psi$-observation is such that $\phi$

In English, there is a syntactic form which is used with the intent to express this meaning. This will be referred to as the Actual form.
(2) i. Actual Form: $\quad \mathrm{C}=\Omega s / p$
ii. Modal Form: $\quad \Omega s=p$

The modal form of expression requires equality between $(\Omega s)$ and $(p)$ however, the actual form merely requires that the quantification of $s$, and participation, are proportional.
(3) $\Omega s=\mathrm{C} p$

If the constant (C) is unknown, then there can be no ratiocination, as has been proposed for the modal form.

The implication under (1) is that the status of the quantification doesn't change when individuals (partitions) are replaced with observations. Therefore, a linguistic quantification is supported by observations when the same quantification as made over individuals, can be made over observations, mutatis mutandis. Another way to state this is that the quantification remains valid despite removing the partition-structure from the subject. The interpretation will be actual iff the quantification made over individual partitions implies that the same quantification can be made over observations.

$$
\begin{equation*}
\Omega^{\text {Partitions }}=\Omega^{\text {Observations }} \tag{4}
\end{equation*}
$$

Let this be called a bare, structural inference from quantification over individuals to quantification over observations, which need not involve any thinking whatsocver. If the status of the quantification remains unchanged despite removing the partition-structure from a class of observations, then the interpretation is completely supported by observations, and therefore it is actual.
(5) i. There is [a cat] [pp in the garden]
ii. $\mathrm{C}=[\Omega s] /[p]$

Any syntactic form in English (5i) that can be described as the epistemo-linguo correspondence in (5ii), will be referred to as Actual. Williams (1984a) provides an analysis of there sentences where the prepositional phrase (PP) in (5i) would be an adjoined coda. The quantificational description under (5ii) only becomes feasible under this kind of syntactic analysis of (5i).

### 4.1 The Strong-Weak Distinction (Milsark, 1974)

Milsark (1974) introduced a contrast between two groups of natural language determiners which is most noticeable in the context of there sentences here said to occupy an actual form. Briefly, the natural language components all, every, most, the, and proper names, are awkward in this form at very best. Milsark distinguished natural language determiners with this property as being "quantificational" or "strong" components, although this terminology is avoided in the explanation of the phenomena that follows.
(6) a. There is a man.
b. There are many men.
c. There are few men
d. There are no men.
e. There are five men.
f. *There is all/every men/man.
g. *There are most men.
h. *There is the man in the garden.
i. *There is Bill in the garden.

Relevant literature on this topic includes Milsark (1974), Barwise and Cooper (1981), De Jong (1983), Keenan (1987), De Jong (1987), Reuland and Ter Meulen (1987), and DeHoop (1995).

### 4.2 A Failed Inference from Partitions to Observations

The correlation between the ban on Milsarkian-strong elements in the actual form can be explained as a failed structural inference from individual-partitions to observations. As above, the structural inference from individuals to observations will fail, if the quantification over individuals is lost when the partition structure is removed from the subject of observations. Under this circumstance, the quantification over individual partitions is not completely supported by observations.

The explanation rests upon the basic requirement that statements made in the actual form must be directly supported by observational knowledge (i.e. actual). It will be shown that statements including the so-called "strong" elements categorically fail structural inference from partitions to observations, and are therefore not justified.

### 4.2.1 Inference Pattern from Individuals to Observations

Below under (7) we find five inferences from individuals to observations which do not hold.
(7) a. \#every $\psi$-individual is such that $\phi \rightarrow$ every $\psi$-observation is such that $\phi$
b. \#most $\psi$-individuals are such that $\phi \rightarrow$ most $\psi$-observations are such that $\phi$.
c. \#the $\psi$-individual is such that $\phi \rightarrow$ the $\psi$-observation is such that $\phi$.
d. \#the $\psi$-individuals are such that $\phi \rightarrow$ the $\psi$-observations are such that $\phi$.
e. \# the $\psi$-individual called Bill is such that $\phi \rightarrow$ the Bill-observations are such that $\phi$.

Primarily, these inferences are not possible because of the weak requirement of participation. An individual-partition will be active under the current definition, given one observation of evidence. Therefore, a partition may be active although the majority of its observations do not reveal the activation property.
(8) a. "Every individual participates" does not imply
"Every observation is of participation"
b. "Most individuals participate" does not imply
"Most observations are of participation"
Specifically, (7b) does not hold because the participants may not be very active (e.g. A Butterfly escaping the Chrysalis). Otherwise, the and proper names (7c, d, e) do not allow the inference because there may be many observations which do not reveal $\phi$.

