SUPERLATIVE MODIFIERS: IGNORANCE AND CONCESSION

BY

YI-HSUN CHEN

A dissertation submitted to the

School of Graduate Studies

Rutgers, The State University of New Jersey

In partial fulfillment of the requirements

For the degree of

Doctor of Philosophy

Graduate Program in Linguistics

Written under the direction of

Veneeta Dayal

and approved by

New Brunswick, New Jersey

October, 2018

ABSTRACT OF THE DISSERTATION

Superlative Modifiers: Ignorance and Concession

by Yi-Hsun Chen

Dissertation Director:

Veneeta Dayal

This dissertation focuses on two intriguing puzzles posed by superlative modifiers (SMs) like English *at least*/ *at most*: the ambiguity puzzle and the morpho-semantic puzzle. The ambiguity puzzle concerns the fact that cross-linguistically, SMs tend to demonstrate an ambiguity between an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. The morpho-semantic puzzle concerns the fact that cross-linguistically, SMs in general involve degree words and quantity adjectives in their morphology.

The central proposal of this dissertation is two-fold. First, it is proposed that the EPI-CON ambiguity results from one unified semantic entry combining with different pragmatic factors such as informativity and evaluativity. That is, the two meanings can be seen as pragmatic variants in natural language. Second, the proposed semantics of SMs can be further decomposed into three pieces: a quantity adjective

(Q-adjective), a superlative component and an existential operator E-OP. In particular, Q-adjectives play a crucial role of encoding a measure function mapping the set of focus alternatives to their corresponding positions ordered along a contextually-given scale. Moreover, a superlative construction is contained in the internal structure of SMs. The superlative construction, expressing a comparison relation between the prejacent and its alternatives along certain contextually-given dimension, not only instantiates the scalar component of a focus particle like SMs but also introduces the bounding property of SMs. Specifically, the non-strict comparison relation of SMs is derived from the focus presuppositions by the squiggle operator ~ (obtaining the prejacent) and the contribution of the superlative (obtaining the higher/ lower alternatives). Finally, E-OP makes an existential statement over a domain that is non-singleton: a set consisting of the prejacent and its higher/ lower alternatives.

This dissertation captures a wide range of linguistic facts: **(a)** why the EPI-CON ambiguity is so pervasive across natural languages and generally shown by one single lexical item (particularly by SMs); **(b)** why multiple lexical items in one single language, as in Chinese, may demonstrate the ambiguity; **(c)** why the two meanings share three common properties: focus-sensitivity, the compatibility with various scales and two scalar effects (the bottom-of-the-scale effect and the top-of-the-scale effect); **(d)** why the availability of the concessive meaning is restricted by the syntactic position of SMs in some languages, as in English and Chinese; **(e)** why and how SMs are parallel with disjunction and epistemic indefinites in natural language; **(f)** why exactly the same Chinese expressions *zui-duo* and *zui-shao*, morphologically consisting of a quantity adjective *duo* 'much'/ *shao* 'little' and the superlative morpheme *zui*, are used as both superlative modifiers and quantity superlatives; **(g)** why Q-adjectives seem to be the core morphological component of SMs.

Acknowledgements

Today, I finally competed this dissertation, but I do not consider this piece of work as complete; instead, this dissertation is simply considered as a starting point to better understand many intriguing puzzles posed in natural language. Various circumstances in life do not allow me to be a student any longer; however, as Steve Jobs puts it in his speech at Stanford: “Stay Hungry, Stay Foolish”; I shall know the fact of my ignorance and never stop fighting for the truth.

This dissertation would not have been completed at this point, without the help from many people. I would like to hereby acknowledge their invaluable contributions. I am extremely lucky to have had my committee members to whom I am heavily indebted: Veneeta Dayal, Simon Charlow, Jane Grimshaw, and Roger Schwarzschild. First and foremost, I would like to thank my advisor, Veneeta Dayal. One of the reasons why I chose to come to Rutgers for my graduate study was because of Veneeta. Six years ago, we first met at a conference on formal syntax and formal semantics in Taiwan. Today, when I look back at my life, I cannot imagine how my graduate study at Rutgers would have been without her.

Veneeta has been an extraordinary mentor in both linguistics and life. She has been immensely supportive and generous with her time. Veneeta has guided me through not only every step of this work but also every phase of my academic development. Her Semantics II and seminars have provided me a solid background to further delve into various topics touching upon genericity, intensionality, plurality, etc. Her exceptional insights in the semantics and pragmatics have been extremely helpful in bringing many of my ideas into shape (in particular, chapter 3 is such a product). Without her serious training, wise guidance, incredible patience and constant support,

this dissertation would not have been possible in the first place, and I would not have gone this far in my academic career today.

Simon Charlow was a committee member of my second qualifying paper and dissertation. Maybe because we are close in our age, I feel Simon is more like an elder brother and a friend than a faculty member. His seminars on alternative semantics and monads have helped me better understand the formal apparatus; his Semantics I was refreshing and has provided me a better control of many basics of formal semantics. Discussions with him have been very helpful to clarify my thoughts.

Jane Grimshaw has played an important role in my graduate career at Rutgers. Jane was not only the chair of my first qualifying paper but also a committee member of my second qualifying paper and dissertation. She has not only guided me how to do a linguistic study (on a top-down perspective), but also taught me how to structure an argument (the logic of argumentation). Two of the most important things that I have learned from her are: "whatever you are arguing for, you must be crystal-clear about it" and "every theory makes predictions", when I stumbled in my first qualifying paper. These two pieces of her advices have shadowed various aspects of my linguistic studies in my later graduate career. I am very grateful to Jane for her effort and help in shaping this dissertation, and more generally, the overall training in the last six years.

Roger Schwarzschild was also one of the reasons why I chose to come to Rutgers for my graduate study. Many aspects of this dissertation are inspired by Roger's suggestions and comments. His curiosity in various aspects of linguistic phenomena has amazed me time and time during his courses at Rutgers and our discussions. Roger's big-picture questions and insights on focus constructions and degree constructions have helped me better understand the intricate complexity between individual examples, empirical generalizations, and the formal apparatus.

I would like to thank all other faculty members who I have always had the great pleasure of taking courses and interacting with in my graduate life at Rutgers. Particularly, Maria Bittner, who taught me Semantics I, has laid a solid foundation for my study of semantics. I am very much impressed by her assignments where compositional processes are designed in a way like a Jigsaw puzzle or Sudoku. Kristen Syrett has broadened my perspective on the experimental method in the study of semantics/ pragmatics. I have very much enjoyed the class discussion on linguistic studies from the perspective of cognitive science. Mark Baker, who taught me Syntax III, has an impressive skill of making a detailed, analytical and insightful presentation of the ideas underlying the assigned readings on the blackboard; this has encouraged me to become not only a good researcher but also a good teacher. Paul de Lacy and Alan Prince, who taught me Phonology II, have greatly inspired my thoughts about how different modules of a natural language should work together and how phonology works like a computational system. Although it was hard and painful at that time, I sometimes miss the time when in the semester, Alan's assignments were uploaded late on Friday evening and due on Sunday (5 pm) every week. I also thank Akinbiyi Akinlabi, Viviane Déprez, Adam Jardine, Troy Messick, Ken Safir, Bruce Tesar; they all have contributed to the completion of this work in one way or another.

Several parts of this dissertation were presented on various occasions. I have benefited tremendously from many people; I am very grateful to Peter Alrenga, Rajesh Bhatt, Ryan Bochnak, Lucas Champollion, Isabelle Charnavel, Gennaro Chierchia, Michael Yoshitaka Elewine, Patrick Grosz, Ray Jackendoff, Hadas Kotek, Chris Kennedy, Manfred Krifka, Jess Law, Haoze Li, Jon Ander Mendia, Marcin Morzycki, Doris Penka, Gregory Scontras, Yael Sharvit, Yasu Sudo, Satoshi Tomioka, Yimei Xiang, Matthijs Westera for invaluable discussions and constructive comments.

I would also like to thank Chen-Sheng Luther Liu, who was my M.A. Thesis advisor at National Chiao Tung University. Without his training, guidance and support in the first place, I would not have been accepted by the linguistics program at Rutgers six years ago, not to mention the completion of this dissertation today.

I have enjoyed my time at Rutgers and greatly appreciate the company and support from my friends and colleagues: Deepak Alok, Ümit Atlamaz, Vandana Bajaj, Matt Barros, Diti Bhadra, Eileen Blum, Yu Cao, Woojin Chung, Nick Danis, Satarupa Das, Vera Gor, Meg Gotowski, Vartan Haghverdi, Sarah Hansen, Luca Iacoponi, Kunio Kinjo, Todor Koev, Nathan Koser, Jess Law, Ang Li, Haoze Li, Shiyue Lin, Mingming Liu, Hazel Mitchley, Morgan Moyer, Lydia Newkirk, Christopher Oakden, Atsushi Oho, Akane Ohtaka, Augustina Owusu, Jeremy Perkins, Yağmur Sağ, Naga Selvanathan, Shu-hao Shih, Ikawa Shiori, Livia Camargo Tavares Souza, Nick Winter, Beiebei Xu, Tadjou-N'Dine Mamadou Yacoubou; thank you all for the joy and fun!

I would like to express my deepest gratitude to my parents. I never forget the day when I sent out the applications for my graduate study, they even considered selling their house just in order to provide me the financial support for temporally two years. I told them I will go only if I receive the fellowship, and I did. Ba and Ma, thank you for your selfless sacrifice, unconditional love and constant support over the years!

I want to thank my wife Hui-Ting Naomi Yang for her love, understanding and support in last six years of my graduate study. Writing a dissertation is hard, but living a decent life for a family is even harder; Naomi, thank you for taking care of everything in my life and always being my side during those ups and downs.

Finally, this dissertation is supported by the Mellon Dissertation Fellowship received during 2017−2018 from The Andrew W. Mellon Foundation and Rutgers Graduate School of Arts and Sciences, to which I am very grateful.

Dedication

To my family.

Table of Contents

Chapter 1 Introduction

This dissertation addresses two intriguing puzzles about superlative modifiers (SMs). The first puzzle concerns the fact that cross-linguistically, SMs in general demonstrate an ambiguity between an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. The second puzzle concerns the fact that cross-linguistically, SMs in general involve degree words and quantity adjectives in their morphology. The central proposal of this dissertation is two-fold. First, it claims that the two meanings can be pragmatic variants in natural languages. In particular, the EPI-CON ambiguity results from one single semantic entry combining with different pragmatic factors such as informativity and evaluativity. Second, it claims that the semantics of SMs can be decomposed into three pieces: a quantity adjective (Q-adjective), a superlative component and an existential operator E-OP. In particular, Q-adjectives encode a measure function mapping the focus alternatives to their corresponding positions ordered along a contextually-given scale. Seen in this light, the scalar component of a focus particle is actually a comparison relation between the prejacent and its alternatives along a certain dimension.

This chapter is organized as follows. Section 1.1 presents an overview of the EPI-CON ambiguity typically demonstrated by SMs across natural languages. Section 1.2 introduces the morpho-semantic puzzle of SMs that degree words and quantity adjectives are typically involved in their morphology across natural languages. It is shown that although there is more than one morpho-semantic route, Q-adjectives seem to be the common core of those different strategies. Section 1.3 lays out my assumptions about the syntax of focus particles (particularly, SMs) and section 1.4 introduces the basics of semantic compositions assumed in this dissertation. Section 1.5 offers an outline of the other chapters in this dissertation.

1.1 The ambiguity puzzle: ignorance and concession

SMs in general demonstrate an ambiguity between an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. Take English *at least* for example. Out of blue, (1) leads to an ignorance inference that the speaker does not know exactly what medal Mary has won. In contrast, (2) has a concessive meaning accompanied with an evaluative flavor: the silver medal Mary has won, while less than optimal, is acceptable. The two examples below are borrowed from Nakanishi and Rullmann (2009; N&R). Note that one crucial difference between (1) and (2) lies in whether the speaker is ignorant about the piece of information indicated by focus, when she uses *at least*.

(1) Mary won **at least** a [silver]$_F$ medal. **EPI**

(2) Mary didn't win a gold medal, but **at least** she won a [silver]$_F$ medal. **CON**

Crucially, the EPI-CON ambiguity is not unique to English *at least*. Chinese presents an interesting case where more than one lexical item is ambiguous between the two meanings in one single language, as shown in (3).

(3) Chinese

Liubei	**zuishao/**	**zhishao/**	**qima**	shi	[fu]$_F$-jiaoshao.	**EPI**, **CON**
Liubei	at least/	at least/	at minimum	be	associate-professor	

'Liubei is at least an associate professor.'

Finally, if we look at languages beyond English and Chinese, the EPI-CON ambiguity is quite pervasive across natural languages. The cross-linguistic counterparts of English *at least* listed in (4) all demonstrate the ambiguity under discussion.

(4) The EPI-CON ambiguity from a cross-linguistic perspective[1, 2]

a. Brazilian Portuguese: ***pelo menos***

b. Czech: ***aspoň***

c. Dutch: ***tenminste***

d. French: ***au moins***

e. Greek: ***tulachiston***

f. Hebrew: ***le-faxot***

g. Hindi/ Magahi: ***kam se kam***

h. Italian: ***almeno***

i. Japanese: ***sukunaku-tomo***

j. Korean: ***cek-eto***

k. Spanish: ***al menos***

l. Turkish: ***en az/ en azindan***

Presumably, (4) is not an exhaustive list of languages with lexical items showing the ambiguity. The cross-linguistic facts above strongly suggest that the EPI-CON ambiguity should not simply be a case of homophony.

Although English *at least* is observed to show the ambiguity, it is not clear

---

[1] See Nakanishi and Rullmann (2009) for discussion of Dutch *tenminste* and Japanese *sukunaku-tomo*; See also Grosz (2011) for discussion of Czech *aspoň*, Greek *tulachiston*, Hebrew *le-faxot* and Spanish *al menos*.

[2] I am very grateful to Deepak Alok (for Hindi and Magahi data), Kunio Kinjo (for Japanese data), Livia Camargo Tavares Souza and Matt Barros (for Brazilian Portuguese data), Luca Iacoponi (for Italian data), Viviane Déprez (for French data), Vera Gor (for Russian data), Woojin Chung (for Korean data), Ümit Atlamaz and Yağmur Sağ (for Turkish data). I also thank Hazel Mitchley and Lydia Newkirk for valuable discussions on English sentences collected in this dissertation.

whether its antonym *at most* also has the concessive reading. On the one hand, it seems that only EPI is detected in (6), forming a sharp contrast to (5) where both EPI and CON are easily detected and differentiated.

(5) Mary **at least** ate [three]$_F$ apples.

(6) Mary **at most** ate [three]$_F$ apples.

The same difficulty in differentiating CON from EPI repeats itself in Chinese *zuiduo* 'at most'. The most salient interpretation of (7) is that the speaker is ignorant about exactly how many apples *Liubei* has eaten, even though the utterance can be interpreted with an additional evaluative flavor: *Liubei*'s eating three apples is the best/ worst situation we can get.

(7) Chinese

Liubei **zuiduo** chi-le [san]$_F$-ke-pinguo.

Liubei at most eat-ASP three-CL-apple

'Liubei at most ate three apples.'

However, there are examples showing that English *at most* does have the concessive reading. Below, (8) is taken from Cohen and Krifka (2014: 75).

(8) This is a bad hotel; **at most**, it is centrally-located.

Two remarks are in order. First, the sentence is interpreted with an evaluative flavor: a set of relevant properties concerning the hotel is evaluated and the hotel's being centrally-located is already the best situation (where the participants ended up with). Second, the speaker is not ignorant: (8) entails that the hotel is centrally-located.

Another intriguing fact concerning the ambiguity is that at least in both English and Chinese, the availability of the two meanings is restricted by the syntactic distribution of SMs. Consider (9) and (10) below.

(9) English

a. Mary won **at least** a [silver]$_F$ medal. √**EPI**, **#CON**

b. Mary **at least** won a [silver]$_F$ medal. √**EPI**, √**CON**

c. **At least** Mary won a [silver]$_F$ medal. **#EPI**, √**CON**

(10) Chinese

a.	Liubei	xie-le	**zhishao**	[san]$_F$-ben-xiaoshuo.	√**EPI**, **#CON**
	Liubei	write-ASP	at least	three-CL-novel	
	'Liubei wrote at least three novels.'				
b.	Liubei	**zhishao**	xie-le	[san]$_F$-ben-xiaoshuo.	√**EPI**, √**CON**
	Liubei	at least	write-ASP	three-CL-novel	
	'Liubei at least wrote three novels.'				
c.	**Zhishao**	Liubei	xie-le	[san]$_F$-ben-xiaoshuo.	**#EPI**, √**CON**
	At least	Liubei	write-ASP	three-CL-novel	
	'At least Liubei wrote three novels.'				

The same distributional restriction shown in (10) holds for the other two Chinese items: *zuishao* 'at least' and *qima* 'at minimum'.

Taken together, we have seen that SMs in general demonstrate the EPI-CON ambiguity across natural languages. However, as shown in English, there is a contrast between the antonymous pair of SMs: the concessive reading of *at least* is detected more easily than that of *at most*. In addition, in both English and Chinese, the availability of the two meanings is restricted by the syntactic distribution of SMs.

Below, (11) is a list of the research questions concerning the ambiguity puzzle that addressed in this dissertation.

(11) **The ambiguity puzzle**

a. Why do SMs in general demonstrate the ambiguity across natural languages? What is the exact nature of the ambiguity?

b. How are the two meanings connected to each other?

c. Why is there a contrast between *at least* and *at most* in detecting the concessive reading?

d. What does the contrast tell us about the semantics of *at least* and *at most*? Can the contrast be derived from the semantics of *at least* and *at most*? Are there other contrasts between *at least* and *at most* following from the discrepancy in their semantics?

e. Why is the availability of the two meanings restricted by the syntactic distribution of SMs?

This dissertation answers these questions and argues for a unified account. The next section introduces a puzzle posed by SMs concerning their morphological makeup.

1.2 The morpho-semantic puzzle

A longstanding and intriguing puzzle about SMs is that degree words and quantity adjectives are involved in their morphology. Take English for example. The same morphological components *most* and *least* are shared by SMs and quantity superlatives (QSs), as illustrated in (12) and (13).

(12) English SMs

a. Mary at **most** invited [Adam and Bill]$_F$.

b. Mary at **least** invited [Adam and Bill]$_F$.

(13) English QSs

a. Mary climbed the **most** mountains.

b. Mary drank the **least** water.[3]

Chinese makes the situation even more puzzling: exactly the same expressions (morphologically consisting of a quantity adjective and the superlative morpheme) *zui-duo* and *zui-shao* are used in SMs and QSs, as illustrated in (14) and (15).

(14) Chinese SMs

a. Liubei mai-le **zui-duo** $[\text{san}]_F$-ke pinguo.

Liubei buy-ASP SUP-many three-CL apple

'Liubei bought at most three apples.'

b. Liubei mai-le **zui-shao** $[\text{san}]_F$-ke pinguo.

Liubei buy-ASP SUP-little three-CL apple

'Liubei bought at least three apples.'

(15) Chinese QSs

a. Liubei mai-le **zui-duo** (ke) pinguo.

Liubei buy-ASP SUP-many CL apple

'Liubei bought more apples than anyone else did.'

b. Liubei mai-le **zui-shao** (ke) pinguo.

Liubei buy-ASP SUP-little CL apple

'Liubei bought fewer apples than anyone else did.'

---

[3] In English, the word *least* occurs in not only quantity superlatives but also quality superlatives, as shown in (i). When appearing in quantity superlatives, the word *least* is only compatible with mass nouns; for count nouns, a different quantity word *fewest* must be used. This is shown in (ii).

(i) Mary climbed the **least** high mountain.

(ii) *Mary ate the **least** apples. cf. Mary ate the **fewest** apples.

See Solt (2009, 2015) and Wellwood (2014, 2015) for an extensive discussion of English Q-adjectives. See also chapter 5 for a brief review of previous analyses of English Q-adjectives.

Finally, if we look at languages beyond English and Chinese, the morpho-semantic puzzle is again found to be cross-linguistically pervasive. In particular, I find that **(a)** there is more than one possible route to the morpho-semantic mapping of SMs in natural languages and that **(b)** despite the variety of strategies, Q-adjectives seem to be the common core across natural languages. Below, (16) − (18) illustrate the point.

(16) **Q-adjectives plus superlatives** (e.g., English, Chinese, Turkish)

Turkish

a.	*en*	*çok*	or	*en*	*fazla*	'at most'
	SUP	many/ much		SUP	many/ much	
b.	*en*	*az*	or	*en*	*az-ın-dan*	'at least'
	SUP	little		SUP	little-3sgposs-ablative(from)	

(17) **Q-adjectives plus comparatives** (e.g., Magahi, Hindi, Russian)

Magahi

a.	*jaadaa*	*se*	*aadaa*	'at most'
	more	than	more	
b.	*kam*	*se*	*kam*	'at least'
	less	than	less	

(18) **Q-adjectives plus even-if** (e.g., Japanese and Korean)

Japanese

a.	*ooku-temo*	'at most'
	many-even.if	
b.	*sukunaku-temo*	'at least'
	few-even.if	

Notice that Chinese is not the only language employing exactly the same expressions in both SMs and QSs. As witnessed by (16), Turkish is another paradigm language. Moreover, (16) − (18) presumably may not be the only three possible combinations in natural languages. These cross-linguistic facts above strongly suggest that degree morphemes and Q-adjectives involved in SMs are not simply a morphological coincidence; instead, they should be intrinsically connected with the semantics.