On the other hand, we have the following well-formed inferences under the assumption that to actually say that an individual participates requires there to have been some observation of that individual engaged in the relevant property.
(9) a. A $\psi$-individual is such that $\phi \rightarrow a \psi$-observation is such that $\phi$
b. Some $\psi$-individual is such that $\phi \rightarrow$ some $\psi$-observation is such that $\phi$
c. Some $\psi$-individuals are such that $\phi \rightarrow$ some $\psi$-observations are such that $\phi$
d. $\psi$-individuals are such that $\phi \rightarrow \psi$-observations are such that $\phi$
e. Many $\psi$-individuals are such that $\phi \rightarrow$ many $\psi$-observations are such that $\phi$
f. Several $\psi$-individuals are such that $\phi \rightarrow$ several $\psi$-observations are such that $\phi$
g. No $\psi$-individuals are such that $\phi \rightarrow$ no $\psi$-observations are such that $\phi$
h. $\mathcal{F} \psi$-individuals are such that $\phi \rightarrow$ (at least) $\mathcal{F} \psi$-observations are such that $\phi$ Intuitively few allows the inference as follows.
(10) Few $\psi$-individuals are such that $\phi \rightarrow$ few $\psi$-observations are such that $\phi$ Here, we see that the distribution of allowable inferences from individuals to observations matches the strong-weak contrast among the determiners.
(11) Strong-Weak Contrast
a. There is *every man in the garden.
b. There are *most men in the garden.
c. There is *the man in the garden.
d. There is *Bill in the garden.
e. There are some men in the garden.
f. There are few men in the garden.
g. There are many men in that garden.
$h$. There is a man in the garden.
i. There are men in the garden.
j. There are no men in the garden.

### 4.3 Actual Form Allows No Enhancement ( $\mathbf{K}^{+}$)

A strong prediction which arises from definition of the actual form, is that the enhancement operation is not allowed. Specifically, since the form requires complete
observational support ceteris paribus, then interpretations which require the removal of contrary observations should not be allowed. This, in turn, requires that the interpretation not be enhanced. Under current assumptions, empirical generalizations are never completely supported by observation, and therefore such interpretations are not expected in the actual form.

In order to materialize this prediction, two domains of evidence will be considered. First, it will be shown that the actual form is reserved for the unexpected/unknown. Second, the ban on I-level predicates in the actual form due to Carlson (1977), will be shown to follow from the lack of enhancement.

### 4.3.1 Actual Form of the Unexpected/Unknown

One prediction of the hypothesis that the actual form means no enhancement, is that expectations are not possible in the actual form. Indeed, there is much evidence to support this prediction. Firstly, it is found that the unexpected is given most naturally in the actual form.
(12) a. i. \#An accident is here.
ii. There is an accident here.
b. i. \#A surprise is here.
ii. There is a surprise here.
c. i. \#A mistake is here.
ii. There is a mistake here.
d. i. \#An error is in your thinking!
ii. There is an error in your thinking!
e. i. \#A strange man is here!
ii. There is a strange man here!

Therefore, surprise is awkward in the modal form. It should be pointed out that surprise and expectation are inverses under negation. The actual form is an expression of surprise, not expectation.
(13) i. Surprise $=$ Not Expected
ii. Not Surprise = Expected

An additional way that Milsarkian-strong determiners may appear in the actual form, is when there is an interpretation of local visual surprise. The cases under (14) are examples of local observations that defy some expectation.
(14) i. A: There is still every plate on the table! (Local visual surprise.)

B: How do you know?
A: Look!
ii. A: There is still every finger on my hand! (After unexpected explosion) B: How do you know?

A: Look!
Therefore in these cases, we see a correspondence between the actual form, and a lack of expectation. Then there are specific cases of local announcement, which can be regarded as little surprises. The strong determiners become more acceptable when observational support is implied.
(15) Local Announcement
a. Look! There is every man standing at attention.
c. Look! There is every omament on the tree.
d. Look! There is every bullet in the gun.

As a final demonstration of the unavailability of enhancement in the actual form, we will consider the contrast due to Carlson (1977), between individual and stage-level predicates.

### 4.3.2 Individual-Stage Level Predicates (Carlson, 1977)

In Carslon (1977), it was shown that properties which do not change over time are not allowed in there sentences.
(16) a. There are firemen available.
b. \#There are firemen *altruistic.
c. \#There are firemen *tall.

As is well known, there are many examples of this contrast that support the Carlsonian distinction.
(17) a. \#There are firemen *brave.
b. \#There are firemen *cowards.
c. \#There are firemen *honest.
d. \#There are firemen *pure.

Therefore it would seem that properties which are able to change over time are allowed in the actual form, while those properties which are invariant under observation are not allowed. Here, we will follow Chierchia's (1995) basic view that I-level predicates are inherently generic. Another way to look at this is that predicates which represent certain qualities are restricted, while predicates which express local surprises about individuals are allowed. Once attributed, qualities that are certain of individuals like, bravery, honesty, and purity etc., are believed to hold of an individual regardless of when you
observe them. Carlson's I-level predicates are believed to hold of an individual inherently, and therefore should be invariant under observation.