In this dissertation, I focus on the superlative strategy observed in English and Chinese. (19) presents the research questions concerning the morpho-semantic puzzle that I investigate.

(19) **The morpho-semantic puzzle**

a. Why do SMs morphologically involve a Q-adjective and the superlative morpheme?

b. What is the role of Q-adjectives and the superlative morpheme inside SMs?

c. How are these morphological pieces connected with the semantics of SMs?

This dissertation takes Chinese SMs as a case study and presents a decompositional analysis showing how those morphological pieces are connected with the semantics. The next section spells out my assumptions about the syntax of focus particles; particularly, the structural positions where SMs are adjoined.

1.3 The syntax of focus particles

In this section, I discuss the view that focus-sensitive particles can be syntactically classified into sentential and constituent-modifying operators, based on their adjunction site (whether they are adjoined to the clausal spine or to non-clausal constituents).[4] Then, I assume a similar distinction holds for SMs and spell out my assumptions concerning the structural possibilities of their adjunction sites.

To being with, in the literature, two different views on where focus particles can be syntactically attached in the structure have been proposed, as shown in (20).

(20) **Two views on where focus particles can be attached**

a. Focus particles are always attached to the clausal spine (e.g., CP, TP, vP, VP): Büring and Hartmann (2001)

b. Focus particles can be attached to either the clausal spine (e.g., CP, TP, vP, VP) or to non-clausal constituents (e.g., DP and PP): Reis (2005), Meyer and Sauerland (2009), Erlewine (2014, 2017), Smeets and Wagner (2018)

In this dissertation, I assume the second view that focus-sensitive particles can be syntactically classified into sentential and constituent-modifying operators, based on whether they are adjoined to the clausal spine or to non-clausal constituents. On this view, the focus particle *only* in English is ambiguous between a sentential focus particle and a constituent focus particle (a case of homophones).[5] Following Jackendoff (1972), F-marking is used to indicate the position of focus, abstracting away from its detailed phonetic realization.

(21) English sentential and constituent *only*

a. Mary **only** likes [John]$_F$. — sentential *only*

b. Mary likes **only** [John]$_F$. — constituent *only*

---

[4] Following Erlewine (2017)'s terminology, in a parallel to the dichotomy between sentential negation and constituent negation, I use *sentential* and *constituent* focus particles for the distinction.

[5] In this respect, according Erlewine (2017), Vietnamese is a language employing two different lexical items for the meaning of *only*: *chỉ* is a sentential 'only', while *mỗi* is a constituent 'only'. Readers are referred to Erlewine (2017) for more details.

As discussed in Erlewine (2017: section 2), sentential *only* takes surface scope, while constituent *only* can lead to scope ambiguity (see also e.g., Rooth 1985: chapter 3).

(22) Sentential focus particles take surface scope: (from Erlewine 2017: (4))

a. They were advised to **only** learn [Spanish]$_F$. *advised* > *only*

b. They were **only** advised to learn [Spanish]$_F$. *only* > *advised*

(23) Constituent focus particles lead to a scope ambiguity: (from Erlewine 2017: (5))

They were advised to learn **only** [Spanish]$_F$. √*advised* > *only*, √*only* > *advised*

In addition to the issue of the adjunction site, it has been argued that a c-command relation must hold between a focus-sensitive operator and the focused constituents it is associated with (focus associates; see Jackendoff 1972, Rooth 1985, among many others). (24) presents the c-command requirement. (25) and (26) illustrates that both sentential and constituent focus particles follow the requirement.

(24) The c-command requirement on association with focus

A focus-sensitive operator must c-command its focus associate(s).

(25) Sentential *only* and association with focus

a. John **only** introduced [Mary]$_F$ to Sue.

b. John **only** introduced Mary to [Sue]$_F$.

c. *[John]$_F$ **only** introduced Mary to Sue.[6]

(26) Constituent *only* and association with focus

a. John introduced **only** [Mary]$_F$ to Sue.

---

[6] The focus particle *even* apparently disobeys the c-command requirement, as witnessed by (i).
(i) [John]$_F$ even introduced Mary to Sue.

Erelwine (2014) argues that in cases like (i), the c-command requirement still holds because *even* is associated with the trace of the subject. Readers are referred to Erelwine (2014) for a detailed discussion of such contrast between *only* and *even*, with respect to their association with focus.

b. *John introduced **only** Mary to $[\text{Sue}]_F$.

c. John introduced Mary **only** to $[\text{Sue}]_F$.

d. John introduced Mary to **only** $[\text{Sue}]_F$.

In (25), the preverbal *only* can associate with any focused constituent in a post-verbal position; in contrast, in (26), *only* in a post-verbal position must associate the focused constituent immediately following it. These patterns of focus association in (25) and (26) are explained by the requirement that a focus-sensitive operator must c-command its focus associates (see (24)), together with different adjunction positions for *only*.

Given the c-command requirement in (24) and the patterns of focus association in (25) and (26), English pre-subject *only* can receive two possible parses, as in (27).

(27) Two syntactic parses for English pre-subject *only*:

Only $[\text{John}]_F$ introduced Mary to Sue.

a. $[_{TP}$ **Only** $[_{TP}$ $[\text{John}]_F$ introduced Mary to Sue.] sentential *only*

b. $[_{DP}$ **Only** $[_{DP}$ $[\text{John}]_F]$ introduced Mary to Sue. constituent *only*

However, notice that the pre-subject *only* cannot associate with any focused constituent that follows the subject, as shown in (28).

(28) a. ***Only** John $[\text{introduced Mary to Sue}]_F$.

b. ***Only** John introduced $[\text{Mary}]_F$ to Sue.

c. ***Only** John introduced Mary to $[\text{Sue}]_F$.

Given what we have seen in (24) − (26), the c-command requirement suggests that the preverbal only in (28) should be a constituent *only*, rather than a sentential *only*.[7]

With the above discussion as our background, let's consider the case of SMs.

[7] Erlewine (2017) offers an interesting analysis explaining how and why the preverbal *only* here cannot be a sentential *only*. His analysis will be discussed in chapter 2 (section 2.4.7).

Recall that the availability of the two meanings (EPI and CON) is restricted by the syntactic distribution of SMs (see (9) and (10)). Because the EPI-CON ambiguity is more easily observed in the case of *at least*, below, I use *at least* for the comparison and illustration. Nothing crucial in this dissertation hinges on this choice.

First of all, following the syntactic classification of English *only* above, I assume that the same dichotomy holds for English *at least*. Crucially, notice that (29a) can be interpreted under either EPI or CON, while (29b) conveys only EPI. This may suggest that concessive *at least* is a sentential focus particle, while epistemic *at least* can be either a constituent focus particle or a sentential focus particle.

(29) English sentential and constituent *at least*

a. Mary **at least** likes [John]$_F$. sentential *at least*

b. Mary likes **at least** [John]$_F$. constituent *at least*

Second, similar to the case of English *only*, sentential *at least* takes surface scope while constituent *at least* can lead to a scope ambiguity, as shown in (30) and (31).

(30) Sentential *at least* takes surface cope

a. They were advised to **at least** learn [Spanish]$_F$. *advised* > *at least*

b. They were **at least** advised to learn [Spanish]$_F$. *at least* > *advised*

(31) Constituent *at least* leads to a scope ambiguity

They were advised to learn **at least** [Spanish]$_F$.

(√*advised* > *at least*, √*at least* > *advised*)

Third, in a parallel with the case of English *only*, both sentential *at least* and constituent *at least* obey the c-command requirement.

(32) Sentential *at least* and association with focus

a. John **at least** introduced [Mary]$_F$ to Sue. √**EPI**, √**CON**

b. John **at least** introduced Mary to [Sue]$_F$. √**EPI**, √**CON**

c. *[John]$_F$ **at least** introduced Mary to Sue.

(33) Constituent *at least* and association with focus

a. John introduced **at least** [Mary]$_F$ to Sue. √**EPI**, **#CON**

b. *John introduced **at least** Mary to [Sue]$_F$.

c. John introduced Mary **at least** to [Sue]$_F$. √**EPI**, **#CON**

d. John introduced Mary to **at least** [Sue]$_F$. √**EPI**, **#CON**

In (32), the preverbal *at least* can associate with any focused constituent in a post-verbal position. In contrast, in (33), the post-verbal *at least* must associate with the focused constituent immediately following it. These patterns of focus association in (32) and (33) are explained by the c-command requirement (see (24)), together with different adjunction positions for *at least*.

Finally, similar to the case of English *only*, the pre-subject *at least* can receive two possible parses, as shown in (34). Notice that the sentence in (34) is ambiguous: it can be interpreted under either EPI or CON.

(34) Two syntactic parses for English pre-subject *at least*:

At least [John]$_F$ introduced Mary to Sue. √**EPI**, √**CON**

a. [$_{TP}$ **At least** [$_{TP}$ [John]$_F$ introduced Mary to Sue.] sentential *at least*

b. [$_{DP}$ **At least** [$_{DP}$ [John]$_F$] introduced Mary to Sue. constituent *at least*

In a sharp contrast to English *only*, English *at least* CAN associate with a focused constituent that follows the subject (see (28)), as witnessed by (35). Furthermore, in contrast to (34), (35) is NOT ambiguous: it must be interpreted under CON.

(35) a. **At least** John [introduced Mary to Sue]$_F$. **#EPI**, √**CON**

b. **At least** John introduced [Mary]$_F$ to Sue. **#EPI**, √**CON**

c. **At least** John introduced Mary to [Sue]$_F$. **#EPI**, √**CON**

Importantly, the dichotomy between sentential and constituent focus particles, together with the distributional restriction of the two meanings, has raised many non-trivial questions not only about the syntax of focus particles but also about the nature of the EPI-CON ambiguity: First, in the case of *only*, how and why is the association with focus impossible in (28)? Second, in the case of epistemic *at least*, how and why is the association with focus impossible in (35)? Third, in the case of concessive *at least*, exactly how and why is the association with focus possible in (35)? Fourth, why does epistemic *at least*, but not concessive *at least*, pattern with *only* with respect to the association with focus?

At this point, a possible answer to the last question is to say that although English has both sentential/ constituent epistemic *at least* and sentential/ constituent *only*, it has only sentential concessive *at least*. This could be a possibility for some languages with dedicated items lexicalizing one of the two meanings (EPI or CON).[8] However, as far as English and Chinese are concerned, the answer presumably brings us to a deeper question: why is it that English lacks a constituent concessive *at least*? Is it a systematic gap or an accidental gap? What exactly is the nature of the distributional restriction on the availability of the two meanings? Notice that the three

---

[8] If we consider it an accidental gap that English lacks a constituent concessive *at least* (and thus the prenominal *at least* receives only an ignorance/ epistemic interpretation), the prediction would be that some languages may have constituent focus particles conveying the concessive meaning. Other things being equal, this in turn suggests that the prenominal restriction on the concessive meaning (observed in English and Chinese) should be absent in those languages. In this respect, German provides a good empirical testing-ground for the nature of the distributional restriction on the two meanings, given its richness of focus-sensitive particles and its dedicated items: *mindestens* 'at least' (exclusively conveying the epistemic/ ignorance reading) and *wenigstens* 'at least' (exclusively conveying the concessive reading), *zumindest* 'at least' (conveying the concessive meaning). See also chapter 6 for a discussion of other potential factors removing the prenominal restriction on the concessive meaning.

Chinese items (*zuishao*, *zhishao* and *qima*) similarly do not receive a concessive interpretation when they are in a prenominal position (see (10)).

To sum up, instead of stipulating the lexical inventory of concessive *at leas*t (and their Chinese counterparts), I assume in this dissertation that SMs such as English *at least* can be a sentential focus particle (adjoined to the clausal spine) or a constituent focus particle (adjoined to non-clausal constituents), as summarized in (36).

(36) English *at least* and two types of focus particles

a. Sentential *at least* are adjoined to the clausal spine (e.g., CP, TP, vP, VP…etc.).

b. Constituent *at least* are adjoined to non-clausal constituents (e.g., DP, PP…etc.).

The next section introduces the basics of semantic compositions.

1.4 The basics of semantic compositions

In this dissertation, I will assume the framework of truth-conditional model-theoretic compositional semantics, as elaborated in Heim and Kratzer (1998) and Chierchia and McConnell Ginet (2000). I will presuppose some familiarity with this formal system and I will not present an extensive introduction of all the conventions. However, I briefly introduce the most important ones pertaining to semantic composition in this dissertation. To begin with, I assume that the input for the semantic module is a syntactic representation generated by the syntactic module. Moreover, this syntactic representation is an unambiguous hierarchical description of a sentence called Logical Form (LF). The semantic module takes an LF and maps it to model-theoretic objects in a compositional way – as a function from the semantics of its atomic parts (the lexicon) and the way these atomic expressions are combined (the structure). The function that maps an LF to its model-theoretic interpretation is the interpretation function, represented as ⟦ ⟧.

Semantic composition is type-driven; for purposes of this dissertation, I assume the following semantic types:

(37) Semantic types

a. D_e := set of individuals;

b. D_t := {False, True} (sometimes written as {0, 1});

c. D_s := set of worlds;

d. D_v := set of events;

e. D_d := set of degrees;

f. If σ and τ are semantic types, then $<\sigma, \tau>$ is a semantic type.

Thus, $D_{<\sigma, \tau>} := \{f: f$ is a function from D_σ to $D_\tau\}$

The compositional rules for semantic interpretations assumed in this dissertation are the following. First, the interpretation of lexical items comes from the lexicon.

(38) Lexical Item

For any terminal node α, ⟦α⟧ is in the lexicon.

Some examples of lexical items and their denotations are illustrated below.

(39) a. ⟦Mary⟧ = Mary

b. ⟦smiled⟧ = λx_e. x smiled

c. ⟦student⟧ = λx_e. x is a student

Second, to compose meanings of expressions more complex than terminal nodes, a compositional rule Functional Application is needed.

(40) Functional Application Rule

If α is a branching node, β and γ are the daughters of α, and $[\![\beta]\!]^g \in D_\sigma$ and $[\![\gamma]\!]^g \in D_{\langle\sigma,\, \tau\rangle}$ (for σ, τ are semantic types), then $[\![\alpha]\!]^g = [\![\gamma]\!]^g([\![\beta]\!]^g)$.

(41) illustrates how the system works. Suppose that the sentence *Mary smiled* has an LF in (41a). The interpretation function $[\![\ ]\!]$ takes the LF as the input and returns the semantic interpretation. Finally, the meaning of the sentence (42b) is obtained by applying the meaning of *smiled* to that of *Mary* via Functional Application Rule.

(41) Mary laughed

a. LF: [Mary [smiled]]

b. $[\![$[Mary [smiled]]$]\!]$ = 1 iff $[\![$smiled$]\!]$($[\![$Mary$]\!]$) = 1 iff Mary smiled

Third, to compose meanings of complex expressions, I also assume a compositional rule Predicate Modification. (43) illustrates the system works.

(42) Predicate Modification Rule

If α is a branching node, β and γ are the daughters of α, and both $[\![\beta]\!]^g$ and $[\![\gamma]\!]^g \in D_{\langle\sigma,\, \tau\rangle}$ (for σ, τ are semantic types), then $[\![\alpha]\!]^g = \lambda x \in \mathrm{D}_\sigma.\ [\![\gamma]\!]^g(x) = [\![\beta]\!]^g(x) = 1$.

(43) $[\![$[city [in New Jersey]]$]\!]$

$= \lambda x \in \mathrm{D}_e.\ [\![\text{city}]\!]^g(x) = [\![\text{in New Jersey}]\!]^g(x) = 1$

$= \lambda x_e.$ *x* is a city and *x* is in New Jersey.

Fourth, I assume that expressions like pronouns and traces left by movement are variables, and they are interpreted via assignment function g – a function that maps numerical indices (pronouns and traces come with) to individuals in the D_e domain.

(44) Pronoun and Trace Rule

If α_i is a pronoun or a trace, $[\![\alpha]\!]^g = g(i)$.

$[\![\ ]\!]^g$ here means that the interpretation function is evaluated relative to the assignment function *g*. As we will see shortly in this dissertation, the interpretation of expressions with variables evaluated relative to a context *c* is treated in a parallel way: $[\![\ ]\!]^c$.

Fifth, to interpret traces and pronouns that are bound by some constituent in their domain, I assume a compositional rule Predicate Abstraction.[9]

(45) Predicate Abstraction Rule

If α is a branching node (of type $<\sigma, \tau>$), with β (of type $<\tau>$) and a binder index λi as its daughter constituents, then $[\![\alpha]\!]^g = \lambda x_\sigma. [\![\beta]\!]^{g[i \mapsto x]}$.

That is, $[\![\alpha]\!]^g$ is the function mapping any $x \in D_\sigma$ to $[\![\beta]\!]^{g[i \mapsto x]}$. Note that $g[i \mapsto x]$ is a minimally modified assignment function such that the only difference from *g* is that $g[i \mapsto x](i) = x$.

(46) illustrates how the rules in (44) and (45) works in the system. Suppose that the sentence *every boy respects his father* has an LF in (46a). (46b) is obtained via Pronoun and Trace Rule and Predicate Abstraction Rule. Assuming that *every* is a generalized quantifier with the semantics in (46c), the meaning of the sentence is derived by applying (46c) to (46b) via Functional Application Rule.

(46) Every boy respects his father.

a. LF: [every boy [$_\alpha$ λ2 [$_\beta$ t_2 respect his_2 father]]]

b. $[\![[_\alpha \lambda 2\ [_\beta\ t_2 \text{ respect his}_2 \text{ father}]]]\!]^g$

$= \lambda x_e.\ x$ respects x's father

c. $[\![\text{every boy}]\!]^g = \lambda P_{<e,t>}. \forall x[\text{boy}(x) \rightarrow P(x)]$

d. $[\![[\text{every boy } [_\alpha \lambda 2\ [_\beta\ t_2 \text{ respect his}_2 \text{ father}]]]]\!]^g$

[9] To simplify the LF somewhat, in this dissertation, I will use [$_\alpha$ λz [$_\beta$ …*z* …]], where *z* is a variable that is bound by the λ-binder, as an LF representation equivalent to [$_\alpha$ λi [$_\beta$ t_i…]], where the binding relation is indicated by the index i. Nothing crucial in my proposal about SMs hinges on this simplification.

= 1 iff for every x who is a boy, x respects x's father.

Finally, a compositional rule particularly relevant to this dissertation is the so-called Pointwise Functional Application rule.

(47) Pointwise Functional Application

If α is a branching node, β and γ are the daughters of α, and $[\![\beta]\!]^g \in D_\sigma$ and $[\![\gamma]\!]^g \in D_{\langle\sigma,\, \tau\rangle}$ (for σ, τ are semantic types), then

$[\![\alpha]\!]^g = \{f(h) \mid f \in [\![\gamma]\!]^g \text{ and } h \in [\![\beta]\!]^g\}$.

This Pointwise Functional Application rule is commonly assumed in alternatives semantics (e.g., Hamblin 1973, Rooth 1985, 1992, Kratzer and Shimoyama 2002, among many others). This rule will be important for us to obtain the interpretation of expressions with focus. Although the rule (47) has some difficulty dealing with cases of binding (see e.g., Shan 2004, Novel and Romero 2010, Charlow 2014, Ciardelli et al. 2017 for discussion), it will not concern us for purposes of this dissertation.

In summary, in the previous sections, I have introduced two puzzles posed by superlative modifiers that this dissertation is dedicated to capturing: the ambiguity puzzle and the morpho-semantic puzzle. Furthermore, I have also spelled out my assumptions concerning the syntax of focus particles and semantic compositions.

1.5 Outline

This dissertation is organized as follows. Chapters 2, 3 and 4 address issues concerning the ambiguity puzzle posed by SMs across languages. In Chapter 2, I take English *at least* as a case study and propose that the two meanings can be pragmatic variants in natural languages: in particular, the EPI-CON ambiguity results from one single semantic entry combining with different pragmatic factors such as informativity and evaluativity. Furthermore, three novel observations about the parallel between the two meanings are presented: they both are focus-sensitive, compatible with various scales and demonstrate two scalar effects (the top-of-the-scale effect TSE and the bottom-of-the-scale effect BSE). Finally, I offer a semantic-pragmatic explanation for why the availability of CON is restricted by the syntactic distribution of SMs. If the current analysis is on the right track, the EPI-CON ambiguity cannot simply be a lexical coincidence in natural languages. Instead, the two meanings are systematically and intrinsically related. This in turn explains **(a)** why the EPI-CON ambiguity is so pervasive across languages and generally shown by one single lexical item (particularly by SMs); **(b)** why multiple lexical items in one single language like Chinese demonstrate the ambiguity; **(c)** why both meanings share the three properties.