From the current perspective it is maintained that nothing empirical is absolutely certain. Therefore, although I-level predicates are thought to hold over a lifetime, there are undoubtedly times when individuals who are truthfully said to be brave, are not so. In short, there are undoubtedly counter-instances. It is for this reason that I-level predication requires exception-handling in the form of enhancement in order to be grammatical. For example, it is of some concern that the contrast goes away when the I-level predicate is placed in a relative clause, which is always found in the modal form. On the current view, such relative clauses would allow enhancement.
(18) a. There are firemen [who are brave].
b. There are firemen [who are cowards].
c. There are firemen [who are honest].
d. There are firemen [who are pure].

Given the modal form, I-level predicates are generally allowed, and the distinction disappears. However, we find Carlson's basic distinction between natural expectations and surprises, in the modal form.
(19) a. Firemen are brave.
b. Firemen are cowards.

Here under (19a), we have a certain (expected) quality of firemen which yields a generalization. In (19b), we have an artifact that is in conflict with common belief, and there is no reading of generalization available if the example is not taken to be false.
(20) There are firemen [who are cowards].

The interpretation of $(20)$ is that the number firemen who participate in the certainquality of cowardice, is not zero. However, the readings of both (19a) and (19b) involve the expression of a property which is probabilistic over firemen, and therefore requires enhancement. This is why the I-level predicate is not allowed in the actual form.
(21) \#There are firemen * cowards.

In short, I-level predicates require enhancement, because they are taken to be certain, inherent properties of individuals. "Certain" qualities should be invariant under observation. However, because there is no actual empirical support for a universal quantification of one's life-times, the well-formed interpretation of such items requires enhancement of inherently imperfect observational knowledge.

A final point is the following. Despite the general commitment to the uncertainty of experience, there appear to be some empirical generalizations that can be made without enhancement. The interpretation of (22a) seems to allow an empirical reading of blue all the time, which is a rare finding in the actual form.
(22) a. There are oceans blue.
b. \#There are skies blue.
c. There are forests green.
d. \#There are fields green.

### 4.4 Empirical Generalization II

In this section, the interpretation type of empirical generalization is separated from other generalization types. Interpretations of empirical generalization are unique because they involve the resolution of a paradox. Empirical generalizations beg certainty, although the interpretation allows special room for counter-examples. Here, the empirical
generalization is distinguished from two other kinds of general interpretation, i.e. the Formal Generalization ${ }^{26}$ and the Reasoned Generalization. The distinction is based on exception-tolerance, justification, and expectation.

### 4.4.1 Expectation and Tolerance of Counter-Examples

Here, the Goodman (1947) and Lewis $(1973,1975)$, counter-factual test for causation will be taken as an indicator for expectation. The example (23c) is relevant because it is taken as a generalization although the vast majority of seeds do not in fact germinate according to Carlson (1977). However, there is no expectation associated with this generalization as can be seen according to the failure of ( 23 cii ).
(23) Goodman-Lewis Counter-Factual Test for Causation.
a. Squares have four sides.
ii. If you were a square, you would have four sides.
b. All dogs bark.
ii. If you were a dog, you would bark.
c. Seeds germinate.
ii. \#If you were a seed, you would germinate.

Given Carlson's information regarding the germination-rate of seeds, these examples show a general correspondence between the epistemic interval $\left(\Omega^{\varepsilon}\right)$ on the one hand, and expectation on the other. Where $\Omega^{\varepsilon}=\left(0 . .^{1 / 2}\right)$ in these examples, no expectation is indicated by the Goodman-Lewis test. According to this test, the Goodman-Lewis intuition of causality relates also to $\Omega^{\varepsilon}$.

[^21]Furthermore, Kratzer (1982) argues that the Goodman-Lewis test separates nonaccident from accident. According to Kratzer, conditions represented by statements like the following, are non-accidental (example from Kratzer).
(24) Every Chinese restaurant gives fortune cookies with the check.

As Kratzer notes, the non-accidental aspect of such an example corresponds to the ability to support counter-factual reasoning.
(25) i. This is an Italian restaurant.
ii. If this were a Chinese restaurant, we would be given fortune cookies with the check.

A contrast arises regarding accidental readings such as under (26) (example Kratzer's).
(26) i. Every person in that room is an artist.
ii. \#If you were in that room, you would be an artist too.

The failure to support counter-factual inference corresponds to an intuitive lack of cause for the local regularity expressed under (26). Here the proposal is that the nonaccidental character of (24) reflects the exclusion of accidental no-shows (i.e. nonparticipating restaurants) from the calculation of quantificational meaning. This idea was introduced above as the optimization of probabilistic knowledge under enhancement. Enhancement may be understood, in modality, as approximation to a non-accidental possible world. This kind of non-accidental possible world would be an entirely causai world, whereby all effects are ultimately reasoned. It may be in this context that the Goodman-Lewis test weighs in. However, in order for enhancement to apply, two basic conditions must be respected. First, the counter-examples must be in the minority. Second, the class of individuals standing as the subject must be unbounded.

### 4.4.2 The Empirical Reason

Although there may be many reasons for a generalization, the empirical trend is the minimal reason.
(27) The minimal reason for generalization is that a Natural regularity has been observed.

In English, there is a syntactic form available for the expression of the minimal reason.
(28) Empirical (Minimal) Reasons
a. Dogs bark, because what dogs do is bark.
b. Dogs bark, because bark is what dogs do.

A fortunate look into the workings of a minimal reason is given by " $j u s t$ ". Here it is shown that the minimal reason (that's what dogs do) is not trivial despite appearances. This is shown by comparing the reason to "just-what dogs do", which is trivial as a justification of the generalization. Furthermore, it is shown that the minimal reason is, in fact, a reason by comparing it to the alternative, which is just a cause.
(29) a. Dogs bark because that's just-what dogs do.
b. Dogs bark just-because that's what dogs do.

It will be shown that the minimal reason, "that's what dogs do" is crucially distinct from just-a-cause namely, "that's just-what dogs do". Primarily, the difference resides in the fact that the minimal reason implies knowledge of the nature of dogs (what dogs do) and suggests the possibility of further reasoned explanation. In contrast, the just-cause indicates that there is no further explanation available.
(30) A: Why do dogs bark?

B: Because that's what dogs do.

A: What do you mean?
B: Well, it's their way of communicating.
(31) A: Why do dogs bark?

B: Because that's just-what dogs do.
A: What do you mean?
B: I mean that's just-what they do. That's it.
Therefore, the empirical reason will be said to be minimal because it is not necessarily reasoned, but provides the basis for further, reasoned explanation. It is an elementary reason.

The meaning of just in these cases is equal to "no-more-than" as is clarified by the following examples.
(32) a. just a little $=$ no more than a little
b. just a minute $=$ no more than a minute
c. *just a lot $=$ *no more than a lot
d. just a lot of silliness $=$ no more than a lot of silliness

While both the minimal reason and just-cause count as being a cause for generalization, only the minimal reason counts as being a reason. While the minimal reason is a cause and a reason, the just-cause is no more than (just) a cause. This is demonstrated by the following that imply a reason, but block just-cause. According to this contrast, we may conclude that just-cause does not count as a reason.
(33) a. That's what dogs do, therefore dogs bark.
b. \#That's just-what dogs do, therefore dogs bark.

Additionally,
(34) a. That's what dogs do, so dogs bark.
b. \#That's just-what dogs do, so dogs bark.

Furthermore, it is clear that the conditional statement is unnatural under just-cause.
a. i. Well, if that's what dogs do, then dogs bark.
ii. \#Well, if that's just-what dogs do, then dogs bark.
b. i. Well, if that's what dogs do, then I can see why dogs bark.
ii. \#Well, if that's just-what dogs do, then I can see why dogs bark.

However, when no reason is implied or required, just-cause is admissible.
a. That's what dogs do, dogs bark.
b. That's just-what dogs do, dogs bark.

These differences show that while what we have been calling a minimal reason is in fact a reason for generalization, the alternative under consideration is just a cause. Within the context of the foregoing discussion, the difference between a reason and simple cause will be taken to reflect the speaker's degree of knowledge and understanding.

It was shown earlier, that just-cause is trivial as an explanation while the minimal reason is not trivial. The evidence shows that an expression like, bark is what dogs do, will be taken both as a cause and as a reason unless it is explicitly indicated that it is no more than a cause. One conclusion to be made from this is that bark is what dogs do is generally taken to be a reason for generalization about dogs and barking, although this reason is clearly not a rationalization. For example, the expression (37a) is not on a par with a reasoned justification like the following (37b).
a. Dogs bark because that's what dogs do.
b. Dogs bark because it helps their circulation.

### 4.4.3 Empirical vs. Reasoned Generalization

The empirical generalization is clearly distinguished from the non-empirical generalization which does not allow an empirical justification. Such generalizations that only allow non-empirical reasons will be called Reasoned generalizations.
(38) Empirical Generalizations Again:
a. Dogs bark because that's what dogs do.
b. Cars rust because that's what happens to cars.
c. A birthday cake has candles because that's how they are.
d. Chinese Restaurants give fortune cookies because that's what they do. The empirical generalizations above in (38) allow a peculiar type of justification, which was called an empirical reason. However, it is also possible that an empirical generalization have an accompanying reasoned explanation. Such generalizations are both empirically and rationally supported which means that they are expressions of frequent and colloquially explained participation, as in (39).
(39) Empirical and Reasoned
a. Babies cry because they desire attention.
b. Cars rust because steel oxidizes naturally.