In chapter 3, I take a close look at discourse properties of concessive *at least* and epistemic *at least*. In the spirit of the unified account presented in chapter 2, I present a formal analysis and illustrate how concessive *at least* and epistemic *at least* are pragmatic variants. One central idea is that although concessive *at least* and epistemic *at least* share one single semantic representation, they have different discourse profiles. Adopting the discourse model (i.e., the conversational scoreboard) originally presented by Farkas and Bruce (2010) and further developed in Malamud and Stephenson (2015) and Beltrama (2018), I argue that the discrepancy in their discourse profiles crucially comes from where propositions are added in the conversational scoreboard. More specifically, assertions with concessive *at least* add associated propositions to the speaker's present commitments, while assertions with epistemic *at least* add associated propositions to the speaker's projected commitments. Furthermore, in this chapter, a number of issues are discussed in detail: What is put forward in the discourse under assertions with concessive/ epistemic *at least*? What is placed on the Table under assertions with concessive/ epistemic *at least*? What are the participants' discourse commitments under assertions with concessive/ epistemic *at least*? What is updated/ added to the discourse under assertions with concessive/ epistemic *at least*? How are assertions with concessive/ epistemic *at least* different from (factual) assertions? Finally, the connection between the unified semantic representation of *at least* (proposed in chapter 2) and their different discourse profiles under the two readings (ignorance vs. concession) is also discussed in this chapter.

Some details of the discourse profile of concessive *at least* and epistemic *at least* are listed below: **(a)** under both readings, the information content of the prejacent is always at-issue; **(b)** under both readings, the information content of the lower alternatives is at-issue; **(c)** under both readings, the information content of the ranking between the prejacent and its alternatives seems to be not-at-issue; **(d)** under the concessive reading, the information content of the higher alternatives can be at-issue or not-at-issue; while under the ignorance reading, it is at-issue; **(e)** the speaker's discourse commitments are different under the two readings: under the concessive reading, the speaker is committed to both the truth of the prejacent and the falsity of the higher alternatives; in contrast, under the ignorance reading, the speaker is *tentatively* committed to the truth of the prejacent or that of the higher alternatives. Put differently, under assertions with epistemic *at least*, the speaker does not *fully* commit herself to the necessary truth of either the prejacent or the higher alternatives. Instead, what the speaker is committed to is the *possibility* that the prejacent is true and the *possibility* that the higher alternative is true in subsequent discourse. To anticipate a bit, this projection of multiple possibilities in subsequent discourse is precisely the pragmatic source of the modal flavor associated with epistemic *at least*.

This in turn suggests that the modal flavor associated with epistemic *at least* is not located at the level of lexical semantics (e.g., a covert epistemic modal in Geurs and Nouwen 2007), but arises at the level of discourse;

In Chapter 4, I extend the unified analysis developed in chapter 2 by taking English *at most* as a case study and proposing a unified semantics for it. Specifically, it is shown that *at most*, like *at least*, similarly gives the EPI-CON ambiguity and the two meanings can be taken as pragmatic variants resulting from one single semantic entry interacting with different pragmatic factors such as informativity and evaluativity. Furthermore, three contrasts between *at most* and *at least* are discussed: First, *at most* shows a mirror image of *at least* with respect to the discrepancy between the two scalar effects TSE and BSE. Second, the concessive reading of *at most* looks like an epiphenomenon of the epistemic reading, while that of *at least* does not. Third, the concessive *at most* gives a "settle-for-less" flavor slightly different than that given by the concessive *at least*. Crucially, these contrasts are not arbitrary, but systematic: they all result from the different semantic bounding properties of *at most* and *at least*.

Chapter 5 shifts the attention to the mapping between the semantics of SMs and their morphology. In this chapter, I take Chinese SMs (*zuishao* 'at least' and *zuiduo* 'at most') as a case study and present a decompositional analysis. There are three crucial pieces in the decompositional analysis: **(a)** a quantity adjective encoding a measure function mapping the focus alternatives to their corresponding position ordered along a contextually-given scale; **(b)** the superlative component introducing the scalarity (i.e., a comparison relation between the prejacent and its alternatives) and serving as a further domain restriction; **(c)** an existential operator E-OP (covert in Chinese) structurally embedding the superlative component and semantically making an existential statement over the non-singleton domain of SMs. It is shown that the proposed decompositional analysis not only explains why the superlative component is involved in SMs across languages and how its contribution is connected with the semantics of SMs, but also captures two types of parallels shown by SMs. On the one hand, resembling epistemic indefinites, SMs always have a non-singleton domain consisting of the prejacent (obtained by the focus presuppositions) and the relevant higher/ lower alternatives (obtained by the superlative component). On the other hand, the semantics of SMs is parallel to that of a disjunction, because an existential claim over a non-singleton domain amounts to a disjunctive statement concerning the elements in the domain. Finally, the current decompositional analysis has three implications. First, an analysis capturing the compositionality below the level of the word is not only tenable but also desirable: the superlative morpheme and quantity adjectives involved in SMs are NOT a morphological coincidence; instead, they are deeply connected with the semantics of SMs. Second, insights and formal tools developed in studies on gradability can be applied to those on scalarity. Recently Greenberg (2016, 2017) has argued for a gradability-based semantics of English *even*. The current study follows the same line of research, given the fact that SMs explicitly involve degree words and quantity adjectives in their morphology across languages. Third, quantity adjectives seem to be the common core in the morphological makeup of SMs across natural languages. In this dissertation, I find that the superlative strategy is not the only route to the morpho-semantic mapping of SMs. At least two other strategies are discovered and documented in this dissertation: **(a)** quantity adjectives plus conditional operators (e.g., Japanese and Korean); **(b)** quantity adjectives plus comparative morphemes (e.g., Hindi, Magahi and Russian). Crucially, quantity adjectives are the common core of these different morpho-semantic strategies.

Under the current analysis, this fact is expected because quantity adjectives semantically encode a measure function, which is the apparatus providing the basis for establishing an ordering/ ranking between the alternatives in the first place.

For purposes of this dissertation, attention is restricted to SMs under unembedded contexts. This move inevitably leaves untouched some other intriguing puzzles about SMs discussed in the literature. Thus, the purpose of chapter 6 is two-fold: the first goal is to address several puzzles that have been observed in the literature but have remained unresolved in this dissertation; the second goal is to discuss some cross-linguistic implications of the current analysis. Chapter 6 is organized as follows. Section 6.1 discusses some interesting interaction between SMs and modals/ universal quantifiers. A particularly puzzling fact is that when SMs are under the scope of existential modals, *at most* leads to a mysterious upper bound inference (which is not predicted by the truth-conditions), while *at least* does not lead to a corresponding lower bound inference. Similar contrast holds for Chinese as well. Section 6.2 presents a puzzling fact that SMs are ill-formed under the scope of negation. This is an intriguing puzzle because nothing in the truth-conditions of SMs explains why they should be ill-formed under negation. However, it has also been observed that SMs are not ill-formed under downward-entailing contexts across the board. For example, it is shown that English SMs are well-formed in the restrictor of a universal quantifier and the antecedent of conditionals. Similar contrast holds for Chinese SMs as well. Section 6.3 compares SMs with a variety of expressions that can result in ignorance inferences (such as free relatives, epistemic indefinites and disjunction) and discusses how the current analysis sheds light on the semantics and pragmatics of ignorance inferences. Section 6.4 discusses some (cross-linguistic) implications of the current analysis. In particular, four issues are addressed: **(a)** the issue of lexicalization; **(b)** the quantificational domain hypothesis for the distributional restriction on CON; **(c)** the variety of SMs and their restrictions; **(d)** the definite determiner inside SMs. Section 6.5 summarizes this dissertation and repeats the crucial pieces of the current analysis of SMs.

Chapter 2 A Unified Analysis of Superlative Modifiers

Cross-linguistically, superlative modifiers (SMs) in general demonstrate an ambiguity between an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. Such EPI-CON ambiguity has been either implicitly or explicitly taken to be a lexical coincidence in previous studies. This chapter argues for a uniform semantic representation of those SMs (e.g., English *at least* and Chinese *zuishao*, *zhishao, qima*) showing the EPI-CON ambiguity in natural languages. Building on Nakanishi and Rullmann (2009)'s observations and Biezma (2013)'s insights, this chapter takes English *at least* as a case study and proposes that the two meanings can be pragmatic variants in natural languages and the EPI-CON ambiguity arises from one unified semantic entry combining with different pragmatic factors such as informativity and evaluativity. Furthermore, three novel observations on the parallel between the two meanings are presented: both are focus-sensitive, compatible with various scales and demonstrate two scalar effects. Finally, I offer a semantic-pragmatic explanation for why the availability of CON is restricted by the syntactic distribution of SMs. If the unified analysis is on the right track, the EPI-CON ambiguity cannot simply be a lexical coincidence in natural languages. Instead, the two meanings are systematically and intrinsically related. This in turn explains **(a)** why the EPI-CON ambiguity is so pervasive across languages and generally shown by one single lexical item (particularly by SMs); **(b)** why multiple lexical items in one single language, as in Chinese, may demonstrate the ambiguity; **(c)** why both meanings share the three common properties.

This chapter is structured as follows. Section 2.1 presents the major data regarding the EPI-CON ambiguity that this dissertation is concerned with. Section 2.2 looks at the pragmatics of the ambiguity and shows how the two meanings can be pragmatic variants. Section 2.3 discusses two views on the nature of the ambiguity and reviews two aspects in which the previous analyses of epistemic *at least* vary. Section 2.4 spells out the core ingredients of my proposal and presents a unified analysis of the ambiguity shown by English *at least*. In particular, a uniform semantic representation of *at least* is offered. Section 2.5 first presents a derivation of ignorance inferences given by *at least*, based on the discussion in section 2.2, and then discusses how the unified analysis captures the similarity between *at least* and disjunction/ epistemic indefinites suggested in previous studies of SMs. Section 2.6 concludes the chapter.

2.1 Basic data: the EPI-CON ambiguity

Nakanishi and Rullmann (2009) (henceforth N&R) observes that English sentences containing *at least* can be ambiguous between two readings (cf. Kay 1992): an epistemic reading (EPI) and a concessive reading (CON). EPI conveys speaker ignorance. For example, (1) conveys that the speaker is uncertain about what medal Mary has won. In contrast, CON conveys speaker concession; (2) conveys that winning a silver medal, while less preferable, is still satisfactory.

(1) Mary won **at least** a [silver]$_F$ medal. **EPI**

(2) Mary didn't win a gold medal, but **at least** she won a [silver]$_F$ medal. **CON**

N&R considers (1) and (2) as a case of lexical ambiguity. That is, the English item *at least* happens to have two independent lexical entries. To my knowledge, Biezma

(2013) is the first study arguing for a unified account of English *at least*. An important insight of Biezma's analysis is that many of N&R's observations that EPI and CON differ can be derived from one unified semantic entry combining with the role of discourse in constructing the set of focus alternatives (I will return to N&R's and Biezma's analyses in section 2.3). The debate between N&R and Biezma (2013) raises many non-trivial questions: What exactly is the nature of the EPI-CON ambiguity? Is it a case of homophony or a case of polysemy? Should we expect the same ambiguity to repeat itself across natural languages?

The rest of this section proceeds as follows. As our point of departure, section 2.1.1 presents some cross-linguistic data showing that the EPI-CON ambiguity is surprisingly pervasive across natural languages. These cross-linguistic facts indicate that the ambiguity in question cannot be simply a lexical coincidence. Section 2.1.2 offers some novel observations on three common properties of EPI and CON: **(a)** both meanings are focus-sensitive; **(b)** both meanings are compatible with various scales, and **(c)** both meanings demonstrate two scalar effects. Section 2.1.3 discusses two aspects in which the two meanings crucially differ: **(a)** CON conveys a "settle-for-less" flavor while EPI is neutral; **(b)** the (un)availability of the two meanings is sensitive to the syntactic distribution of SMs.

2.1.1 A cross-linguistically pervasive phenomenon

To begin with, according to Grosz (2011), Greek *tulachiston*, Hebrew *le-faxot*, Czech *aspoň*, Spanish *al menos* all demonstrate the EPI-CON ambiguity. The Greek and Hebrew examples below are borrowed from Grosz (2011: 581).[1]

[1] Besides those languages with lexical items showing the EPI-CON ambiguity, Grosz (2011: 577) also observes that there are languages employing dedicated particles exclusively for EPI or CON (e.g.,

(3) Greek

a. EPI

Sto	aftokinitistiko	atixima,	skotothikan	**tulachiston**	pende	anthropi.
in-the	car	accident	were.killed	at.least	five	people

‘There were at least 5 casualties in the traffic accident.’

b. CON

I	Maria	dhen	kerdhise	chriso	metalio,	ala	**tulachiston**	kerdhise
the	Maria	not	won	gold	medal	but	at.least	won

arjiro/asimenjo	(metalio).
silver	medal

‘Mary didn’t win a gold medal, but at least she won a silver medal.’

(4) Hebrew

a. EPI

Be-te’unat	ha-drax-im	hayu	**le-faxot**	xamiSa	harug-im.
in-accident	the-way-pl	exist.masc-pl	at-least	five	casualties

‘There were at least 5 casualties in the traffic accident.’

b. CON

Mary	lo	zaxta	be-medalyat	zahav,	aval	**le-faxot**	hi	zaxta	be-medalyat
Mary	not	won	in-medal	gold	but	at-least	she	won	in-medal

kesef.
silver

‘Mary didn’t win a gold medal, but at least she won a silver medal.’

Furthermore, it is worth emphasizing that Greek, Hebrew, Czech and Spanish are not

Finish, German, Polish, Romanian and Russian). This means that the EPI-CON ambiguity is not universally attested. Please see chapter 5 (section 5.4) for discussion of issues concerning lexicalization.

the only languages with lexical items showing the ambiguity. The same EPI-CON ambiguity is found with Brazilian Portuguese *pelo menos*, Dutch *tenminste*, Hindi *kam se kam*, Italian *almeno*, French *au moins*, Japanese *sukunaku-tomo*, Korean *cek-eto*, Magahi *kam se kam* and Turkish *en azindan*.[2]

Finally, Chinese makes the EPI-CON ambiguity even more puzzling: more than one lexical item in one single language is ambiguous between the two meanings.[3, 4] Consider the Chinese example (5) and the two contexts (6) and (7).

(5) Chinese

Liubei	**zuishao/**	**zhishao/**	**qima**	na-le	$[\text{yin}]_F$-pai.	**EPI**, **CON**
Liubei	at least/	at least/	at minimum	take-ASP	silver-medal	

'Liubei at least got a silver medal.'

(6) Speaker A: What medal did *Liubei* get?

Speaker B: I don't know…

(7) Speaker A: What medal did *Liubei* get? Did *Liubei* get a gold medal?

Speaker B: No, but…

The Chinese sentence (5) is ambiguous: speaker B can felicitously use it as a continuation in context (6) or (7). More specifically, the very same utterance (5) conveys the speaker's ignorance about what medal *Liubei* has got in the context (6),

---

2 I am very grateful to Deepak Alok (for Hindi and Magahi data), Kunio Kinjo (for Japanese data), Livia Camargo Tavares Souza and Matt Barros (for Brazilian Portuguese), Luca Iacoponi (for Italian data), Viviane Déprez (for French data), Vera Gor (for Russian data), Woojin Chung (for Korean data), Ümit Atlamaz and Yağmur Sağ (for Turkish data). I also thank Hazel Mitchley and Lydia Newkirk for valuable discussions and constructive comments on English sentences collected in this dissertation.

3 An anonymous NELS reviewer points out that English *at minimum*, like *at least*, is also ambiguous in conveying speaker ignorance and speaker concession. This means that Chinese is not the only language with multiple lexical items showing the EPI-CON ambiguity. This English fact, again, reinforces our point that the EPI-CON ambiguity cannot simply be a lexical coincidence in natural languages.

4 Abbreviations in this chapter: pl: plural marker; ASP: aspect marker; CL: classifier.

while it conveys the speaker's concession about the fact that *Liubei* got a silver medal in the context (7). Thus, the three lexical items *zuishao* 'at least', *zhishao* 'at least' and *qima* 'at minimum' all demonstrate the familiar EPI-CON ambiguity.

In short, these cross-linguistic facts strongly suggest that the two meanings EPI and CON should be intrinsically related and the ambiguity in question cannot simply be a case of homophony in some languages. In the following two subsections, I first present my novel observations on three common properties of EPI and CON, and then discuss two important differences between EPI and CON observed in N&R.

2.1.2 Three common properties of EPI and CON

The first common property is that both meanings are focus-sensitive. The position of focus associate makes explicit the piece of information that the speaker's ignorance or concession is about.[5] Consider a scenario where there are three individuals in the discourse: Adam, Bill and Chris. There is a priority ranking on the invitation list: Adam $\succ$Bill $\succ$Chris. In this scenario, (8a) is felicitous and conveys the speaker's ignorance about *who John invited*. In contrast, (8b) is infelicitous because the conveyed ignorance inference is about *what John did*, rather than *who John invited*. It is well-established in the literature that English *only* is focus-sensitive and conventionally associates with focus (in the sense of Beaver and Clark 2008). The parallel contrast between (8) and (9) indicates that EPI is also focus-sensitive.

(8) Who did John invite?

a. John **at least** invited [Bill]$_F$.

[5] The fact that EPI is focus-sensitive is not new. Readers are referred to Krifka (1999), Beaver and Clark (2008), Coppock and Brochhagen (2013) and Mendia (2016) for discussions and arguments. The novel observation here is that CON is also focus-sensitive. That is, both meanings are conventionally associated with focus.

b. #John **at least** [invited Bill]$_F$.

(9) Who did John invite?

a. John **only** invited [Bill]$_F$.

b. #John **only** [invited Bill]$_F$.

The example (10) shows that CON is also focus-sensitive. First, (10a) is felicitous while (10b) is not. Second, in (10a) the conveyed speaker concession is about *who John invited* while in (10b) it is about *what John did*. Recall that the given priority ranking is Adam $\succ$Bill $\succ$Chris.

(10) Who did John invite? Did John invite Adam?

a. No, but he **at least** invited [Bill]$_F$.

b. #No, but he **at least** [invited Bill]$_F$.

The same focus-sensitivity of EPI and CON is observed in Chinese as well.[6] Recall that the three Chinese items *zuishao* 'at least', *zhishao* 'at least' and *qima* 'at minimum' all demonstrate the EPI-CON ambiguity. In (11), the piece of information that the speaker's ignorance or concession is about varies with the position of focus associate: in (11a) the conveyed speaker ignorance or concession is about ***who Liubei invited***, while in (11b) it is about ***what Liubei did***.

(11) Chinese

a. Liubei **zuishao/ zhishao/ qima** yaoqing-le [Bill]$_F$. **EPI**, **CON**

Liubei at least/ at least/ at minimum invite-ASP Bill

'Liubei at least invited Bill.'

[6] In English, focused constituents usually bear prosodic prominence. One may wonder whether Chinese focused constituents bear similar prosodic realizations as English. Although I did not conduct any phonetic experiment, the focus effect in Chinese can be clearly observed in the question-answer congruence between focused constituents and their discourse questions. I leave for future study issues concerning how Chinese focused constituents are phonetically realized.

b. Liubei **zuishao/ zhishao/ qima** [yaoqing-le Bill]$_F$. **EPI**, **CON**

Liubei at least/ at least/ at minimum invite-ASP Bill

'Liubei at least invited Bill.'

The second common property of EPI and CON is that both meanings are compatible with various scales. This is evidenced by sentences from both English and Chinese below.[7, 8]

(12) English

a. **Numeral Scales** (a contextual ranking: 4 ≻3 ≻2)

John **at least** wrote [three]$_F$ novels. **EPI**, **CON**

b. **Plurality Scales** (a contextual ranking: adam⊕bill⊕chris ≻adam⊕bill ≻adam)

John **at least** hired [Adam and Bill]$_F$. **EPI**, **CON**

c. **Lexical Scales** (a contextual ranking: gold medal≻silver medal≻bronze medal)

John **at least** got a [silver]$_F$ medal. **EPI**, **CON**

d. **Pragmatic Scales** (a contextual ranking: cherries ≻apples ≻bananas)

John **at least** bought [apples]$_F$. **EPI**, **CON**

(13) Chinese

a. **Numeral Scales**

Liubei **zhishao** xie-le [san]$_F$-ben-xiaoshuo. **EPI**, **CON**

Liubei at least write-ASP three-CL-novel

'Liubei at least wrote three novels.'

---

7 To save space and avoid repetition, in what follows, I use *zhishao* 'at least' as the representative item in the relevant Chinese examples for demonstration. The arguments established in this paper do not depend on this choice and they are in principle applicable to the other two items: *zuishao* 'at least' and *qima* 'at minimum'. A detailed discussion of Chinese data is presented in Chapter 4.

8 The fact that EPI is compatible with various scales is not new. See for example, Coppock and Brochhagen (2013) and Mendia (2016) for discussion. The novel observation here is that CON is also compatible with various scales. That is, besides their focus-sensitivity, another parallel holds between the two meanings.

b. **Plurality Scales**

Liubei **zhishao** guyong-le [Adam he Bill]$_F$. **EPI**, **CON**

Liubei at least hire-ASP Adam and Bill

'Liubei at least hired Adam and Bill.'

c. **Lexical Scales**

Liubei **zhishao** na-le [yin]$_F$-pai. **EPI**, **CON**

Liubei at least take-ASP silver-medal

'Liubei at least got a silver medal.'

d. **Pragmatic Scales**

Liubei **zhishao** mai-le [pingguo]$_F$. **EPI**, **CON**

Liubei at least buy-ASP apple

'Liubei at least bought apples.'

Two remarks are in order. First, the sentences in (12) and (13) are all well-formed and built on different types of scales. Notably, the numerical scale and the plurality scale are based on semantic strength (i.e., an entailment relation), while the lexical scale and the pragmatic scale are based on pragmatic strength (i.e., a non-entailment relation). Second, the sentences in (12) and (13) are all ambiguous between the two meanings. Taken together, the English and Chinese facts above indicate that both meanings are compatible with various scales.

Moreover, a related observation here is that there is a discrepancy between scales based on semantic strength vs. pragmatic strength. By manipulating the context, it is easy enough to reverse the ordering between the alternatives in the case of pragmatic scales or lexical scales.[9] In contrast, however, it does not seem possible to reverse the

[9] For lexical scale, although context manipulations are not impossible, they are indeed harder because the ordering is based on our common world knowledge. Instances of lexical scale are such as *gold medal* $\succ$ *silver medal* $\succ$ *bronze medal*, and *full professor* $\succ$ *associate professor* $\succ$ *assistant professor*.

ordering in those scales based on semantic strength, such as numerical scales or plurality scales, even with some contextual effort.

(14) Context: Adam, Bill and John are planning to buy some fruit for their party tonight. There are three types of fruit available to them: cherries, apples and bananas. However, they are poor and do not have enough money to buy everything. For them, bananas are the optimal because they are the cheapest; apples are less optimal but acceptable because they are still cheaper than cherries.

The contextual ranking (in terms of price): bananas ≻apples ≻cherries

(15) Context: John is planning to hire some people. There are three applicants in the discourse: Adam, Bill and Chris. But the budget is limited. If three people are all hired, John needs to pay a great amount of money for their salary. If only two people (say, Adam and Bill) are hired, the situation is better, but John still pays more than he does in hiring just one person. The best situation for John is simply to hire only one person (say, Adam) while getting all the work done.

The **intended** contextual ranking:
only adam ≻only adam&bill ≻only adam&bill&chris

Under the context (14), the utterance with *at least* in (12d) is understood to convey that John bought apples or bananas (given the contextual ranking: bananas ≻apples

---

One possible scenario reversing the ordering between the three types of medals is provided below: There is a race competition. People who win a gold medal will not get any prize. People who win a silver medal will get just a small amount of prize. However, people who win a bronze medal will get a great amount of prize. Of course, people who do not win any medal will get nothing. Ali is poor but his sister is sick. He needs money to pay the medal expense. In Ali's case, the best situation for him is to win the bronze medal. A situation that is acceptable while less optimal is to win the silver medal. The worst situation for him is to win the gold medal because he won't get any money. Now, given this scenario, the ordering is reversed: *bronze medal ≻silver medal ≻ gold medal*, as shown below.

(i) CON: Although Ali did not win the bronze medal; he **at least** won the silver medal.
EPI: Ali **at least** won a silver medal; so in any case, he will get some money to pay for his sister.

$\succ$cherries). This means that the original ranking (cherries $\succ$apples $\succ$bananas) in (12d) is now **reversed**. In contrast, the utterance with *at least* in (12b) cannot be understood to be that John hired only Adam and Bill, or hired only Adam, even with the contextual massage in (15). This indicates that the original ranking (adam&bill&chris $\succ$adam&bill $\succ$adam) in (12b) **cannot** be reversed. The same observation applies to numeral scales. I leave it for readers to verify the case of numerical scales.[10]

Finally, besides the two common properties above; another novel observation is that both EPI and CON demonstrate two scalar effects: the top-of-the-scale effect (TSE) and the bottom-of-the-scale effect (BSE). This is the third common property of the two meanings. More specifically, TSE demands that the associate cannot be the element at the top of the scale while BSE requires that the associate cannot be the element at the bottom of the scale. Consider the scenario (16) below. Note that (20a) is intended to convey ignorance about what number Chris has got (EPI), and (20b) is intended to convey concession about the number Chris has got (CON).

(16) Scenario: Adam, Bill and Chris are playing dice. In each round, whoever gets a bigger number wins; scores are not cumulated. A dice has six numbers on it: ***Six*** is the upper bound and ***one*** the lower bound on the possible results. Chris threw the dice but Adam missed the result. During his turn, Adam asks about the result.

(17) Adam: What number did Chris get?

a. Bill: I don't know…

b. Bill: Although he didn't get 2 or any number bigger than 2…

(18) #Chris **at least** got [one]$_F$. [**Bottom-of-the-Scale Effect** (**BSE**)]

[10] This discrepancy between scales also holds in Chinese. A detailed discussion of Chinese data is deferred until Chapter 4, where I take Chinese SMs as a case study and present a detailed decompositional analysis of SMs.

Given the scenario, (18) is infelicitous as a continuation to either utterance (17a) or (17b). Intuitively, the infelicity arises because (18) is contextually uninformative: it is already in the common ground that the number *one* is the lower bound on the six possible results and it is also known that Chris has rolled the dice and got a number.

Crucially, in the same scenario(16), (19) is also deviant.[11] In contrast to (18), my informants consider the sentence (19) plainly unassertable in the given scenario. Such contrast raises many questions concerning the two scalar effects: Why should such contrast between (18) and (19) exist? What exactly is the nature of the two scalar effects? How are the two scalar effects connected to the semantics of *at least*? Let's keep these puzzles in mind and assure ourselves that the two scalar effects do not arise from any lexical idiosyncrasy of English *at least*.

(19) #Chris **at least** got [six]$_F$. [**Top-of-the-Scale Effect** (**TSE**)]

Interestingly, BSE and TSE are both attested in Chinese as well. Under the same scenario (16), neither of Bill's utterances in (21) is felicitous as an answer to Adam's question, regardless of whether EPI or CON is intended.

(20)	Adam:	Chris	shai-le	shenme	shuzi?
		Chris	dice-ASP	what	number
		'What number did Chris get?'			

(21) a.	#Chris	**zhishao**	shai-le	[yi]$_F$.	[**Bottom-of-the-Scale Effect (BSE)**]
	Chris	at least	dice-ASP	one	
	'Chris at least got one.'				

[11] As we will see later, I argue that the two scalar effects BSE and TSE are of different nature. In the case of *at least*, BSE arises from discourse uninformativity while TSE semantic vacuity. Specifically, TSE is ruled out as a consequence of a general ban on vacuous quantification in natural language. Therefore, only BSE can be *pragmatically repaired*, when the speaker intentionally flouts the maxim of quantity. I will return to this point in section 2.4.3.

b. #Chris **zhishao** shai-le [liu]$_F$. [**Top-of-the-Scale Effect (TSE)**]

Chris at least dice-ASP six

'Chris at least got six.'

The above English and Chinese facts together indicate that the two scalar effects BSE and TSE are intrinsically connected to the two meanings of SMs: they are not language-specific and do not arise from lexical idiosyncrasy.

To sum up, in this section, we have seen three common properties of the two meanings: the focus-sensitivity, the compatibility with various scales and the discrepancy between scales based on semantic strength vs. pragmatic strength, the two scalar effects BSE and TSE. The next section discusses N&R's two observations where EPI and CON crucially differ: **(a)** the "settle-for-less" flavor; **(b)** the distribution of the two meanings.

2.1.3 Two differences between EPI and CON

One important observation by N&R is that CON conveys a "settle-for-less" flavor while EPI can be neutral. (22) is borrowed from N&R (2009: slide 9).[12]

(22) EPI: Phelps won **at least** eight gold medals. Neutral flavor

CON: **At least** Phelps won eight gold medals. A "settle-for-less" flavor

Under CON, winning eight gold medals falls short of an intended goal or standard. In contrast, it is neutral under EPI.

Another important observation by N&R (2009: slide 6) is that in English, the two meanings can be syntactically distinguished based on the distribution of *at least*.

[12] In this section, I focus on English *at least* but the empirical patterns of EPI and CON discussed here hold for Chinese *zhishao* and potentially for their cross-linguistic counterparts in section 2.1.1.

Notably, CON is missing when *at least* appears at the prenominal position, while EPI is missing when *at least* appears at the sentence-initial position, as illustrated below.[13]

(23) a. Mary won **at least** a [silver]$_F$ medal. √**EPI**, **#CON**

b. Mary **at least** won a [silver]$_F$ medal. √**EPI**, √**CON**

c. **At least** Mary won a [silver]$_F$ medal. **#EPI**, √**CON**

Crucially, the cross-linguistic facts again indicate that the semantic-syntactic correlation is not due to any language-specific property of English or lexical idiosyncrasy of *at least*.[14]

(24) Chinese

a. Liubei xie-le **zhishao** [san]$_F$-ben-xiaoshuo. √**EPI**, **#CON**
Liubei write-ASP at least three-CL-novel
'Liubei wrote at least three novels.'

b. Liubei **zhishao** xie-le [san]$_F$-ben-xiaoshuo. √**EPI**, √**CON**
Liubei at least write-ASP three-CL-novel
'Liubei at least wrote three novels.'

c. **Zhishao** Liubei xie-le [san]$_F$-ben-xiaoshuo. **#EPI**, √**CON**
At least Liubei write-ASP three-CL-novel
'At least Liubei wrote three novels.'

The fact that the distributional restriction is cross-linguistically attested strongly

[13] N&R also discusses the post-verbal position in English. As a mirror image of the pre-verbal case, the sentence with *at least* appearing post-verbally is ambiguous between EPI and CON: *Mary won a* [*silver*]$_F$ *medal at least*. However, Chinese SMs are ill-formed in the post-verbal position, because Chinese generally disallows post-verbal adjuncts. In this dissertation, I focus on the three syntactic positions shared by English and Chinese SMs for my illustrations: the prenominal position, the pre-verbal position and the sentence-initial position.

[14] For reasons of space, I use only Chinese for illustrations. Beyond English and Chinese, similar distributional restriction seems attested in other languages (see Grosz 2011). Please see chapter 5 (section 5.4) for discussion of the distributional restriction from a cross-linguistic perspective.

suggests that it is not simply be a difference at the level of the lexicon; instead, it is intrinsically related to the mapping between syntax and semantics-pragmatics of the two meanings. It is worth noting that any purely pragmatic account would leave the observed distributional restriction unexplained. The reason is that we could always create a context where all the pragmatic requirements of CON are fulfilled, even when English *at least* (or Chinese *zhishao*) appears at the prenominal position. Put differently, a purely pragmatic account predicts that CON should be available whenever the relevant pragmatic requirements are satisfied and thus **insensitive** to the syntactic distribution of SMs. But this is not what we have seen in English or Chinese.

At this point, it is worth noting that Biezma (2013) proposes a purely pragmatic account and argues that CON is available for the prenominal *at least*.

(25) The track and field coaches are looking at the statistics and discussing the results of the last competition.

Coach 1: The competition was awful.

Coach 2: Yes, but Mary won **at least** that gold medal [pointing at the data in the statistics]

However, native speakers that I have consulted seem to disagree with Biezma's example. My English consultants find the prnominal *at least* under CON degraded in the example above. Given these considerations and the fact that similar distributional restrictions are also observed in Chinese, in this dissertation I stick to N&R's observation: CON is not available when SMs appear at the prenominal position.

To sum up, in this section 2.1, we have seen that the EPI-CON ambiguity is cross-linguistically pervasive. This suggests that the ambiguity cannot simply be a lexical homophony. Moreover, we have also seen that the two meanings share three common properties: the focus-sensitivity, the compatibility with various scales and the discrepancy between scales based on semantic strength vs. pragmatic strength, and two scalar effects BSE and TSE. Finally, the two meanings crucially differ in two respects: **(a)** CON has a "settle-for-less" flavor while EPI is neutral; **(b)** the (un)availability of the two meanings is sensitive to the syntactic distribution of SMs.

The next section looks at the pragmatics of the two meanings and presents empirical data showing that **(a)** the ignorance inference given by SMs is pragmatic, not semantic; **(b)** while under EPI the relevant higher alternatives are left open to be true or not, they must be contextually known to be false under CON. These two pieces are established as the first step toward my analysis of the EPI-CON ambiguity, where a uniform semantic representation of SMs is maintained while the two meanings are rendered as pragmatic variants (e.g., whether the relevant higher alternatives are left open or known to be false in a given discourse).

2.2 The pragmatic source of the ambiguity

It has been proposed in the literature that the ignorance inference conveyed by English *at least* is **pragmatic** in nature (e.g., Büring 2008, Cummins and Katsos 2010, Coppock and Brochhagen 2013, Mayr 2013, Westera and Brasoveanu 2014, Kennedy 2015, Schwarz 2016a, Mendia 2016a, Buccola and Haida 2017).[15] In what follows, I first briefly review some empirical data supporting the pragmatic approach, and then offer one novel piece of evidence for the pragmatic nature of EPI.[16] Finally, I present two core ingredients in the unified analysis proposed by Biezma (2013): **(a)** when

[15] In addition to the debate on whether ignorance inferences are part of conventional meaning or the result of certain pragmatic process, a number of different proposals have been put forth in the literature. For example, SMs have been analyzed as modals (Geurts & Nouwen 2007), as minima and maxima operators (Nouwen 2010), as inquisitive expressions (Coppock & Brochhagen 2013), as meta-speech act operators (Cohen & Krifka 2014) and as epistemic indefinites (Nouwen 2015).

[16] Readers are referred to Coppock and Brochhagen (2013: section 4) for a detailed review and an extensive discussion on different proposals of SMs in previous studies.

constructing the set of alternatives, the speaker takes into consideration the discourse, in particular, the speaker's goals and the interlocutors' interests in a given discourse; **(b)** EPI and CON are pragmatic variants; CON arises when the relevant higher alternatives are contextually known to be false.

To begin with, Mendia (2016a) provides three pieces of evidence that the ignorance inference conveyed by English *at least* should be pragmatic (rather than semantic): **(a)** it can be cancelled; **(b)** it can be reinforced; **(c)** it disappears when the maxim of quantity is deactivated in the context (this third observation is built on the discussion of disjunction in Fox 2014). The examples below are borrowed from Mendia (2016a: (4) - (6)).

(26) **Cancellability**

Context: Bill has four kids. Yesterday he saw a sign at a supermarket: "Discounts for parents. To qualify you must have at least three kids." Bill reasoned as follows.

I qualify: I have four kids, so I do have **at least** three kids.[17]

---

[17] There are cases where ignorance inferences associated with *at least* seem difficult to cancel.

Context: In order to qualify for a tax rebate, one needs to have at least three kids.

(i) Q: Do you have three kids?
A: ??Yes, I have at least three kids.

(ii) Q: How many kids do you have?
A: #I have at least three.

Similar facts are observed with disjunction.

(iii) Q: How many kids do you have?
A: #I have three or more.

Crucially, the general consensus seems to be that ignorance inferences associated with disjunction are pragmatic, despite examples like (iii). At this point, my response to examples like (i) and (ii) is that the answer to *whether* ignorance inferences with *at least* are cancellable is different from the answer to *when* ignorance inferences with *at least* can be cancelled. Put differently, in some cases, there may be some factors making them difficult to cancell; in this respect, whether the subject is the first person is presumably an important factor, given that the speaker is usually considered to be competent and know how many kids they have. A related difference between Media's example and (i)/ (ii) is that the latter is a cross-speaker conversation, while the former is more like a monologue. Taken together, I believe that

(27) **Reinforceability**

Bill has **at least** three kids, I don't know how many exactly.

(28) Context: In a game, my friend has to guess the number of marbles that I have hidden. <u>I know how many I have hidden and she knows that I have that information</u>. I provide the following clue:

I have **at least** five marbles.

~> no ignorance about the number of marbles that I have

(26) and (27) show that the ignorance inference is cancellable and reinforceable, which are the two hallmarks of conversational implicatures (Grice 1989). (28) shows that when the maxim of quantity is deactivated in the context, the ignorance inference does not arise. This, again, confirms the pragmatic nature of the ignorance inference.

Another piece of evidence comes from Westera and Brasoveanu (2014)'s observation that whether the ignorance inference arises depends primarily on whether a precise answer is requested or not (i.e., the question-under-discussions (QUDs) in the sense of Roberts 1996/ 2012, see also Büring 2003, Beaver and Clark 2008).

(29) Ignorance Inference

A: Exactly how many students took Experimental Pragmatics?

B: **At least** ten students took Experimental Pragmatics.

(30) No Ignorance Inference

A: Did at least ten students take Experimental Pragmatics?

B: **At least** ten students took Experimental Pragmatics.

The *wh*-question in (29) requires a precise answer about the number of students who

---

Media's evidence on cancellability is compatible with examples like (i) and (ii).

took the course while the polar question in (30) does not. Therefore, the ignorance inference arises in the former but not in the latter. Some acute readers may wonder whether the absence of the ignorance inference in (30) is somehow due to the repetition of *at least* (in the question and the answer). Although the original examples discussed in Westera and Brasoveanu (2014) do raise this concern, I show below that the lack of ignorance inference does not rely on the repetition of *at least*.

(31) No Ignorance Inference

Adam: Do you know whether Experimental Pragmatics is offered this semester?

Bill: Yes, **at least** ten students have signed up for Experimental Pragmatics. **In fact/ To be precise**, 13 students have signed up for the course.

The dialogue in (31) is felicitous and can be justified in the following scenario. Adam is interested in auditing Experimental Pragmatics, but he does not know how many students have signed up for the course. Bill knows that it is 13 students that have signed up for experimental pragmatics. It is known that a course is offered in a semester only when (at least) ten students registered for it. Thus, while Adam's question amounts to asking <u>whether (at least) ten students sign up for experimental pragmatics</u>, it does not involve *at least* in the form of his question.[18] Taken together, (30) and (31) show that the lack of ignorance inference does not rely on the repetition of *at least*.

In short, what (31) shows is that the ignorance inference is absent, as evidenced by the *in fact/ to be precise* phrase, even though *at least* is missing in Adam's question.

[18] Notice that for Adam's purposes, whether there are exactly ten students registering for the course is not relevant, given that the course is still offered even if only ten students registered for it. As we will see in this chapter, the calculation of ignorance inferences associated with *at least* requires a set of exhaustified alternatives. In light of this, ignorance inferences are expected not to arise when the set of exhaustified alternatives is not present/ relevant in the given discourse, as in this scenario where Adam's question is raised.

This means that the repetition of *at least* is not necessary for the absence of ignorance inference. The above scenario is provided simply to make the number *ten* **relevant** for Adam to felicitously initiate his question in the first place. Crucially, Adam's question is not asking for exactly how many students taking the course; for Adam's purpose, any number *n* above the threshold ten would suffice.[19]

Building on Westera and Brasoveanu's insight on the connection between QUDs and the presence of ignorance inference, I provide one novel piece of evidence. The observation is that the ignorance inference conveyed by English *at least* is justified in (32) when a *wh*-question is interpreted exhaustively (i.e., a precise answer is requested), while it is not justified in (33) when a *wh*-question is interpreted non-exhaustively (as indicated by the partiality marker: *for example*).[20, 21]

---

[19] Here I use bare numerals for my illustration. The same argument can be replicated for modified numerals like *more than ten*: the absence of ignorance inference does not rely on the repetition of *at least* in the question.

(i) No ignorance inference
  Adam: Did more than ten students pass the midterm?
  Bill: Yes, at least ten students passed the midterm. In fact/ To be precise, thirteen students passed the midterm.

[20] Building on Fox (2012)'s idea that mention-some readings are actually situations where multiple maximally informative answers are allowed, Xiang (2016) proposes a unified account of mention-some and mention-all questions while excluding examples like (33) from her study of mention-some questions for purposes not relevant here. In this dissertation, I remain neutral on whether the observed contrast is due to the distinction between mention-some and mention-all questions.

[21] Veneeta Dayal (p.c.) suggests that the phrase *for-example* should not be a marker for mention-some questions, because it is compatible with mention-all questions marked by the quantifier *all*.

(i) Who all did John invite?
(ii) Who all did John invite, for example?

Because of the presence of *all*, (i) is understood as a mention-all question requesting for the information about all the individuals John has invited. Now, if the phrase *for-example* is a marker for mention-some questions, it should be incompatible with a mention-all question containing the quantifier *all*, contrary to the fact (see (ii)). In this dissertation, I am neutral to whether the phrase *for-example* is a marker for mention-some questions. Having said that, Veneeta Dayal (p.c.) also points out that the ignorance inference given by *at least*, intriguingly, is similarly not justified in a mention-some question like (iii).

(iii) Context: There are three individuals Adam, Bill and Chris. Any one of them can sign the paper.
  a. Who can sign the paper?
  b. #At least Bill.

(32) A: Who did John invite?

B: John invited **at least** [Adam and Bill]$_F$. Felicitous: Ignorance Inference

(33) A: Who did John invite, **for example**?