However, there are those empinical generalizations that may have no clear reasoned explanation.
(40) Empirical Only (Perhaps no rationale)
a. Cats flick their tails when resting.
b. Birthday cakes have candles.
c. Car buyers kick the tires.
d. Viruses kill their hosts. (c.f. How vs. Why)

On the other hand, the following generalizations are only reasoned; they are not empirically motivated.
(41) Non-Empirical Reasoned Generalizations
a. Skunks make good house pets, because they are not aggressive.
b. Belgians are good weight-lifters, because of their diet.
c. Mammals bear live-young, because that's how it works with mammals. These generalizations cannot be empirically motivated because there is no frequency of observed participation that supports them. As expected, we see that they are not generalizations of certainty, and do not generate an expectation.
(42) a. \#If you were a skunk, you would make a good house-pet.
b. \#If you were Belgian, you would be a good weight-lifter.
c. \#If you were a mammal, you would bear live-young.

Furthermore, the Reasoned generalization does not allow the empirical minimal justification as do the empirical generalizations. None of these generalizations are supported by likelihood based on observation.
(43) ai. \#Skunks make good house pets, because that's what skunks do.
ii. \#Skunks make good house pets, because that's how skunks are.
bi. \#Belgians are good weight-lifters, because that's what Belgians do.
ii. \#Belgians are good weight-lifters, because that's how Belgians are.
ci. \#Mammals bear live-young, because that's what mammals do.
ii. \#Mammals bear live-young, because that's how mammals are. ${ }^{27}$

[^22]There is also a major class of reasoned generalizations supported by reasons of purpose, or reasons for being.
(44) a. Rubber gloves prevent contamination.
b. Canals let the water flow-out.

Presumably, these are supported by reasons of purpose. Included among these kinds of generalizations are examples like the following.
(45) Seeds germinate because that's their purpose.

Carlson (1977) suggests that the overwhelming majority of seeds do not in fact germinate. Therefore, (45) may be taken as a generalization that holds despite many, perhaps a majority of, counter-examples. According to Carlson's information about seeds, it is understandable that the generalization under (45) cannot be expressed as a certainty (46a), nor as a probability (46aii). Instead, the interpretation is possibilistic (46aiii), and there is no associated expectation.
(46) a. \#If you were a seed, you would certainly germinate.
ii. \#If you were a seed, you would probably germinate.
iii. If you were a seed, you might possibly germinate.
b. \#If you were a Mammal, you would certainly bear live-young.
ii. \#If you were a Mammal, you would probably bear live-young..
iii. If you were a Mammal, you might possibly bear live-young..

The reasoned generalization is not supported on probabilistic knowledge based on observation. In the case of seeds, it is because seeds do not readily germinate, or else they germinate underground where they are not generally seen. Instead, the generalization
arises from a belief regarding the function or purpose of seeds. It is found that (45) allows the following justification under (47).
(47) Seeds germinate because that's what seeds do.

However, the term do has an alternate meaning of purpose, and it turns out that all generalizations of purpose allow this justification. The following under (48) may be said in a case where the canals have only been used once, during a great flood. Otherwise, the canals are always are empty. However, it does appear that at least one observation of the purpose must be made to demonstrate the possibility.
(48) a. Those canals let the water flow out. That's what they do.
b. Those canals let they water flow out. That's what they're for. (if not confirmed) In this case, the reason for generalization is not empirical because "that's what they do" means "that's what they're for". These cases may be separated with the observation that purpose reasons do not allow emphasis of abundance.
(49) a. Dogs bark because that's what dogs do.
ii. Brother! Dogs bark.
b. Seeds germinate because that's what seeds do.
ii. \#Brother! Seeds germinate.

Therefore, in cases where the reason is of purpose, then the reason, that's what they do may be used. However, this reason is not the empirical reason. Alternative forms of reasoned justification include, but are not confined to, the following.
(50) Some Reasoned Justifications
a. Functional Reason (because, that's how it works)
b. Causal Reason (because, that's what happens when...)
c. Reason of Meaning (because, that's what it means)
d. Folk-Reason (because, that's what they say)

### 4.4.4 The Unbounded Subject

Here the term subject refers to the set of individuals that are denoted by the subject noun of the sentence. An unbounded interpretation of the subject is found to be available for any natural language determiner. In the following, the class of dogs is unbounded.
(51) a. All dogs bark.
b. Every dog barks.
c. Most dogs bark.
d. A dog barks.
e. Dogs bark.
f. Few dogs climb trees.
g. No dog sleeps standing-up.
h. The dogs in China sleep standing-up.
i. Three dogs bark, (when placed together).
j. Some dogs get hit by cars every year.

As can be verified according to the irrelevance of bounding questions How many?, When?, and Where? to (almost) every artifact under (51), the best interpretation of the expressions are unbounded. ${ }^{28}$ Interestingly, even examples (51f) and (51j) do not refer to any particular number. Precisely, the question of number is entirely irrelevant to

[^23]quantification in this example. In connection with the foregoing arguments, an unbounded interpretation is associated with an expectation.
(52) a. All dogs bark. If you were a dog, you would bark too.
b. Every dog barks. If you were a dog, you would bark too.
d. A dog barks. If you were a dog, you would bark too.
e. Horses sleep standing-up. If you were a horse, you would sleep standing-up.
g. No dog sleeps standing-up. If you were a dog, you would not sleep standing.
h. The dogs in China sleep standing-up. If you were a dog in China...
i. Three dogs bark, (when together). If there were three dogs together ...