B: #John invited **at least** [Adam and Bill]$_F$. Infelicitous: Ignorance Inference

Suppose that there are three individuals relevant in the discourse: Adam, Bill and Chris. The ignorance inference on <u>whether John invited Adam, Bill and Chris</u> is justified in (32) because the *wh*-question is requesting for a precise answer on the individuals that John invited and the use of *at least* signals the failure of providing the maximally informative unique answer. In contrast, the infelicity of (33) intuitively comes from the fact that the speaker B's response is **over-informative**. Informally put, any non-exhaustive answer would suffice, but the speaker B is trying to signal that there is one maximally informative unique answer and he fails to provide that particular answer. Thus, the ignorance inference is not justified in (33).[22]

Given our discussion above, a generalization regarding EPI suggests itself:

(34) **Informativity and Speaker Ignorance**

a. Ignorance inferences arise in responses to *wh*-questions but not (necessarily) to polar questions.

b. Ignorance inferences are justified when *wh*-questions are interpreted exhaustively, but not when they are interpreted non-exhaustively.

I take (34) to indicate that English *at least* under EPI addresses the issue of informativity: ignorance inferences arise to justify the failure of providing the maximally informative unique answer. As we will see in a moment, in contrast, English *at least* under CON addresses a different issue: the issue of evaluativity.

[22] The contrast here suggests that ignorance inferences have a non-trivial relation with the exhaustivity of *wh*-questions. I will return to this connection in section 2.5.

Now, let's shift our attention to two core ingredients in Biezma (2013)'s analysis and reconsider the "settle-for-less" flavor given by concessive *at least*. The first ingredient of Biezma's analysis is that when constructing the set of alternatives, the speaker takes into consideration the speaker's goals and the interlocutors' interests in a given discourse. (35) is adapted from Biezma (2013: (17)).

(35) Tom dated with someone he met online. He got home to find his friend Jim.

Jim: How was your date?

Tom: It was ok, **at least** she was smart.

As Biezma puts it: "In principle, out of context, there are many ways Tom could answer Jim's questions. Certainly, Jim is considering alternatives that would present an **evaluation** of Tom's date regarding whether she was good-looking, funny or smart (for example). Understanding the question amounts to understanding the goals of the speaker when asking the question and hence identifying what are the possible answers…". More specifically, in giving an answer evaluating his date, Tom actually identifies some possible aspects that he infers Jim is interested in.

The second ingredient of Biezma's analysis is that CON arises when the relevant higher alternatives are known to be false in the discourse. This is a crucial piece for understanding why CON is a pragmatic variant of EPI. Putting the two ingredients together, under CON, Tom's utterance indicates that with *all things* considered, his date was ok (although his date could have been better), given that his date could have been worse. This gives the "settle-for-less" flavor observed in N&R. For expository purposes, let's assume that the possible evaluative aspects are ordered as in (36).

(36) Great: She was tall, smart and beautiful

Good: She was tall and smart, or She was smart and beautiful, or She was tall and beautiful

Ok: She was tall, or She was smart, or She was beautiful

Bad: She was not tall and She was not smart and She was not beautiful

Building on Biezma's insight, I add a novel observation to the second pragmatic ingredient: the information that the relevant higher alternatives are false may not reside in the common ground, while it must be known to the speaker.

(37) Emily: What medal did John get? Did he win a gold medal?

Frank: No, but **at least** he won a $[\text{silver}]_F$ medal.

(38) Emily: What medal did John get? Did he win a silver medal?

Frank: Yes, **at least** he won a $[\text{silver}]_F$ medal.

Assuming that the set of focus alternatives is {John won a gold medal, John won silver medal, John won a bronze medal}, the polar question in (37) targets the higher alternative (i.e., John won a gold medal), while the polar question in (38) targets the prejacent (i.e., John won a silver medal). That is, the information that John didn't win a gold medal is an at-issue content in (37) but a non-at-issue content in (38). In the latter case, given Frank's use of *at least*, Emily may or may not need to accommodate the information that John didn't win a gold medal, depending on the context.[23] This

[23] von Fintel (2004) discusses a "hey, wait a minute" test that targets the presupposition but not the entailment (cf. Shannon 1976), as in (i). If we apply the test, as shown in (ii), we see that the information that the relevant higher alternative is false (John didn't invite all the three persons) can behave like a presupposition. I will return to this point in chapter 3, where a formal analysis of the discourse profile of concessive *at least* and epistemic *at least* is presented.

(i) Mary's boyfriend is visiting today.
a. #Hey, wait a minute. I didn't know Mary's boyfriend is visiting today.
b. Hey, wait a minute. I didn't know Mary has a boyfriend.

(ii) Context: there are three individuals relevant for John's invitation: Adam, Bill and Chris.
Emily: Who did John invite for the party? Did John invite Adam and Bill?
Frank: Yes, **at least** he invited a $[\text{Adam and Bill}]_F$.
Emily: Hey, wait a minute. John didn't invite all of them/ Chris?

It is worth emphasizing that although the falsity of the relevant higher alternatives is one crucial shows that the falsity of the relevant higher alternatives may not be in the common ground of the interlocutors, at the moment when the speaker uses concessive *at least*.

Furthermore, when the speaker knows that the relevant higher alternatives are not necessarily false in the context, the use of *at least* under CON is infelicitous. For example, (39) is deviant in expressing Frank's concession on the medal that John won. This strengthens Biezma's first pragmatic ingredient into a case of necessity: CON (speaker concession) arises only when the speaker knows that the relevant higher alternatives are false in a given discourse.

(39) Emily: What medal did John get? Did he win a gold medal?

Frank: #Maybe, and/ but **at least** he won a [silver]$_F$ medal.

Taken together, building on Biezma (2013)'s insights, I consider that English *at least* under CON addresses the issue of evaluativity: the set of answers is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse.

(40) **Evaluativity and Speaker Concession**

a. The "settle-for-less" flavor arises when **(i)** the set of answers is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse; and **(ii)** the relevant higher alternatives are known to be false.

b. The "settle-for-less" flavor conveys speaker concession: given the evaluation, the asserted content is true and although it is not the best situation, it is not the worst situation either.

To sum up, we have seen that the ignorance inference given by *at least* is

---

requirement of the concessive meaning, it need not be encoded in the lexical semantics of *at least*. For one thing, we have seen in (37) and (38) that the requirement can be fulfilled in terms of assertion or presupposition. For another, hard-wiring such a requirement into the lexical meaning of *at least* would inevitably force us to accept that the two meanings of *at least* are a lexical coincidence, thus leaving the EPI-CON ambiguity and the observed cross-linguistic facts unexplained.

pragmatic rather than semantic in its nature because **(a)** it can be cancelled and reinforced; **(b)** it disappears in a discourse where the maxim of quantity is deactivated; **(c)** it arises to justify the failure of providing the maximally informative unique answer when a *wh*-question is interpreted exhaustively. I take these facts to indicate that the pragmatics of EPI addresses the issue of informativity (see (34)). Furthermore, we have also seen that the "settle-for-less" flavor of CON comes from the fact that the set of answers is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse. I take this fact to indicate that the pragmatics of CON addresses the issue of evaluativity (see (40)). Finally, let's summarize the main facts that any linguistic theory of SMs must explain:

(41) a. **The EPI-CON ambiguity**: Cross-linguistically, SMs in general demonstrate an ambiguity in giving an ignorance inference and a concessive inference.

b. **Focus-sensitivity**: the semantic contribution of SMs under both meanings (EPI and CON) depends on the position of their focus associate.

c. **Scale types and their discrepancy**: SMs under both meanings (EPI and CON) are compatible with various scales (based on semantic strength or pragmatic strength). However, in contrast to lexical scales and pragmatic scales, the ordering between focus alternatives in numerical scales and plurality scales cannot be reversed.

d. **Two scalar effects (TSE and BSE) and their discrepancy**: SMs under both meanings (EPI and CON) demonstrate two scalar effects, but there is a contrast in the type of infelicity that arises. For English *at least* (and Chinese *zhishao*, etc), BSE may be pragmatically repaired while TSE may not.

2.3 Previous studies of SMs concerning EPI and CON

Sections 2.1 and 2.2 have introduced the empirical facts concerning the ambiguity. This section looks at the formal details of previous studies concerning the ambiguity and the semantics of *at least*. Section 2.3.1 reviews N&R's two lexical entries for *at least* and Biezma (2013)'s unified entry. Section 2.3.2 discusses two dimensions along which the previous analyses of epistemic *at least* vary: **(a)** what kind of scales SMs are thought to operate on, and **(b)** how the ignorance inference is derived.

2.3.1 Two views on the EPI-CON ambiguity

To begin with, recall that N&R proposes a non-uniform account of English *at least*. That is, the English item *at least* happens to have two independent lexical entries: one for EPI and the other for CON. In the case of EPI, N&R (2009: slide 16) adopts Geurts and Nouwen (2007)'s view that the speaker ignorance conveyed by English *at least* stems from a covert epistemic modal hidden in its semantics.[24, 25]

(42) Epistemic *at least*

a. Truth conditions

$\exists q \in C\ [q \geq p \wedge q(w) = 1]$

[24] Although both studies assume a covert epistemic modal in the semantics of English *at least*, N&R crucially differs from Geurts and Nouwen (2007) in that the former considers it as a conventional implicature, while the latter takes it to be part of the truth-conditions. Note that both proposals fail to explain why the ignorance inference given by *at least* behaves like a conversational implicature, as shown in section 2.1.

[25] Kay (1992: 311) distinguishes three uses of *at least* in English: a scalar use, an evaluative use and a rhetorical use. The three uses of *at least* are illustrated in (i).

(i)	a. Mary received calls from **at least** three soldiers.	Scalar
	b. **At least**, this one is cooked.	Evaluative
	c. I see her every day, **at least** when I'm in town.	Rhetorical

His scalar use corresponds to N&R's EPI and his evaluative use corresponds to N&R's CON. This dissertation is concerned with the first two uses of *at least* (and its cross-linguistic counterparts): EPI and CON. But I am confident that the analysis to be proposed can be extended to the rhetorical use.

"There is a proposition q which ranks higher than or as high as the prejacent p, and which is true"

b. Conventional implicature

$\exists w'[\text{Epist}(w, w') \wedge \exists q \in C\ [q > p \wedge q(w') = 1]$

"It is epistemically possible that some proposition q that ranks higher than p is true"

For CON, N&R (2009: slide 18) assigns a different lexical entry to English *at least*.

(43) Concessive *at least*

a. Truth conditions

$p(w) = 1$

"The prejacent proposition p is true"

b. Conventional implicatures

i. $\forall r, r' \in C\ [r' > r \leftrightarrow r'$ is preferred to $r]$

"The scalar ranking reflects a preference ranking"

ii. $\exists q \in C\ [q > p]$

"There is a proposition q that ranks higher than p"

iii. $\exists q \in C\ [q < p]$

"There is a proposition q that ranks lower than p"

It is worth emphasizing that given N&R's proposal, there is **no** intrinsic relation between EPI and CON. This lack of intrinsic connection between the two meanings makes it difficult to explain many empirical facts we have seen: **(a)** the EPI-CON ambiguity is cross-linguistically pervasive; **(b)** more than one lexical item is ambiguous between the two meanings in one single language such as Chinese; **(c)** both meanings share the three common properties, in particular, the two scalar effects and their discrepancy. To anticipate, the table in (44) illustrates a comparison between N&R's proposal and mine in this dissertation, with respect to the two scalar effects.

(44) The predicted nature of TSE and BSE

	N&R's proposal		My proposal	
	EPI	CON	EPI	CON
TSE	!	!	#	#
BSE	Δ	**!**	Δ	Δ

# indicates semantic vacuity, Δ discourse uninformaitvity, and ! the failure of conventional implicature

Crucially, N&R's proposal predicts that EPI (see 42b), but not CON (see 43bii-iii), would demonstrate a discrepancy between BSE and TSE. However, by contrast, I suggest that both meanings should demonstrate a discrepancy between the two scalar effects. As we will see in section 2.4, the two scalar effects are of different nature and only BSE (crucially not TSE) can be *pragmatically repaired* by certain conversational strategies, where the speaker intentionally flouts the maxim of quantity. That is, N&R's proposal wrongly predicts that there is **no** discrepancy between TSE and BSE under the concessive meaning.

Under N&R's proposal for the concessive meaning, the relevant higher alternatives are always more preferable (see 43bi). A potential challenge may come from the concern that N&R is not explicit about what it means to be more preferable.

(45) Context: Killing people is a crime more serious than stabbing people.

Adam did not kill Mary, but he **at least** intentionally [stabbed]$_F$ her.

(46) Context: Stealing the jewelry is a crime more serious than stealing the cash.

Adam did not steal the jewelry, but he **at least** stole the [cash]$_F$.

I believe that both utterances can be felicitously used in a context where the speaker is trying to argue and persuade people that Adam has committed a crime and must be held accountable. Now, is killing a person or stealing the jewelry more preferable? N&R does not explicitly say what it means for a higher alternative to be more preferable than the prejacent. In N&R's original examples concerning sports competition, given our common world knowledge, winning a gold medal is (straightforwardly) more preferable than winning a silver medal. Thus, the answer can be no, if we are simply considering our common world knowledge. However, if the set of relevant alternatives is evaluated and ranked against the speaker's goals and the interlocutors' interests in a particular type of discourse, the answer would be yes. Taken together, (45) and (46) indicate that whether a given focus alternative is more preferable (i.e., higher ranked than the prejacent) should be decided with reference to the speaker's goals and the interlocutors' interests in a given discourse, rather than on the basis of the lexicon or the world knowledge.

In contrast to N&R, Biezma (2013) argues that the two meanings of *at least* can actually be unified and thus proposes a single lexical entry as shown below.[26]

(47) Let p be a proposition, and $[p]_{A,i}$ the set of alternatives of p ordered according to $\leq_i$, where $\leq_i$ is a contextually salient order of alternatives and $\forall \pi \in [p]_{A,i}$, $\pi \in$QUD:

$$[\![\textit{at least } p]\!] = \lambda w.\exists q, r \in [p]_{A,<i}, s.t.\ r <_i p <_i q \ \&\ [p(w) \vee q(w)]\ \&$$
$$\forall s \in [p]_{A,i}, s <_i p[\neg s(w) \vee p \text{ entails } s]$$

Crucially, under Biezma's proposal (contra N&R), there **is** an intrinsic relation

[26] In fact, Biezma (2013) provides two unified lexical entries for *at least* (to take care of the fact that it can occur in different syntactic positions): one is propositional and the other is non-propositional. Only the propositional version is illustrated here. Readers are referred to her paper for details.

between EPI and CON: whether the relevant higher alternatives are left open or known to be false in a given discourse. Biezma captures this intrinsic connection between the two meanings by encoding a disjunction into the semantics of *at least* (i.e., $p(w) \vee q(w)$).[27] Biezma's disjunctive treatment of *at least* raises many non-trivial questions about the relation between SMs and disjunction in general. I will return to this point in the next section. For now, let me point out how Biezma's analysis explains many empirical facts discussed in sections 2.1 and 2.2. First, the EPI-CON ambiguity can be systematically derived from one single semantic entry. Second, the focus-sensitivity of both meanings follows because the unified entry operates on a set of focus alternatives sensitive to different QUDs. Third, the compatibility of different scales under both meanings follows because the relevant ordering is contextually-valued by an assignment function in the unified entry. Fourth, the "settle-for-less" flavor is derived from the two pragmatic ingredients: **(a)** when the set of focus alternatives is evaluated and ranked against the speaker's goals and interlocutors' interests in the discourse, and **(b)** when the relevant higher alternatives are known to be false. Crucially, Biezma's unified entry also derives the fact that the prejacent is entailed under CON (cf. 43a). Given the disjunctive statement made by *at least* (i.e., $p(w) \vee q(w)$)), when the relevant higher alternative is false (i.e., $q(w)$ is false), then the fact that the prejacent p is entailed is derived (i.e., $p(w)$ is true).

Although Biezma's unified analysis is insightful and successfully derives many empirical facts, there remain some empirical challenges. First, it is unclear how the discrepancy between scales based on semantic strength vs. pragmatic strength can be captured, if the relevant ordering is given by an assignment function. Why is it that

[27] To my knowledge, Büring (2008) is the first study suggesting the striking parallel between *at least* and disjunction. Since then, many studies on SMs have followed this path. It is worth noting that although Büring (2008) suggests that *at least* can be understood as a disjunction for purposes of implicature calculation, it does not hard-wire any disjunctive component into the semantics of *at least*.

the relevant ordering in numerical scales and plurality scales cannot be reversed, even with some contextual support? Why is the assignment function sensitive to certain types of scales, but not to contexts? Second, it is unclear how Biezma's analysis explains the two scalar effects BSE and TSE and their discrepancy. Why and how do BSE and TSE arise with both meanings of *at least*? What do the two scalar effects inform us about the semantics of *at least*? Why is there a judgment contrast between BSE and TSE under the two meanings? Notice that under Biezma's analysis, the requirement $r <_i p <_i q$ in (47) may explain why both meanings of *at least* demonstrate TSE and BSE, but it fails to explain the discrepancy between the two scalar effects. In particular, the requirement $r <_i p <_i q$ would predict that both TSE and BSE are *equally infelicitous*, contrary to the facts. Third, it is unclear why the availability of the two meanings should be restricted by the distribution of *at least*.

Given these considerations, I propose an alternative unified analysis while incorporating Biezma's insights in section 2.4. The next section discusses two aspects where the previous analyses of epistemic *at least* vary: **(a)** what kind of scales SMs are thought to operate on, and **(b)** how the ignorance inference is derived.

2.3.2 Two dimensions where previous analyses of *at least* vary

To start with, it is worth mentioning that most of the previous studies on SMs are dedicated to the semantics of epistemic *at least* and/ or to the derivation of the ignorance inference given by *at least*. Various proposals have been put forth in the literature (e.g., Krifka 1999, Geurts and Nouwen 2007, Büring 2008, Nouwen 2010, Cummins and Katsos 2010, Coppock and Brochhagen 2013, Mayr 2013, Cohen and Krifka 2014, Kennedy 2015, Schwarz 2016a, Mendia 2016a-c, among others). For example, SMs have been analyzed as modals (Geurts & Nouwen 2007), as disjunction

(Büring 2008), as minima and maxima operators (Nouwen 2010), as inquisitive expressions (Coppock & Brochhagen 2013), as meta-speech act operators (Cohen & Krifka 2014) and as epistemic indefinites (Nouwen 2015). For reasons of space, I will not be able to review the details of all the analyses here. However, I would like to point out that despite the variety of proposals, they can be generally classified into two approaches, depending on what kind of scales SMs are thought to make reference to: a degree-based approach and a discourse-based approach. The degree-based approach considers SMs as degree operators and invoke a scale of degrees (Nouwen 2010, Kennedy 2015). (48) is taken from Kennedy (2015: (42)).

(48) a. $[\![\textit{at least}]\!] = \lambda m_{<d>}\, \lambda P_{<d,\,t>}.\ max\{n \mid P(n)\} \geq m$

b. $[\![\textit{at most}]\!] = \lambda m_{<d>}\, \lambda P_{<d,\,t>}.\ max\{n \mid P(n)\} \leq m$

In contrast, the discourse-based approach invokes scales of pragmatic strength, which are not restricted to numerals and may not even respect entailment (e.g., Krifka 1999, Geurts and Nouwen 2007, Büring 2008, Coppock and Brochhagen 2013). An example lexical entry following the discourse-based approach is shown in (49).[28]

---

[28] To cover cases where SMs are adjoined to non-propositional constituents, a non-propositional version of the entry can be obtained by type-shifting such as the Geach rule and the backward Geach rule. The Geach rule converts a function f with type $<a, b>$ into a function f' with type $<<c, a>, <c, b>>$ of the form $\lambda R.\lambda x.f(R(x))$, where R is of type $<c, a>$ and x of type $<c>$. See a similar application of the Geach rule in Coppock and Brochhagen (2013) for the lexical entries of English *at least* and in Coppock and Beaver (2014) for the lexical entries of English *only*. Below, π is any type.

(i) The case of *at least*

By the Geach rule (the prejacent = $\alpha(\beta)$)

$[\![\textit{at least}(C)]\!]^w = \lambda\alpha_{<\pi,st>}.\lambda\beta_{<\pi>}.\exists\alpha' [\alpha'\in C \wedge \alpha'(\beta)(w) \wedge \alpha'(\beta) \succeq_i \alpha(\beta)]$

By the backward Geach rule (the prejacent = $\beta(\alpha)$)

$[\![\textit{at least}(C)]\!]^w = \lambda\alpha_{<\pi>}.\lambda\beta_{<\pi,st>}.\exists\alpha' [\alpha'\in C \wedge \beta(\alpha')(w) \wedge \beta(\alpha') \succeq_i \beta(\alpha)]$

(ii) The case of *at most*

By the Geach rule (the prejacent = $\alpha(\beta)$)

$[\![\textit{at most}(C)]\!]^w = \lambda\alpha_{<\pi,st>}.\lambda\beta_{<\pi>}.\forall\alpha' [\alpha'\in C \wedge \alpha'(\beta)(w) \wedge \alpha'(\beta) \preceq_i \alpha(\beta)]$

By the backward Geach rule (the prejacent = $\beta(\alpha)$)

$[\![\textit{at most}(C)]\!]^w = \lambda\alpha_{<\pi>}.\lambda\beta_{<\pi,st>}.\forall\alpha' [\alpha'\in C \wedge \beta(\alpha')(w) \wedge \beta(\alpha') \preceq_i \beta(\alpha)]$

(49) a. $[\![\textit{at least (C)}]\!]^{w;\, g} = \lambda p_{<s,\, t>}.\exists q\ [q{\in}C \wedge q(w) \wedge q \succeq_{\mathrm{i}} p]$

b. $[\![\textit{at most (C)}]\!]^{w;\, g} = \lambda p_{<s,\, t>}.\forall q\ [q{\in}C \wedge q(w) \wedge q \preceq_{\mathrm{i}} p]$

Crucially, a non-strict comparison relation between the prejacent and its focus alternatives is proposed in the semantics of SMs under both approaches.[29] However, if we pay attention to the morphology of SMs, the non-strict comparison relation becomes puzzling. In particular, English SMs (apparently) involve a superlative morpheme and a quantity adjective in their morphology. Crucially, superlatives are traditionally analyzed as involving a strict comparison relation (e.g., Heim 1985, 1999, Farkas and Kiss 2000, Sharvit and Stateva 2002, among others), as shown below.

(50) a. Adam climbed the highest mountain.

b. Relative reading:

$\forall y[y{\in}C \wedge y \neq \text{adam} \rightarrow \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z){\geq}d \wedge$

$\text{adam climbed } z]) > \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z){\geq}d \wedge y \text{ climbed } z])$

(51) a. Adam climbed the least high mountain.

b. Relative reading:

$\forall y[y{\in}C \wedge y \neq \text{adam} \rightarrow \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z){\geq}d \wedge$

$\text{adam climbed } z]) < \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z){\geq}d \wedge y \text{ climbed } z])$

Where does the non-strict comparison relation come from? What is the nature of the non-strict comparison relation? Is it a semantic primitive? Or can it be derived from other semantic components? These questions boil down to a long-standing and intriguing morpho-semantic puzzle posed by SMs: How exactly is the semantics of SMs connected to their degree morphology? Of course, the analyses mentioned above do not tackle the morpho-semantic issue of SMs. However, the fact that the non-strict

[29] A non-strict comparison is a relation that includes the equality relation such as $\succeq$ or $\preceq$. In contrast, a strict comparison is a relation that does NOT include the equality relation such as $\succ$ or $\prec$.

comparison relation is assumed in both approaches indicates that it is a core component in the semantics of SMs. I will return to this point in section 2.4 where a single semantic representation of *at least* is proposed. Crucially, under the proposal, the non-strict comparison is not a semantic primitive; it would be derived from focus presuppositions along with the superlative component in the semantics of *at least*.[30]

Another dimension along which previous analyses of epistemic *at least* vary is how the ignorance inference is derived. Again, the field of approaches is too broad for me to do justice to the details of all the previous proposals. Moreover, the exact mechanism is still an ongoing debate in the literature. Below, I will briefly mention two strategies in the pragmatic camp.[31] One influential strategy suggested by Büring (2008) for generating the ignorance inference is to consider SMs as being akin to disjunction; in the same way that 'Φ or Ψ' conveys that both Φ and Ψ are epistemically accessible options. Thus, *at least n* is interpreted as the disjunction *exactly n* or *more than n*, where *n* is a numeral. Büring's idea is illustrated in (52).

(52) 'at least n P Q' := 'exactly n P Q $\vee$ n' P Q', where $n' > n$ (Büring 2008: (5))

Representatives of this strategy include Mayr (2013), Kennedy (2015), Mendia (2016a) and Schwarz (2016a). In particular, Schwarz (2016a) assumes that the calculation of ignorance inferences given by *at least* requires two sources of

---

[30] In this chapter, I focus on what the uniform semantic representation of *at least* looks like and how the proposed entry combining with different pragmatic factors results in the EPI-CON ambiguity. The morpho-semantic puzzle of how exactly the semantics of SMs are connected to their degree morphology is tackled in chapter 4, where a detailed decompositional analysis is presented.

[31] The semantic camp includes Geurts and Nouwen (2007), N&R and Nouwen (2010). A common assumption of these studies is that the ignorance reading observed with SMs is because of a covert epistemic possibility modal. However, the three studies differ in where and how the covert epistemic possibility modal appears. For example, Geurts and Nouwen (2007) directly puts it in the truth-conditions of *at least* and *at most*; N&R discusses only *at least* and puts it in the conventional implicature of *at least*. Nouwen (2010) relies on a "reinterpretation" process inserting the epistemic possibility modal. However, as we have seen in section 2.2, the ignorance inference given by *at least* is controlled by contextual factors and is not part of the truth-conditions. Therefore, I purse a pragmatic view in this dissertation. Readers are referred to Coppock and Brochhagen (2013: section 4) and Kennedy (2015) for a detailed discussion of some serious empirical challenges to the semantic camp.

alternatives: one source comes from the fact that *at least* and *only* form a Horn-scale, and the other source is the set of alternatives induced by focus. Taken together, the alternatives generated for an expression like *at least three* are *at least three*, *at least four*, *at least five*, etc., and *only three*, *only four*, *only five* etc. The resulting structure of alternatives is *symmetric* and ignorance inferences are generated through Fox (2007)'s idea of innocent exclusion.[32]

Another strategy for generating the ignorance inference is to consider SMs as being akin to epistemic indefinites like *irgendein* analyzed in Kratzer and Shimoyama (2002), or like *algún* analyzed in Alonso-Ovalle and Menédez-Benito (2010). One proponent of this strategy is Coppock and Brochhagen (2013). They consider SMs as an alternative-introducing expression akin to *irgendein* and couch the strategy in the framework of inquisitive semantics, where declaratives, on a par with interrogatives, denote a set of ordinary propositions, in the fashion of Hamblin's treatment of questions. For example, their analysis for *Adam bought at least three apples* can be schematically represented as follows:

(53) Adam bought {3, 4, 5…} apples.

These alternatives expand upward as in Kratzer and Shimoyama (2002)'s analysis of *irgendein*. Thus, the denotation of *Adam bought at least three apples* is a set of propositions, one for each number: {*Adam bought three apples*, *Adam bought four apples*, …}. For Coppock and Brochhagen (2013), the ignorance inference is generated through the Maxim of Interactive Sincerity, a novel pragmatic principle

[32] In section 2.5, I present a derivation of ignorance inferences based on Mendia (2016a-c)'s and Schwarz (2016a)'s insights on using the idea of symmetric alternatives. Crucially, unlike Schwarz, I do not claim that *at least* and *only* form a Horn-scale. Instead, on my view, the set of *only*-like alternatives is a set of exhaustified Hamblin alternatives (answers) and results from the exhaustivity of the *wh*-question (the CQ). As we will see later, this nuanced perspective not only preserves Schwarz (2016a)'s insights about the mechanism deriving ignorance inferences and their quantity-based nature, but also avoids the empirical challenge that the counterparts of *at least* and *only* in some languages such as German and Chinese do not have the same syntactic distribution as in English.

couched in the framework of inquisitive semantics, which basically says: "Don't raise an issue that you know how to resolve".[33, 34] Crucially, asserting a sentence whose denotation is a set of propositional alternatives amounts to raising an issue, and this is exactly the case of a declarative sentence containing *at least* because *at least* is an inquisitive expression.

Nouwen (2015) is another proponent of the strategy seeing SMs as being akin to epistemic indefinites, though in a different way than Coppock and Brochhagen (2013). According to Nouwen (2015), *at least* imposes an anti-specific presupposition on its domain, on a par with *algún* analyzed in Alonso-Ovalle and Menédez-Benito (2010). Below, the semantics of *algún*, borrowed from Alonso-Ovalle and Menédez-Benito (2010: 19), is presented in (54). Nouwen (2015)'s analysis of *at least* is illustrated in (55). Note that f is a function that selects the domain of quantification for the existential quantifier and $\boldsymbol{B}$ is a universal doxastic modal in an assertion.

(54) $[\![algún]\!] = \lambda f_{<<e,t>, <e, t>>}$: anti-singleton (f). $\lambda P_{<e, t>}.\lambda Q_{<e, t>}.\exists x_{<e>}[f(P)(r) \wedge Q(r)]$

(55) Adam wrote at least $[\text{two}]_F$ pages.

a. Assertion: $\boldsymbol{B}[\ |\lambda x.\text{page}(x) \wedge \text{wrote (adam}, x)| \in f(\{\text{n: n}\geq 2\})]$

b. Presupposition: $|\ f(\{\text{n: n}\geq 2\})| > 1$

The idea is that *at least* patterns with *algún* in requiring an anti-singleton domain and the ignorance inference given by *at least* can be generated in the same mechanism

---

[33] In a sense, the maxim of interactive sincerity is similar to the spirit behind the Gricean maxim of quality ("Don't say what you believe to be false") in that both principles are more speaker-based. Seen in this light, questions are raised as to what exactly the nature of ignorance inferences given by SMs is. Is it a quality-based implicature or a quantity-based implicature? This issue is non-trivial and is not yet settled in the literature, because the exact mechanism deriving the ignorance inference is still under debate. Readers are referred to Schwarz (2016b) for a discussion of the two different perspectives.

[34] Schwarz (2016b) argues that the maxim of interactive sincerity proposed in Coppock and Brochhagen (2013) fails to derive all the desired inferences for sentences with *at least* and also ignorance inferences for disjunction. He suggests that an account in terms of Gricean quantity implicature (whether couched in inquisitive semantics or otherwise) is a better analytical option.

based on the reasoning about the domain alternatives, as proposed in Alonso-Ovalle and Menédez-Benito (2010) for the ignorance inference associated with *algún*.[35]

To sum up, in this section, we have seen how the previous analyses of epistemic *at least* vary along two aspects: what kind of scales SMs operate on and how the ignorance inference is derived. The next section presents a unified semantic representation of *at least* while explaining how the EPI-CON ambiguity and the three common properties of the two meanings follow from the current unified analysis. Section 2.5 shows how the ignorance inference is derived, and discusses why and how epistemic *at least* is parallel to disjunction and epistemic indefinites under the proposed analysis.

2.4 The proposal: a unified analysis

In this section, I spell out my unified analysis of the EPI-CON ambiguity shown by English *at least* (and potentially its cross-linguistic counterparts discussed in section 2.1). In a nutshell, the idea is that **(a)** one uniform semantic representation of *at least* can be maintained; **(b)** the two meanings arise from the semantic core interacting with different pragmatic factors such as informativity (see (34)) and evaluativity (see (40)). Below, (56) is repeated as a summary of the relevant facts that this dissertation attempts to capture.

(56) a. **The EPI-CON ambiguity**: Cross-linguistically, SMs in general demonstrate an ambiguity in giving an ignorance inference and a concessive inference.

[35] Crucially, SMs and epistemic indefinites are not fully parallel: in particular, ignorance inferences associated with *algún* or *irgendein* seem to be obligatory (i.e., not cancellable), while those given by SMs can be cancelled, especially when the maxim of quantity is deactivated (see section 2.2). It remains to be seen in the anti-specific approach, regarding how to capture the similarity between SMs and epistemic indefinites in giving ignorance inferences while explaining why those given by SMs may be cancelled in certain contexts but those given by epistemic indefinites may not across the board.

b. **Focus-sensitivity**: the semantic contribution of SMs under both meanings (EPI and CON) depends on the position of their focus associate.

c. **Scale types and their discrepancy**: SMs under both meanings (EPI and CON) are compatible with various scales (based on semantic strength or pragmatic strength). However, in contrast to lexical scales and pragmatic scales, the ordering between focus alternatives in numerical scales and plurality scales cannot be reversed.

d. **Two scalar effects (TSE and BSE) and their discrepancy**: SMs under both meanings (EPI and CON) demonstrate two scalar effects, but there is a contrast in the type of infelicity that arises. For English *at least* (and Chinese *zhishao*, etc), BSE may be pragmatically repaired while TSE may not.

The rest of this section is structured as follows. Section 2.4.1 presents my theoretical assumptions. In particular, I assume Rooth (1985, 1992)'s focus semantics and Beaver and Clark (2008)'s focus principle and QUD-based discourse model. Section 2.4.2 offers the semantic entry of *at least*. Section 2.4.3 shows how the semantic core combining with different pragmatic ingredients leads to EPI and CON. Section 2.4.4 explains why the two scalar effects arise and demonstrates how they are connected to the semantics of *at least*. Section 2.4.5 discusses some cases where BSE apparently disappears in the discourse. Section 2.4.6 illustrates how the "settle-for-less" flavor and the distribution of CON together follow from the current unified analysis. Section 2.4.7 looks at the distribution of EPI and suggests some directions for future research.

2.4.1 Theoretical assumptions

To begin with, for purposes of this dissertation, I assume Rooth (1985, 1992)'s focus semantics that every expression φ has an ordinary semantic value and a focus semantic value. For an unfocused constituent, its focus semantic value is a singleton set containing the ordinary value of that expression. For a focused constituent, its focus semantic value is a set of alternatives: a set of objects that have the same semantic type as the focused constituent. The set of alternatives induced by focus is computed recursively (essentially as in Rooth 1985, 1992). Furthermore, the semantic contribution of a focus-sensitive operator depends on the focus semantic value of its sister. The set of focus alternatives projects until they meet the focus operator where they are interpreted by a squiggle operator ~ and restricted by a contextual variable C. The definition of ~ in (57) is drawn from Rooth (1996: (20)). The composition of association with focus is illustrated by English *only* in (58). Below, ONLY abbreviates the contribution of *only*.

(57) Where φ is a syntactic phrase and C is a syntactically covert semantic variable, $\varphi \sim C$ introduces the presupposition that C is a subset of $\|\varphi\|^f$ containing $\|\varphi\|^o$ and at least one another element.

(58) a. $[_{IP}$ John $[_{VP2}$ **only** (C) $[[_{VP1}$ won a $[\text{silver}]_F$ medal$]$ $\sim C]]]$

b. $[\![\text{VP1}]\!]^o = \lambda x \lambda w.\ x\ \text{won}_w$ a silver medal

c. $[\![\text{VP1}]\!]^f = \{\lambda x \lambda w.\ x\ \text{won}_w$ a gold medal, $\lambda x \lambda w.\ x\ \text{won}_w$ a silver medal,

$\lambda x \lambda w.\ x\ \text{won}_w$ a bronze medal$\}$

d. $[\![\textit{only}\ \text{VP1}]\!] = \lambda x \lambda w$: $[\![\text{VP1}]\!]^o(y)(w).\ \forall P \in C\ [P_w(y) \rightarrow [\![\text{VP1}]\!]^o(y) \subseteq P(y)]$

e. $[\![\text{VP2}]\!] = \lambda x \lambda w.\text{ONLY}(\text{won}_w\ (x, \text{a silver medal}))$

f. $[\![\text{IP}]\!] = \lambda w.\text{ONLY}(\text{won}_w\ (\text{John, a silver medal}))$

In (58), C is contextually restricted and is presupposed to be a subset of the focus semantic value of VP1, the sister node of the focus-sensitive operator *only*.

Furthermore, *C* provides the quantificational domain for *only*. Under the current implementation, crucially, the quantificational domain of a focus-sensitive operator varies with its (syntactic) position. As we will see in section 2.4.4, this provides a way of understanding why the (un)availability of EPI and CON is sensitive to the distribution of SMs.

Next, building on Roberts (1996/ 2012)'s work, I assume with Beaver and Clark (2008) that discourse evolves by interlocutors continually raising and answering questions (see also Büring 2003). According to Beaver and Clark, a question that is proffered (in the sense of Roberts 1996/ 2012) and mutually accepted by the interlocutors as the most immediate goal of the discourse becomes the Current Question (CQ). More specifically, we assume with Beaver and Clark (2008) the following principles:

(59) **Current Question Rule**: The Current Question (CQ) must contain at least one true alternative and contain multiple alternatives which are not resolved as true or false in the common ground.[36]

(60) **Discourse Principle**: Utterances should be maximally relevant to the CQ.[37]

(61) **Focus Principle**: Some part of a declarative utterance should evoke a set of alternatives containing all the Rooth-Hamblin alternatives of the CQ.

The focus principle (61) differs from the one in Roberts (1996/ 2012) in two respects. First, following Rooth, the congruence relation between the alternative set and the CQ denotation is a subset relation (Roberts takes it to be an equality relation). Specifically,

[36] This principle determines whether the CQ remains open in a discourse.

[37] The notion of relevance between questions and answers is defined in Beaver and Clark (2008) as follows: "for a discourse move to be relevant it must address the CQ, which Roberts (1996/ 2012) takes to mean either the move introduces a partial or total answer to the CQ, or that it is part of a strategy to answer the CQ".

the CQ may have fewer alternatives than the focus semantic value of the answer because the CQ contains only alternatives that are consistent with the common ground while the focus semantic value of the answer is computed blindly (taking only semantic type into consideration). Second, Robert requires the complete declarative sentence to be congruent to the CQ, (61) demands only part of it to be congruent.[38]

Finally, to establish the connection between Beaver and Clark's QUD-based discourse model and Rooth's representation of focus, we follow Rooth's idea that the variable *C* is contextually restricted and assume that *C* is constrained by the choice of CQ in the discourse. Given the assumptions above, the relation between the quantificational domain of a focus operator, the denotation of a question, and the focus value of an answer to the question can be understood as follows.

(62) a. $[\![\mathrm{Q}]\!]^{o} \subseteq [\![\mathrm{Ans}]\!]^{f}$ the question-answer congruence

b. $C = [\![\mathrm{Q}]\!]^{o}$ the domain restrictor *C* is anaphoric

What (62) means is that **(a)** the discourse congruence requires the denotation of a question to be a subset of the focus value of the answer; **(b)** the quantificational domain of a focus operator is contextually restricted by the question in the discourse.[39] Taken together, the contribution of focus particles like English *only* can be understood as imposing further restrictions on the answer space. The dialogue in (63) and its relevant representations in (64) illustrate the point.

(63) Context. There are three individuals in the discourse: Adam, Bill, Chris

[38] As discussed in Beaver and Clark (2008: section 2), under their model, the sentence *I think [Mary]$_F$ laughed* can answer the CQ *Who laughed?*, whereas for Roberts it would be congruent to the CQ *Who do you think laughed?*. Readers are referred to chapter 3 and chapter 5 in their book for more discussions on this point.

[39] An alternative way of capturing the relation between the quantificational domain of a focus particle and the question in a discourse is to assume a subset relation: $C \subseteq [\![\mathrm{Q}]\!]^{o}$, rather than an identity relation.

A: Who left? B: Only Adam left.

(64) a. C = $[\![Q]\!]^{o}$ = {Adam, Bill, Chris}

b. LF: [[$_{DP}$ Only(*C*) [$_{DP}$ [$_{DP}$ Adam]$_{F}$ ~*C*]] λx [x left]]

The use of *only* narrows down the answer space because it excludes the other two individuals (*Bill* and *Chris*) from being in the answer. Notice that the focus particle *only* is syntactically adjoined to the DP subject *Adam* and its quantificational domain *C* (anaphoric to the question) is a set of individuals {Adam, Bill, Chris}, not a set of propositions.[40] It is a natural consequence of (62) that a *wh*-question like *who left* should in principle be able to denote a set of short answers. This is consistent with Xiang (2016)'s hybrid categorial approach where the denotation of a *wh*-question can be either a set of full answers (a set of propositions) or a set of short answers, but contrasts with the Hamblin-Karttunen approach where a *wh*-question always denotes a set of propositions (Hamblin 1973, Karttunen 1977, among others).[41] It is worth noting that this dissertation leaves open issues concerning how short answers are generated. They can be directly obtained by the semantics of *wh*-questions (see Jacobson 2016 for a proposal) or syntactically derived from ellipsis (see Weir 2014 and references therein for proposals).

The next section offers the uniform semantic representation of *at least* and illustrates how the ambiguity arises from the semantic core combining with different pragmatic factors: the issue of informativity in the case of EPI and the issue of evaluativity in the case of CON.

[40] In this case, the particle *only* is a two-place focus operator: it takes the DP *Adam* as its first argument and the constituent represented by the lambda-construct as its second argument. I assume with Coppock and Beaver (2014) that such lexical entry of English *only* can be obtained by type-shifting such as the Geach rule. See also Wagner (2006) for discussion.
[41] See Xiang (2016: chapter 1) and Dayal (2016: chapter 2) for a recent review and discussion of different theories of questions and answers.

2.4.2 The semantics of *at least*

Let us recall that the two meanings share three common properties: the focus-sensitivity, the compatibility with various scales and the two scalar effects TSE and BSE. Seen in this light, I propose that the semantic core of the two meanings is scalarity. In particular, I suggest that scalarity can be understood as (65).

(65) **Scalarity** (the semantic core of EPI and CON)

The set of focus alternatives (the set of answers addressing the CQ) is ordered along a contextually given scale.

Furthermore, I suggest that a uniform semantic representation of the two meanings should encode scalarity. Given these considerations, I propose that English *at least* has the following semantic representation.[42, 43]

(66) **A propositional version of *at least***

$$[\![at\ least(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$$[44]

Several remarks are in order. First, C represents the contextual restriction as in Rooth's representation of focus. Second, μ_c is defined as a measure function (of type $<\eta, d>$; η in principle could be any type), mapping the focus alternatives to their corresponding positions along a contextually-valued scale. Third, the ordering between alternatives is represented in terms of a (strict) comparison relation between the prejacent α and its alternatives along a contextually-given scale: $\mu_c(\alpha) < \mu_c(\beta)$.

[42] Other things being equal, the proposed semantic representation of *at least* in principle applies to its cross-linguistic counterparts showing the EPI-CON ambiguity (see section 2.1).