However, the probabilistic determiners most and few require a probabilistic caveat.
(53) a. Most dogs are friendly.
i. \#If you were a dog, you would be friendly too.
ii. If you were a dog you would probably be friendly too.
b. Few dogs climb trees.
i. \#If you were a dog you wouldn't climb trees either.
ii. If you were a dog you probably wouldn't climb trees either.

Therefore, expectations that arise under probabilistic determiners will be called probabilistic expectations. The determiner some may give rise to an expectation under modal weakening.
(54) Some dogs get hit by cars every year.
i. \#If you were a dog you would certainly get hit by a car.
ii. \#If you were a dog you would probably get hit by a car.
iii. If you were a dog you might possibly get hit by a car.

In (54), the interpretation still concerns the natural class of dogs. The quantified interpretation of (54) expresses a positive lower-bound on the occurrence. The mere imposition of a positive lower-bound requires that the quantified interpretation occupy a value on the possibility interval. Constraint to the possibility interval yields the weakest quantified interpretation.
(55) Possibility Interval: (0..1]

Many interpretations will be discussed that include some level of expectation from the following list under (56).
(56) Expectation and Possibility
i. Expected at Certainty: [1] or [0]
ii. Expected at Probability: $\left(0 . .^{1} / 2\right)$ or $\left(\frac{1}{2} . .1\right)$
iii. Possibility: (0..1]

These may be compared with other interpretations that appear to be general, but do not allow counter-factual inference, even under the probabilistic caveat.
(57) Mammals bear live-young.
i. \#If you were a mammal you would bear live-young.
ii. \#lf you were a mammal, you would probably bear live-young.
iii. If you were a mammal you might bear live young.

Therefore, we see that some generalizations commonly referred to as generics (See Cohen 1999 for review), neither support counter-factual inference at the level of certainty, nor at the level of probability. Therefore, while the interpretation is unbounded, there is no associated expectation. It will be maintained that such interpretations are not empirical.

It is of theoretical interest to associate the unbounded Natural subject with the set of possible members, such that Natural interpretations are modal. Presently, what can be said is that the unbounded status of the interpretation is assessed according to the relevance of bounding questions ( $\S 1.8$ ). Further investigation is required to satisfactorily demonstrate modal characteristics of a Natural interpretation.

### 4.5 Higginbotham (1983), and Davidsonian Events

In Barwise and Perry (1981), the fact that the following perceptual report (58) implies the truth of Mary's crying, is called Veridicality. The perceptual report under (58) also implies that someone cried, which they call the Exportation of an existential quantifier.
(58) John saw Mary cry.
(59) i. If John saw Mary cry, then Mary cried. (B\&P Veridicality)
ii. If John saw Mary cry, then there is someone who cried. (B\&P Exportation) In defense of a first-order extensional semantics, Higginbotham notices that the two inferences themselves, follow from the existence of an event of Mary's crying. He then connects with Donald Davidson's theory of action sentences (Davidson, 1967) in which there is always a hidden existential quantification over events to be found.
(60) $\quad \exists x: x$ is an event $\&$ cry $($ Mary, $x)]$ John see $x$.

While it is true that these properties follow from the existence of an appropriate event, they also follow from the existence of an appropriate observation. Therefore, if the following can be understood as implying an observation, then Barwise and Perry's properties follow as well.
(61) John saw Mary cry implies there was an observation such that Mary cry

Higginbotham recognizes that examples like the following, appear to flout Veridicality, and takes measures to explain this fact. For instance, just because John saw no one leave, does not mean that no one left.
(62) John saw no one leave. does not imply No one left.

However, assuming inference to observations, the sentence simply requires that there was no observation made of someone leaving. Therefore as desired, there is no claim regarding whether anybody actually left, as indicated by the failure of $B \& P$ veridicality.

### 4.6 Some Things About to Happen

A natural class of observations does not exist for things that have no observationpotential. Therefore, a natural class of observations is not anticipated for objects that are not expected, in principle, to be seen.
a. John saw a ghost.
b. John saw [an argument about to happen].
c. \#John saw [an earthquake about to happen].

In a sense that is easy on the intuition, it seems that at least arguments about to happen have a non-null observation potential. However, there is no clear sense in which ( 63 c ) is possible. Still more examples show that quantification over such things is not intuitively acceptable.
(64) a. An argument about to happen is usually stressful.
b. \#An earthquake about to happen is usually stressful.

However, when the earthquake is somehow known or planned, then the quantification is possible.
a. \#Every earthquake about to happen is stressful.
b. Every earthquake about to happen is simulated.

This effect is attributed to the assumption of known private-class of earthquakes (i.e. fake-ones). On this view it is predicted that the private-class marker the is acceptable.
(66) a. The earthquake about to happen is stressful.
b. The earthquakes about to happen are stressful.