[43] A fully decompositional analysis of SMs is presented in chapter 4, illustrating how this semantic representation is connected to the morphological pieces of *at least*. For now, I focus on how the EPI-CON ambiguity results from this semantic entry interacting with different pragmatic factors.

[44] I thank Roger Schwarzschild for drawing my attention to the issue of presuppositions. This semantic representation is inspired by his original suggestion adding the semantic component $\exists\gamma[\gamma\in C \wedge \gamma_w]$ as a presupposition in the semantic entry under the discourse-based approach (discussed in section 2.3.2).

Fourth, a superlative component, $\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)$, is contained in the semantic representation of *at least* and imposes an additional restriction on the answer space: all the alternatives that are non-identical to the prejacent are ranked above the prejacent; this amounts to excluding the lower alternatives from the domain C. Fifth, the domain C further restricted by the superlative now denotes a set consisting of the prejacent (obtained by the focus presuppositions) and its relevant higher alternatives (obtained by the superlative component). Putting these pieces together, the semantics of *at least* in (66) asserts that there is one proposition γ in the domain consisting of the prejacent and its relevant higher alternatives such that the proposition γ is true. Crucially, the semantic representation in (66) not only captures the bounding property of *at least*: the prejacent is the lower bound among the set of focus alternatives ordered along a contextually-valued scale, but also preserves many insights of Biezma (2013)'s analysis: **(a)** it yields a disjunctive statement without hard-wiring a disjunction into the semantics, because an existential claim over a set amounts to a disjunctive statement of the elements in that set; **(b)** it leaves open whether the relevant higher alternatives are true in a given discourse, which crucially leaves a room for pragmatics to play a role in delivering the ambiguity. Finally, to cover cases where *at least* is syntactically adjoined to constituents that are not propositional, I assume that the following two entries can be obtained by type-shifting (see Coppock and Beaver 2014 for similar treatment of English exclusive particles).

(67) a. **A non-propositional version (by the Geach rule)**

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta, st>}\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

b. **A non-propositional version (by the backward Geach rule)**

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\ st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) < \mu_c(P_w(\beta))]]$$

Crucially, under the current analysis, the non-strict comparison relation (discussed in section 2.3.2) is only illusive: it is not a semantic primitive, but derived from focus presuppositions and the superlative component in the semantics of *at least*.

Finally, it is worth noting that the semantic core (66) already predicts two common properties of the two meanings: the focus-sensitivity and the compatibility with various scales. More specifically, the focus-sensitivity follows because the unified entry operates on a set of focus alternatives and imposes further restriction on the answer space (i.e., excluding the relevant lower alternatives). The compatibility with various scales follows because the unified entry requires a contextually-valued scale and the set of focus alternatives is ordered along that scale.[45, 46] The next section shows how the EPI-CON ambiguity arises from the unified semantic entry interacting with different pragmatic factors: informativity vs. evaluativity.

2.4.3 Explaining the EPI-CON ambiguity

Let's consider the case of EPI first. In section 2.2, we have seen that the ignorance inference is pragmatic and arises when a precise answer is requested in a given

[45] The traditional analysis of English *even* invokes the scale of likelihood. However, this traditional view has been recently challenged by Greenberg (2016, 2018). Specifically, Greenberg (2016, 2018) argues that *even* invokes a contextually-given scale and then gives a gradability-based analysis of *even*. In this respect, this dissertation shares a similar line of research in applying formal tools from previous studies on gradability to the phenomena of scalarity shown by SMs. See chapter 4 for more details.

[46] The discrepancy between scales based on semantic strength vs. pragmatic strength will be explained in chapter 4, where the formal property of the measure function μ_c is discussed. To anticipate, the leading idea is that the measure function μ_c presents a structure-preserving mapping between the focus alternatives and their positions along a contextually-given scale. More specifically, when the set of alternatives has its own internal structure such as plurality scales (partial ordering) or numerical scales (total ordering), the ordering between alternatives is structurally-preserved and cannot be altered even with contextual manipulations.

discourse. This is exactly what the unified entry in (66) leads us to expect. First, the speaker ignorance is not lexicalized in the semantics of *at least*. This means that the ignorance inference does not come directly from the semantics. Second, the ignorance inference arises from pragmatics because the contribution of *at least* provides a partial answer to the CQ in a discourse where the precise answer is requested. Below, (68) presents the relevant context and conversation; (69) illustrates the computation of the utterance with epistemic *at least* and (70) the truth-conditions. Sup abbreviates the contribution of the superlative component in the entry (see (66) and (67)).

(68) Context: John won a gold medal. Emily knows that he won a medal but she doesn't know what kind of medal he has won. She asks John's friend Frank.

Emily: What medal did John win?

Frank: John won **at least** a $[\text{silver}]_F$ medal.

(69) a. LF: $[[_{DP}$ **at least**$(C)[_{DP}[_{DP}$ a $[\text{silver}]_F$ medal$]\sim C]$ λz [John won z]]

b. C = {a gold medal, a silver medal, a bronze medal}

c. Sup = {a gold medal}

d. $C \cap$ Sup = {a gold medal, a silver medal}

e. The prejacent: a silver medal

(70) $[\![(69a)]\!]^{w,\,c} = 1$

$$\text{iff } \exists\gamma[\gamma\in C \wedge \gamma_w(\lambda z.\text{John won}_w\, z) \wedge$$
$$\forall\beta[\beta\in C \wedge \beta \neq [\![\text{a silver medal}]\!]^{w,\,c}$$
$$\rightarrow \mu_c([\![\text{a silver medal}]\!]^{w,\,c}(\lambda z.\text{John won}_w\, z) < \mu_c(\beta(\lambda z.\text{John won}_w\, z))]]$$

Assume that Frank obeys the Gricean maxims (Grice 1989) and understands that Emily's question is requesting information about the medal John won; Frank's answer apparently fails to provide that maximally informative unique answer: John won a gold medal. Given the semantic entry, Frank's utterance conveys that there is one element in the set represented by $C \cap$ Sup such that John won that element. Put differently, by using *at least*, Frank's utterance conveys that John won a gold medal or John won a silver medal. An ignorance inference arises to justify Frank's failure of providing the unique answer. Crucially, given that Frank obeyed the Gricean maxims, if he had known the unique answer, he would have uttered it. In this line of reasoning, the proposed semantic entry leads to ignorance inferences only under certain contexts where a precise answer is requested (as discussed in section 2.2). Taken together, I propose that SMs under EPI is addressing the issue of informativity, as defined below.

(71) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.[47]

Next, let's consider the case of CON. Recall that N&R observes that CON has a "settle-for-less" flavor. Recall also that Biezma (2013)'s recipe of CON requires two pragmatic ingredients: **(a)** the relevant higher alternatives are contextually known to be false; **(b)** the set of alternatives is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse. Although Biezma's original example involves an evaluative *how*-question as the CQ (see section 2.2), I generalize the idea of evaluativity (her second pragmatic ingredient of CON) to other types of

[47] The notion of "the maximally informative unique answer" can be defined in terms of the relation between the answerhood operator ANS and the denotation of the *wh*-question (abbreviated as Q) below, as proposed in Dayal (1996, 2016).

(i) $\text{ANS}_{\text{Dayal}}(Q)(w)$ is defined iff $\exists p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$.
When defined, $\text{ANS}_{\text{Dayal}}(Q)(w) = 1$ iff $\iota p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$

In words: the answerhood operator $\text{ANS}_{\text{Dayal}}$ is defined if and only if there is one proposition p in the denotation of the question Q (which is a set of possible answers/ propositions) such that p is true and p entails all the true propositions q in Q; When defined, $\text{ANS}_{\text{Dayal}}(Q)(w)$ is true if and only if the unique proposition p in Q is true (i.e., the uniqueness) and p entails all the true propositions q in Q (i.e., the maximal informativity). Readers are referred to Dayal (2016: chapter 2) for a detailed discussion on the existential presupposition and the uniqueness requirement of *wh*-questions.

wh-questions involving *what* and *who* as the CQ. For instance, imagine a scenario that John has some bronze medals but he has never got a gold medal or a silver medal in a swimming competition. So, John has been practicing very hard and hopes to get a gold medal. Given John's situation, winning a gold medal is definitely the best but winning a silver medal is also satisfactory. Emily wonders whether John has successfully achieved his goal. In answering Emily's question, Frank takes into consideration Emily's interests, evaluates and ranks the set of alternatives against her interests (e.g., how successful John's plan is). (72) presents the relevant conversation and (73) illustrates the computation of concessive *at least*.

(72) Emily: What medal did John win? Did he win a gold medal?

Frank: No, but **at least** John won a $[\text{silver}]_F$ medal.

(73) a. LF: $[_{IP}$ **at least**$(C)[_{IP}[_{IP}$ John won a $[\text{silver}]_F$ medal$]{\sim}C]]$

b. C = {John won a gold medal, John won a silver medal,

John won a bronze medal}

c. Sup = {John won a gold medal}

d. $C \cap$ Sup = {John won a gold medal, John won a silver medal}

e. The prejacent = John won a silver medal

(74) $[\![(73a)]\!]^{w,c} = 1$

iff $\exists\gamma[\gamma\in C \wedge \gamma_w \ \wedge\forall\beta[\beta\in C \wedge \beta \neq [\![\text{John won a silver medal}]\!]^{w,c}$

$\rightarrow \mu_c([\![\text{John won a silver medal}]\!]^{w,c}) < \mu_c(\beta)]]$

In (73), the unified entry of *at least* in (66) is applied: there is one element in the set represented by $C \cap$ Sup such that the element is true. Crucially, in the case of CON, the two pragmatic requirements are fulfilled: **(a)** the relevant higher alternatives are contextually known to be false; **(b)** the set of alternatives is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse. The first condition ensures that the speaker Frank knows that the relevant higher alternative is false: John did not win a gold medal. By using *at least*, the speaker signals the fact that while excluding the lower alternative, the prejacent is the only alternative that is true in the set represented by $C \cap \text{Sup}$. The "settle-for-less" flavor arises from the fact the prejacent is the only true alternative in the domain represented by $C \cap \text{Sup}$, despite some relevant alternative ranked higher than the prejacent with respect to the interlocutors' interests in the discourse. Taken together, I propose that *at least* under CON is addressing the issue of evaluativity, as defined below.

(75) **CON and the issue of evaluativity**

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is true; while it is not the best situation, it is not the worst situation either.

Before leaving this section, it is worth emphasizing that given the current unified analysis, the semantic core underlying the EPI-CON ambiguity is the notion of scalarity (defined in (65)) along with the bounding property of *at least*: under both meanings, the prejacent is set up as the lower bound among the set of focus alternatives (the set of answers addressing the CQ). Depending on how this semantic core combines with different pragmatic factors, such as whether the relevant higher alternatives are left open or known to be false in the discourse, an ignorance inference or a concessive interpretation may arise.[48]

The next section illustrates how the third common property, the two scalar effects

[48] The main purpose of this chapter is to present empirical facts calling for a unified account of the EPI-CON ambiguity, providing a single lexical entry of *at least* and showing how the common properties of the two meanings are captured by the unified semantics. In chapter 3, I will discuss in more detail about the pragmatics of speaker concession and address issues concerning how the assertion with concessive *at least* interacts with the dynamics of the discourse.

(TSE and BSE) and their discrepancy, follows from the current analysis.

2.4.4 Deriving TSE and BSE

Recall that under both meanings, *at least* demonstrates two scalar effects. The top-of-the-scale effect (TSE) demands that the associate cannot be the element at the top of the scale and the bottom-of-the-scale effect (BSE) that the associate cannot be the element at the bottom of the scale. The two utterances (76) and (77) are infelicitous in a dice-playing scenario where it is known that a dice has six numbers and that the number *six* is the upper bound and the number *one* the lower bound on the possible results. Moreover, there is a discrepancy at the intuitive level; for *at least*, TSE seems *strictly infelicitous* while BSE is not.

(76) #Chris **at least** got [six]$_F$. [**Top-of-the-Scale Effect** (**TSE**)]

(77) #Chris **at least** got [one]$_F$. [**Bottom-of-the-Scale Effect** (**BSE**)]

Are these two scalar effects qualitatively the same? Why and how do they arise? What do they tell us about the semantics of *at least*? In what follows, I show that the proposed semantic entry of *at least* not only predicts the two scalar effects but also predicts them to be different in nature. In particular, in the case of *at least*, TSE arises from **semantic vacuity** and is semantic in nature; in contrast, BSE arises from **discourse uninformativity** and is pragmatic in nature. Therefore, only BSE can be *pragmatically repaired* by certain conversational strategies.

To begin with, let's consider TSE. Al Khatib (2013: 17) observes that the use of English *only* cannot be vacuous. The examples (78) and (79) illustrate his point.

(78) #Adam **only** saw [every student]$_F$.

(79) a. #Of Mary and Sue, Adam **only** saw [Mary and Sue]$_F$.

b. Of Mary and Sue, Adam **only** saw [Mary]$_F$.

c. Of Mary and Sue, Adam **only** saw [Sue]$_F$.

Intriguingly, the same contrast is observed with *at least* under the two meanings.

(80) #Adam **at least** saw [every student]$_F$.

(81) a. #Of Mary and Sue, Adam **at least** saw [Mary and Sue]$_F$.

b. Of Mary and Sue, Adam **at least** saw [Mary]$_F$. √EPI, √CON

c. Of Mary and Sue, Adam **at least** saw [Sue]$_F$. √EPI, √CON

Note that (80) is plainly infelicitous, regardless of whether EPI or CON is intended. Along with these contrasts, I propose that in the dice scenario, the utterance (76) is infelicitous because the use of *at least* is vacuous. Informally put, the use of *at least* is vacuous when no relevant higher alternatives exist in the first place because the associate (the number *six*) is the upper bound. Formally, (82) illustrates the relevant pieces of the computation. In particular, **semantic vacuity** arises because the contribution of the superlative component Sup is vacuous. Recall that Sup requires all the alternatives non-identical to the prejacent to be ranked above the prejacent (i.e., $\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)$). When no higher alternatives exist in the first place, the contribution of Sup becomes vacuous. This is illustrated by the fact that $C \cap$ Sup is a singleton set of an element precisely identical to the prejacent: *Chris got six*.

(82) a. The LF of (76): [$_{IP}$ **at least**(C)[$_{IP}$ [$_{IP}$ Chris got [six]$_F$]~C]]

b. C = {Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}

c. $C \cap$ Sup = {Chris got six}

d. The prejacent = Chris got six

Thus, TSE arises as a consequence of violating a general constraint against vacuous quantification in natural language: no semantic operators can be used vacuously.

Next, let's consider BSE. I propose that the utterance (77) is infelicitous because it is **contextually uninformative**: it is already known in the common ground that only six results are possible and the number *one* is the lower bound. Below, (83) illustrates the relevant pieces of the computation. In particular, discourse uninformaitvity is illustrated by the representation that the domain $C \cap$ Sup is exactly the same set as C: {*Chris got one*, *Chris got two*, *Chris got three*, *Chris got four*, *Chris got five*, *Chris got six*}. What this means is that an assertion with *at least* does not substantially remove any possible results from the original discourse (in a more dynamic term, the assertion with *at least*, if accepted, does not *update* the common ground).[49]

(83) a. The LF of (77): $[_{\text{IP}}$ **at least**$(C)[_{\text{IP}}[_{\text{IP}}$ Chris got $[\text{one}]_{\text{F}}]{\sim}C]]$

b. C = {Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}

c. $C \cap$ Sup = {Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}

d. The prejacent = Chris got one

---

[49] One way to define discourse uninformativity is in terms of assertion and context set (Stalnaker 1978, 1998, 2002; among others), as sketched below.

(i) A context set C is a set of propositions that the interlocutors have publically committed to.
$C =_{\text{def}} \{w \mid w \in \cap\{p_{<s,t>} \mid \text{the interlocutors have publically committed to } p\}\}$

(ii) An assertion of utterance U is informative in a discourse D if it updates the context set C.
$C[U] =_{\text{def}} \{w \in C: U^{w}=1\}$

(iii) An assertion of utterance U is uninformative in a discourse D if it doesn't update the context set C.
$C[U] =_{\text{def}} \{w \mid w \in C\}$

The idea is that discourse (un)informativity depends on whether asserting a proposition p updates the context set (i.e., remove those worlds where p is false from C). For our current purposes of the distinction, the definitions in (i)−(iii) should suffice. However, a more correct way should be to characterize an assertion as a discourse move in proposing an update of the context set: an update happens only when the discourse move is accepted (e.g., Farkas and Bruce 2010, among others).

Thus, BSE arises as a consequence of violating the maxim of quantity: be as informative as required. Crucially, seen in this light, the utterance (77) becomes *felicitous* once it is understood in a way that the speaker is joking or being sarcastic about Chris: that is, the speaker is being *intentionally uncooperative* and *flouting* the maxim of quantity (Grice 1989).

In short, the unified semantic entry predicts TSE and BSE to be different in nature. In the case of *at least*, TSE results from **semantic vacuity** while BSE from **discourse uninformativity**. This is evidenced by the fact that BSE, but not TSE, can be *pragmatically repaired* by certain conversational strategies.

To sum up, we have seen **(a)** how the EPI-CON ambiguity arises from the uniform semantic representation of *at least* interacting with different pragmatic factors such as informativity and evaluativity; **(b)** how the three common properties of the two meanings follow from the unified entry. In the next section, I briefly discuss some cases where BSE apparently disappears in the discourse.

2.4.5 BSE and negative responses

We have seen that both meanings demonstrate BSE: the focus associate cannot be the element at the bottom of the scale. However, there are cases where BSE is apparently missing. Suppose that Adam, Bill and Chris are the relevant individuals in the discourse. Consider the examples below; (84) illustrates the case of epistemic *at least* and (85) the concessive *at least*.

(84) Epistemic *at least*

A: Who did John invite?

B: John invited **at least** [Adam]$_F$.

(85) Concessive *at least*

A: Who did John invite? Did John invite (both) Adam and Bill?

B: No, but **at least** he invited [Adam]$_F$.

Given BSE, the felicitous use of *at least* in examples like (84) and (85) are surprising because there seems to be no bottommost element ranked below the singular individual *Adam* along a plurality scale. Put differently, what is the bottommost element in these two examples? Note that the issue here is not specific to the plurality scale. The same point can be made for the numerical scale: what is the bottommost element in (86) and (87)?

(86) Epistemic *at least*

A: How many apples did John buy?

B: John bought **at least** [one]$_F$ apple.

(87) Concessive *at least*

A: How many apples did John buy? Did John buy more than two apples?

B: No, but **at least** John bought [one]$_F$ apple.

So, what is going on in these examples? I suggest that the answer lies in the discourse status of negative responses. Consider the following examples.

(88) A: Who did John invite?

B: No one/ John invited no one.

(89) A: How many apples did John buy?

B: John didn't buy any apple/ John bought no apples.

Two remarks concerning negative answers are in order. First, these negative responses are felicitous. Second, depending on one's assumption about the semantics of constituent questions, these negative responses may or may not be part of the denotation of the question (i.e., "the set of answers" to the question). For examples, under Groenendijk and Stokhof (1984)'s partition semantics, negative responses like *no one* constitute one cell of the partition and is thus part of the denotation of the question (see also Groenendijk 1999, among others).

Another line to approach the status of those negative responses is to consider that they are NOT part of the semantics of questions, but allowed under certain discourse conditions.[50] It has been argued that a constituent question semantically imposes an existential presupposition (e.g., Dayal 2016: chapter 2; among others). Roughly speaking, when the speaker asks a question like *who left?*, the speaker imposes a soft presupposition that *someone left* in his/ her asking the question. An important observation from Dayal (2016: section 2.3.4) is that those negative responses are mostly allowed in a cross-speaker conversation (see (97) and (91)), while questions with a cleft typically do not brook those negative responses (see (92)).

(90) #I am not sure whether Mary likes any student. Which student does she like?[51]

(91) Speaker A: Who left?

Speaker B: No one/ no one left.

(92) Who was it that left the party?

#No one/ no one left the party.

The conclusion suggested in Dayal (2016) is that the ordinary constituent question has a soft presupposition while the question with a cleft has a hard presupposition (that

---

[50] See Biezma and Rawlins (2012) for a similar point and discussion on the negative response *neither* in the case of alternative questions.

[51] As discussed in Dayal (2016), it is possible for a speaker to ask a question while overtly suspending the existence commitment.

(i) Who, if anyone, does Mary like?

comes from the cleft). (93), taken from Dayal (2016: 52), illustrates her point.

(93) a. Who left?

Presupposes: $\text{I}_{\text{SPEAKER}}$ assume that someone left.

b. Who was it that left?

Presupposes: $\text{We}_{\text{SPEAKER+HEARER}}$ believe that someone left.

In this dissertation, I assume with Dayal (2016) that negative responses like *no one* are NOT part of the semantics of a constituent question, but only allowed under certain discourse conditions (typically a cross-speaker conversation). With this background in mind, I propose that the reason why BSE apparently disappears in examples like (84) − (87) is because negative responses can serve as the bottommost element (in an out-of-blue context and a cross-speaker conversation).