However, without the assumption of a private-class, quantification on such things is not possible.
a. \#Earthquakes about to happen are stressful.
b. \#Every earthquake about to happen is stressful.
c. \#No earthquake about to happen is stressful.
d. \#Some earthquakes about to happen are stressful.
e. \#Most earthquakes about to happen are stressful.
f. \#Few earthquakes about to happen are stressful.
g. \#Many earthquakes about to happen are stressful.

In conclusion, things with no observation potential such as an earthquake about to happen, do not support the existence of a so-named natural-class of observations. Therefore, if there will be well-formed quantification, a private-class (such as a familiar class) must be assumed. Importantly, this analysis separates observations from events theoretically. Simply, there are events which have no observation potential. According to the data in this section, it appears that grammatical judgments are sensitive to observations not events.

### 5.0 Conclusion

This investigation of Natural Language quantification posits interpretation as the optimization of the description of language. An account of the optimization of quantificational description is given within the optimality framework of Prince and Smolensky (1993). A strict dominance hierarchy of constraints on the goodness of a quantified description is proposed. Firstly, a quantificational description is an expression of epistemo-linguo correspondence. Secondly, a description is not bound to any particular set of individuals, but depicts an empirical trend concerning all possible members of the subject. Next, a description is not probabilistic, but certain. Then, a description is actual for incorporating all relevant knowledge, despite conflicting exceptions. Explicit ranking arguments are provided for the strict ranking of these rules. An additional constraint is proposed to account for phenomena such as partitivity, and the article "the". Under this constraint, a description is public or, based on widespread knowledge/belief.

An important goal of the theory is to explain the possibility of generalization despite counter evidence in natural language. This is typically cast as a problem of genericity (Carlson, 1995). Under the current view, genericity arises when limited counter-examples are disregarded in order to achieve a true but non-actual description of the sentence. Optimization to a non-accidental possible world addresses a tendency in the literature to explain generic interpretation under the intuition of normalcy (Cohen, 1995).

An important part of the current approach is the recognition of the empirical generalization. The empirical generalization is distinguished from the formal and reasoned generalization types. It is argued here that only empirical generalizations are truly generic; a simplification which greatly clarifies the phenomenon.

A number of open questions remain. Firstly, a substantial connection is lacking between the demonstrable unbounded status of an interpretation, and the modal status of an interpretation. An interpretation may be shown to be "unbounded" according to the relevance of bounding questions. However, although the modal character of such unbounded interpretations seems to be theoretically essential, the material connection between these two aspects of quantification requires further explication. Secondly, certain "idiosyncratic" characteristics of English determiners have been set aside. Primarily this includes the differences between the determiners "all" vs. "every", and "bare-plural vs. indefinite article "a". Also, questions remain regarding the role of aspect in quantified interpretation. Furthermore, issues regarding the implicit/default model of bare-plural subjects require further scrutiny. Currently, default models and default epistemic intervals are assumed at possibility (0..1], in order to satisfy the fundamental requirement of meeting the description. However, at present this approach remains unconstrained, and therefore remains at best, theoretically inadequate.

An important issue in OT that is left unaddressed here is alternate rankings of the constraints and possible implications. One possible area of further investigation is raised by Edwin Williams (p.c.) regarding communicative intentions and alternative constraint rankings. Accordingly, the optimization of the description of speaker's interests, attitudes, intentions, as well as knowledge, may prove to be a rewarding direction of further research.

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## Curriculum Vita

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Education:
2004 Ph.D. Linguistics, Rutgers University, New Brunswick NJ.
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Principle Occupations:
2003-2004 Software Engineer
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1997-1999 University Lecturer
1994-1997 Graduate Fellow
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1991-1993 Language Instructor
Publications:
1999 "Optimal models", Proceedings from the conference of the North East Cognitive Science Society.
1998 "Simultaneous depictive and resultative modification", Department of Linguistics Working Papers, Rutgers University.


[^0]:    ${ }^{1}$ order irrelevant.
    ${ }^{2}$ As a model of $\Omega^{\varepsilon}, \Omega^{\delta}$ is expected to reflect the value and constitution of $\Omega^{\varepsilon}$. As a model however, the initial value of $\Omega^{\delta}$ is not often the same as the epistemic ratio $\Omega^{\varepsilon}$ according to any particular interpreter. The symbols $\Omega^{\delta}$ and $\Omega^{\epsilon}$ will be used interchangeably to represent a value, or else to represent the minimal interval which can be said to contain the value. For example, $\Omega^{\varepsilon}$ may be the minimal interval that is thought to contain the epistemic ratio.

[^1]:    ${ }^{3}$ Individual participation refers to the simple number of individuals $p$ among $S$ that participate in $\varphi$.

[^2]:    ${ }^{4}$ Compare with the unacceptability of, "All dogs are good ones."

[^3]:    ${ }^{5}$ Minimally, let a single observation of barking confirm the dog's participation in the behavior.

[^4]:    ${ }^{6}$ A taxonomic-category property like "animal" can never serve as the criterion for being a dog.

[^5]:    ${ }^{7}$ Counter examples involving death and the necessary properties of higher taxonomic categories miss the point. A micro-characteristic for dog-ness may be sub-visual.

[^6]:    ${ }^{8}$ Excluding the reading, "Dogs can do math."

[^7]:    ${ }^{9}$ Allowing the probabilistic opinion ( $\omega$ ) to represent the former epistemic ratio. The epistemic interval ( $\Omega^{\varepsilon}$ ) is the minimal interval thought to contain $\omega$.

[^8]:    ${ }^{10}$ For each of the two intervals of $(\%)$ exception, there is a corresponding interval which represents the participants. However, the system is based on the concept of exceptionality.
    ${ }^{11}$ Such examples as (i) below show that a lack of knowledge may allow a default to the epistemic interval of allowed possibility (0..1]. Otherwise, such a default may be optimistic of positive probability $(1 / 2.1)$, or else pessimistic $(0 . .1 / 2)$.

[^9]:    ${ }^{i 2}$ some, numeral, and the to be discussed in chapter 3 .

[^10]:    ${ }^{13}$ As as result many perhaps problematic intervals never arise as a salient quantification. For instance, intervals which contain chance ( $1 / 2$ ) cannot represent probabilistic knowledge, and will therefore never correspond to the intervals $(1 / 2.1),(0 . .1 / 2),[1],[0]$, nor sub-intervals thereof.

[^11]:    ${ }^{14}$ Then correspondence $\left(\Omega^{e}=\Omega^{\delta+}\right)$ is assured under $\mathrm{M}^{+}$, if required. Of course this implies the failure of the interpretation to be natural and to be actual.

[^12]:    ${ }^{16}$ According to Bruce Tesar (pc) the strict implementation of Faithfulness in this account, where faithfulness violations are incurred for any departure from the initial candidate ( $I_{0}$ ), is not relevant to all OT analyses. This is the case especially in Phonology where "faithful" divergence from the input form may occur in the form of syllabification etc.

[^13]:    ${ }^{17}$ The operations of $\mathrm{K}^{+}$and TRV are mutually exclusive because enhancement requires an unbounded subject.

[^14]:    ${ }^{18}$ This raises a question regarding the source of the input model $\Omega^{\delta}$. In cases where there is no overt determiner of any kind (29b), how is the model acquired? In this case the current general approach of interpretation under quantificational description provides a possible solution. Because the interpretation only obtains if a specific description of the sentence is achieved, it may be possible to assume a default model (possibility model) for the purposes of meeting the desired description of the sentence. Such a capability would need to be governed by principled constraint, although this is a topic which requires further investigation. Presently, the default model of possibility (0..1] is assumed in such cases (§ 3.2.2).

[^15]:    ${ }^{19}$ A constraint ranking $\left.\alpha\right\rangle \beta$ in Optimality Theory implies a categorical dominance relation between the constraints. This means that no amount of violation of the constraint Actual will outweigh the benefit of some higher constraint satisfaction (here Natural). This aspect of OT allows the possibility of gradiently violable constraints.

[^16]:    ${ }^{20}$ The results of $\mathrm{M}^{+}$must progress towards a more restricted state toward positive certainty from $I_{0}$. (0. ${ }^{+}(x .11$ ( ( 3.3.2).

[^17]:    ${ }^{21}$ (i) will be true if there are ten children playing. But this is not the optimal interpretation which is upwardly bounded at five for Certainty. In general, Certainty will favor minimal subject cardinality when a lower bound is placed on $p$. The optimal interpretation must be bounded for Truth because the Public set of children must be more than five.
    (i) Five children are playing.

[^18]:    ${ }^{23}$ One's observations, as representations of space-time regions, do not overlap. However, a location may be observed at various times.

[^19]:    ${ }^{24}$ Partial modality is the set of possibilities by unbounded dimensions.

[^20]:    ${ }^{25}$ (§4.4)

[^21]:    ${ }^{26}$ Formal generalization refers to definitional type statements such as, "All squares have four sides." Unlike Empirical generalizations, formal generalizations express necessary mundane observable properties over the possible members of a set of individuals, as indicated by the acceptability of sentences like (i).
    (i) If it didn't have four sides, it would not be a square.

[^22]:    ${ }^{27}$ No such truthful generalization can be made over all individuals that may be called a mammal. Mammals do not bear live-young, only female mammals do. (See Cohen 1999, for citations). Therefore, regarding the

[^23]:    ${ }^{28}$ Example ( 52 i) requires the group be of cardinality three but does not require, or refer to, any specific three dogs. However this example is odd, and was included simply to show that the outer limits of the interpretation type are pressed, when specific numbers or bounded quantities are brought into question.