(94) a. The case of (84) and (85):

no one ≺adam, bill, chris ≺ adam⊕bill…≺ adam⊕bill⊕chris

b. The case of (86) and (87):

no/ zero apples ≺ one apples ≺ two apples…[52]

Crucially, when negative responses are not possible in a given discourse (i.e., when the focus associate is genuinely the bottommost element), BSE shows up again. This is evidenced by (95) and (96).

(95) Context: It is already known that John has invited Adam for his party.

a. Epistemic *at least*

A: Who did John invite?

B: #John invited **at least** $[\text{Adam}]_F$.

[52] In principle, the word "zero" can be analyzed as a prenominal numeral or a generalized quantifier like *no*. But see Bylinina and Nouwen (2018) for the view that *zero* is a proper numeral. I do not take a stand in this dissertation.

b. Concessive *at least*

A: Who did John invite? Did John invite (both) Adam and Bill?

B: #No, but **at least** he invited $[\text{Adam}]_F$.

(96) Context: It is already known that John has bought some number of apples.

a. Epistemic *at least*

A: How many apples did John buy?

B: #John bought **at least** $[\text{one}]_F$ apple.

b. Concessive *at least*

A: How many apples did John buy? Did John buy more than two apples?

B: #No, but **at least** John bought $[\text{one}]_F$ apple.

In (95), it is already in the common ground that Adam is invited. By asking the question, speaker A would like to know whether John also invited someone else; speaker B's utterance is infelicitous because it is uninformative. Similarly, BSE arises in (96) because speaker B's utterance is uninformative, given that it is already in the common ground that John bought some apples. Note that examples like (95) and (96) also support Dayal's view that once the interlocutors are committed to the existence presupposition, negative responses are not possible (see (92) and (93)).

To sum up, I have discussed cases where BSE apparently disappears and argued that in those cases, some negative responses serve as the bottommost element in the discourse. Crucially, when those negative responses are not possible (i.e., the existence commitment is part of the common ground), BSE shows up again.

In the next section, I tackle the distribution of CON, in particular, the question of why and how CON is precluded from the prenominal position.

2.4.6 Deriving the distribution of CON

In sections 2.4.3 − 2.4.4, I have been using the preverbal position for illustration. Recall that according to N&R, the concessive reading is unavailable when *at least* appears at the prenominal position. The relevant examples are repeated below.

(97) a. Mary won **at least** a [silver]$_F$ medal. √EPI, #**CON**

b. Mary **at least** won a [silver]$_F$ medal. **√EPI, √CON**

c. **At least** Mary won a [silver]$_F$ medal. #EPI, √**CON**

I consider the observed distributional restriction as an important property of CON because it apparently holds across languages (see section 2.1; see also Grosz 2011). Building on the idea that *at least* under CON addresses the issue of evaluativity, I propose that the distributional restriction of CON can be understood as a sister phenomenon with the "settle-for-less" flavor. In a nutshell, the idea is that in giving an evaluative answer to the CQ, what the speaker evaluates is some possible circumstances addressed by different propositions, namely, different sets of situations or worlds. This semantic-pragmatic view leads us to expect that concessive *at least* requires a propositional domain. Specifically, this quantificational domain hypothesis requires four ingredients: **(a)** speaker concession conveys an evaluative answer to the CQ (a *wh*-question as the super-question); **(b)** the sub-questions addressing the CQ (a set of polar questions) serves as the relevant dimensions of the evaluation; **(c)** evaluativity requires the congruence between the CQ and an evaluative answer that is based on the set of evaluative dimensions; **(d)** the syntactic position of *at least* determines the relevant quantification domain of the evaluative answer.

To begin with, let us reconsider Biezma's example, repeated below as (98). As we have seen, in giving an answer evaluating his date, Tom identifies some possible aspects that he infers Jim is interested in. That is, based on some relevant dimensions evaluating the date, Tom provides an evaluative answer to Jim's question.

(98) Jim: How was your date?

Tom: It was ok, **at least** she was smart.

Now, what are the relevant dimensions evaluating the date? Suppose that Jim is interested in whether she was tall, whether she was smart and whether she was beautiful, the relevant dimensions evaluating the date are then a set of sub-questions addressing the CQ (a set of polar questions). Specifically, the answers to the set of polar questions count toward an evaluation of the date. Consider (99) and (100).

(99) The relevant dimensions toward an evaluation of the CQ *How was the date?*

Was she tall? Was she smart? Was she beautiful?

(100) Great: She was tall, smart and beautiful

Good: She was tall and smart, or She was smart and beautiful, or She was tall and beautiful

Ok: She was tall, or She was smart, or She was beautiful

Bad: She was not tall and She was not smart and She was not beautiful

Because the evaluation is based on the answers to the set of polar questions, I suggest that the quantificational domain of an evaluative answer must be propositional (i.e., sets of situations or worlds). That is, what the speaker evaluates is some possible circumstances addressed by different propositions. Furthermore, as discussed in section 2.4, under Rooth's representation of focus, the syntactic position of *at least* crucially determines its quantificational domain. Taken together, I propose that CON is unavailable in the prenominal position because the relevant quantificational domain is not propositional. The representations in (101) − (103) illustrate the point.[53]

(101) **The unavailability of CON** (prenominal position)

a. The LF of (97a): $[_{DP}$ **at least**$(C)[_{DP}\ [_{DP}$ a $[\text{silver}]_F$ medal$]\sim C]\lambda z$ [Mary won z]]

b. C = {a gold medal, a silver medal, a bronze medal}

(102) **The availability of CON** (preverbal position)

a. The LF of (97b): $[_{IP}$ **at least**$(C)[_{vP}\ [_{vP}$ Mary won a $[\text{silver}]_F$ medal$]\sim C]]$[54]

b. C = {Mary won a gold medal, Mary won a silver medal,

Mary won a bronze medal}

(103) **The availability of CON** (sentence-initial position)

a. The LF of (97c): $[_{IP}$ **at least**$(C)[_{IP}\ [_{IP}$ Mary won a $[\text{silver}]_F$ medal$]\sim C]]$

b. C = {Mary won a gold medal, Mary won a silver medal,

Mary won a bronze medal}

In (101), CON is unavailable because the relevant quantificational domain is a set of generalized quantifiers over individuals. By contrast, in (102) and (103), CON is available because the relevant quantificational domain is a set of propositions.

(104) summarizes the hypothesis pursued in this dissertation:

[53] Our analysis rejects Rooth (1985)'s view that focus particles always take propositional scope (via quantifier-raising) and thus have a propositional domain, regardless of where their surface positions are. From our perspective, Rooth (1985)'s view seems too simplistic. N&R's observation concerning the distributional restriction on the two meanings strongly suggests that the surface position of *at least* should play a role in its interpretations. See also Roberts (1996/ 2012: section 2.2.1) and Zimmermann (2017) for more detailed discussion of challenges to the view that focus particles always have a propositional domain.

[54] In this dissertation, I assume (a) *at least* is adjoined to vP in the case of the preverbal *at least*; (b) the subject is generated at Spec, vP and thus vP is propositional (Kratzer 1996). For simplicity, I further assume that the subject reconstructs back to its base position at Spec, vP, for interpretative purposes at LF. This assumption regarding the reconstruction is simply to avoid unnecessary complications such as calculating the lambda-abstract created by the movement of the subject from Spec, vP to Spec, IP. Nothing crucial hinges on the assumption of the reconstruction.

(104) The Quantificational Domain Hypothesis

The quantificational domain of concessive *at least* must be (minimally) propositional (i.e., a set of propositional alternatives).

Crucially, the quantificational domain hypothesis further predicts a novel observation: CON should not arise with short answers. Consider (105).

(105) Context. There are three individuals in the discourse: Adam, Bill and Chris.

Emily: Who did John invite?

Frank: **At least** [Adam and Bill]$_F$ **√EPI**, **#CON**

cf. **At least** John invited [Adam and Bill]$_F$ **#EPI**, **√CON**

In (105), Frank's utterance conveys an ignorance inference about whether John invited all the three individuals: Adam, Bill and Chris. In contrast, Frank's utterance **cannot** be understood as conveying a concessive inference: although John didn't invite all the three individuals, he invited Adam and Bill. The proposed quantificational domain hypothesis readily explains why CON is not available with short answers: the concessive reading is missing because the relevant quantificational domain of *at least* is a set of individuals, rather than a set of propositions.

It is worth noting that (105) is also compatible with the view that short answers are derived by movement plus PF deletion (e.g., Merchant 2004). A possible syntactic derivation is illustrated in (106), where *at least* is generated at a prenominal position and the phrase *at least Adam and Bill* (i.e., the fragment answer) moves to Spec, CP. Finally, the whole TP complement undergoes PF deletion.

(106) [$_{CP}$ **at least** Adam and Bill [$_C$ [$_{TP}$ ~~John invited at least Adam and Bill~~]]]

Although a derivation like (106) may explain why the concessive meaning is not available with short answers (see (105)), unfortunately, it brings us back to where we started: why is the concessive meaning of *at least* unavailable in a prenominal position?

Crucially, (105) cannot be analyzed as in (107), where *at least* is at a clausal-initial position and everything except for the phrase Adam and Bill is deleted.

(107) *[$_{TP}$ **at least** [$_{TP}$ ~~John invited~~ [$_{DP}$ Adam and Bill]]]

The impossibility of a derivation like (107) is also witnessed by (108), an example borrowed from Jacobson (2016: 341).

(108) A: Who left?

B: *Carefully, John. (compare to *Carefully, John left*.)

Finally, I would like to point out that besides (105), another intriguing contrast between long answers and short answers is observed in Jacobson (2016: 350): short answers convey exhaustiveness while long answers do not. Consider (109).

(109) Q: Who all left the party at midnight?

A: Adam, Bill and Chris.

A: Adam, Bill and Chris left the party at midnight.

According to Jacobson (2016), A's utterance with the short answer conveys only the exhaustive reading: the speaker is committed to the proposition that these three left the party at midnight and that only these three did. In contrast, A's utterance with the long answer does not necessarily express the exhaustiveness.

To sum up, in this section, I have provided a semantic-pragmatic explanation for why and how CON is unavailable in the prenominal position. Specifically, I pursue a quantificational domain hypothesis and anchor the issue of evaluativity to a QUD-based discourse structure and Rooth's representation of focus. Briefly put, the idea behind the hypothesis is that in giving an evaluative answer addressing the CQ, concessive *at least* requires its quantificational domain to be propositional because what the speaker evaluates are some possible and contextually relevant circumstances addressed by different propositions. The quantificational domain hypothesis in (104) gives us a single explanation for **(a)** why the concessive meaning is unavailable when *at least* is at a prenominal position (see (101)), and **(b)** why the concessive meaning is unavailable with short answers (see (105)). Although (105) indicates an intriguing contrast between long answers and short answers, I leave it open as to exactly how short answers are generated (see Weir 2014, Jacobson 2016 and references therein).

2.4.7 Notes on the distribution of EPI

In this section, I briefly look at the distribution of EPI. Recall that according to N&R, EPI is unavailable when English *at least* appears in the sentence-initial position. At this point, I do not have a good explanation yet, as to why EPI should be restricted as it is. However, I would like to point out some key observations that may be helpful for future research. I observe that the distributional restriction on EPI is not unique to English *at least*. Other focus particles like *only* and *even* in English (and potentially their counterparts in other languages as well) are subject to the same restriction.

(110) EPI: #**At least** John won a [silver]$_F$ medal.

cf. EPI: **At least** [John]$_F$ won a silver medal

(111) #**Only** John won a [silver]$_F$ medal.

cf. **Only** [John]$_F$ won a silver medal.

(112) #**Even** John won a [silver]$_F$ medal.

cf. **Even** [John]$_F$ won a silver medal.

The parallel between (110) and (111)/ (112) indicates that the distributional restriction on EPI is part of the long-standing puzzle why the association with focus is not possible when focus particles apparently occur in a sentence-initial position, as shown in (111) and (112). The traditional wisdom is that the association with focus in (111) and (112) is impossible because English *only* and *even* are directly adjoined to the DP subject (rather than to a clausal position) and thus fail to c-command their associates.[55] However, concessive *at least* poses a serious empirical challenge to this traditional view.[56] In particular, in sharp contrast to epistemic *at least*, the association with focus from the clausal position **IS** possible, as evidenced by (113). This means that a clausal position, *a priori*, does **NOT** preclude the possibility of focus association.

(113) CON: **At least** John won a $[\text{silver}]_F$ medal.

Furthermore, concessive *at least* is not the only focus particle that allows focus association from a clausal position. In English, another expression *at the very least*, conveying an ignorance reading, similarly allow the association with focus from a clausal position, as witnessed by (114).

(114) EPI: **At the very least**, John bought $[\text{three}]_F$ apples.

The parallel between (113) and (114) makes two points. First, focus association from a clausal position is possible. Second, the (im)possibility of focus association from a clausal position does not depend on the meaning of focus particles (e.g., EPI vs. CON). Taken together, a theory of focus particles should tell us why the focus

[55] To my knowledge, Jackendoff (1972) is the first study suggesting that a focus operator must c-command its focus associate(s).

[56] A related issue here is the debate on whether focus particles are always adjoined to the clausal spine and banned from adjoining to nominal arguments. Readers are referred to Büring and Hartmann (2001), Reis (2005), Smeets and Wagner (2018) for discussion.

association is impossible in cases like (110) − (112), while possible in cases like (113) and (114).

Recently, Erlewine (2017) provides an interesting syntactic analysis accounting for why the focus association is impossible in cases like (110) − (112), based on Vietnamese. Very briefly, instead of stipulating that focus particles are directly adjoined to the subject DP (contra the traditional view), focus particles are systematically excluded from being merged at the sentence-initial position because the relevant derivation is blocked in syntax. A crucial component of Erlewine (2017)'s analysis is that syntactic derivations are done by multiple phases (derivation-by-phase). The core of Erlewine's analysis is summarized in (115), along with his idea in (116).

(115) Erlewine (2017)'s analysis of Vietnamese exclusive particles:

Sentential focus particles (focus-sensitive sentential modifiers) must be **as low as possible** while **c-commanding** their focus associate, within a given phase.

(116) Erlewine (2017)'s idea in a nutshell:

The syntactic derivation for clause-initial focus particles is **blocked** by that of preverbal ones.

To see how Erlewine's analysis works, consider the two syntactic derivations below. Assume that the derivation is in a bottom-up fashion. Suppose that there is a focus associate (abbreviated as **associate**) inside the predicate and adjoining focus particles (abbreviated as **Foc-adv**) to TP is **in competition with** adjoining focus particles to vP, within the construction of the CP-phase (compare steps (b) and (c) in (117) and (118)). Then, the consequence is that the syntactic derivation of preverbal focus particles systematically blocks that of clause-initial ones; that is, (117) blocks (118)).

(117) The preverbal case

a. [$_{vP}$ …**associate**] within the vP-phase

b. [$_{vP}$ **Foc-adv** [$_{vP}$ **associate**]] within the CP-phase

c. [$_{TP}$ Subject [$_{vP}$ **Foc-adv** [$_{vP}$ **associate**]]] within the CP-phase

(118) The sentence-initial case

a. [$_{vP}$ …**associate**] within the vP-phase

b. [$_{TP}$ Subject [$_{vP}$ **associate**]] within the CP-phase

c. [$_{TP}$ **Foc-adv** [$_{TP}$ Subject [$_{vP}$ **associate**]]] within the CP-phase

Erlewine's analysis elegantly explains why focus association is impossible in cases like (110) − (112). However, it only explains half of our puzzle. The remaining puzzle is: How and why exactly is the focus association in cases like (113) and (114) possible? Note that if we blindly apply Erlewine's analysis, we would predict that the focus association in (113) and (114) is impossible, contrary to the facts.

Assuming that Erlewine (2017) is on the right track, how do we reconcile the conflict between the analysis and the facts shown by concessive *at least* and the expression *at the very least*? One possible line of thinking is to acknowledge that multiple phase heads exist in the rich left-periphery of CPs, given the CP-Split Hypothesis (e.g., Rizzi 1997; See also Krifka 2014 on the syntactic layer of specch acts). In this way, we can explain the possibility of focus association in (113) and (114) by proposing that concessive *at least* and the expression *at the very least* can be structurally higher than focus particles like epistemic *at least*/ *only*/ *even* (crucially, in a higher phase than the first CP phase). The consequence is that unlike epistemic *at least*/ *only*/ *even*, they may NOT participate in the syntactic competition with their preverbal cases. Therefore, the focus association is possible in (113) and (114), even though they appear at a sentence-initial position.

I believe that this is a promising line of investigation on the syntax of focus particles in the future, given the richness of the CP-periphery. At this point, a piece of evidence for the claim that concessive *at least* can be structurally higher comes from the fact that it may scope over speech acts such as imperatives.

(119) **At least**, close the door, please![57]

Notice that to obtain the concessive meaning, the set of relevant alternatives must be of an imperative form such as {*turn off TV*, *close the door*, *leave the room*…}, and certain ranking relation exists between the elements in the set: *turn off TV* ≻ *close the door* ≻ *leave the room*, with respect to the interlocutors' goals in the discourse.

To conclude, so far, we have seen **(a)** what the uniform semantic representation of *at least* is; **(b)** how the EPI-CON ambiguity and the three common properties of the two meanings follow from the current analysis; **(c)** why the concessive meaning has a "settle-for-less" flavor and is unavailable in the prenominal position. Crucially, the two properties of CON are not independent of each other; instead, both can be traced back to the evaluative nature of concessive *at least*. To complete the analysis, the next section illustrates how the ignorance inference is derived, based on the idea that epistemic *at least* addresses the issue of informativity.

2.5 Deriving ignorance inferences

Recall that in section 2.2, we have seen that epistemic *at least* gives an ignorance inference only when a precise answer is requested, for instance, when a *wh*-question serving as the CQ is interpreted exhaustively. Based on the relevant facts, I propose that epistemic *at least* addresses the issue of informativity, as repeated below.

[57] I am grateful to Jane Grimshaw and Roger Schwarzschild for drawing my attention to the interaction between concessive *at least* and speech act operators.

(120) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

In what follows, I present a derivation for ignorance inferences based on the idea in (120), while incorporating the insights from Mendia (2016a-c)'s two scale analysis (see also Schwarz 2016a).[58, 59] To begin with, let's consider a toy context in which epistemic *at least* gives an ignorance inference. Suppose that there are three relevant individuals in the discourse: Adam, Bill and Chris. Consider the question-answer pair in (121) and their corresponding domains shown in (122).

(121) a. Emily: Who left?

b. Frank: **At least** $[\text{Bill}]_F$ left.

(122) a. The domain of (121a): {a, b, c, a⊕b, b⊕c, a⊕c, a⊕b⊕c}[60]

b. The domain of (121b): {b, a⊕b, b⊕c, a⊕b⊕c}

---

[58] In fact, various candidates for the Horn-scale mate with *at least* have been proposed in previous studies: *exactly*, *more than*, *at most*, etc. These are the so-called "two-scale" analysis in Schwarz (2016a). Readers are referred to Schwarz (2016: section 3.2) for discussion.

[59] The precise mechanism of how epistemic *at least* leads to ignorance inferences is an ongoing debate. See Mihoc (2018) for a recent proposal couched in the grammatical approach (Chierchia 2004, 2013, Chierchia et al. 2012, Fox 2007, among others) and a critical review of the previous analyses. An important aspect of Mihoc (2018)'s analysis is that it simultaneously captures the similarity and the difference between bare numerals, comparative modifiers (*more than n*/ *less than n*) and superlative modifiers (*at least n*/ *at most n*), with respect to their (in)ability leadings to scalar implicatures and ignorance inferences under both unembedded environment and embedded environments (universal quantifiers and deontic modals). The choice of the pragmatic approach here is simply for illustration. In this dissertation, I do not take a stand on the debate between the pragmatic approach and the grammatical approach to ignorance inferences/ scalar implicatures. I am confident that the idea here that epistemic *at least* addresses the issue of informativity can be recast and implemented under the grammatical approach without further ado. However, under my view, some challenges to Mihoc (2018) come from the semantics she assigns to SMs (and comparative modifiers); in particular, she considers both types of numeral modifiers as a generalized quantifier denoting a relation between two sets of individuals (*P* and *Q*). This yields the difficulty of extending her analysis to non-numeral cases (e.g., plurality scales, lexical scales and pragmatic scales). Moreover, she assumes a non-strict comparison relation in the semantics of SMs. Overall, I believe that a promising line for future research is to combine the semantics of SMs proposed in this dissertation with Mihoc (2018)'s analysis of ignorance inferences and scalar implicatures for SMs. I leave this line of research for another occasion.

[60] Here, I use the lowercase *a*, *b*, *c* to represent the three individuals Adam, Bill and Chris respectively. In addition, I assume Link (1983)'s semi-lattice structure of plural individuals.

Given the discourse and Emily's question in (97a), the power set of the three individuals (Adam, Bill and Chris) are all relevant. However, after Frank's response in (121b), only four individuals remain in the domain: {b, a⊕b, b⊕c, a⊕b⊕c}. Now, given Frank's response, the hearer is invited to reason about the alternatives in the domain selected by Frank's answer. Following Mendia (2016a-c)'s and Schwarz (2016a)'s two-scale analysis, I assume that the calculation of ignorance inferences given by SMs requires two sources of alternatives. One source of alternatives comes from the set of alternatives induced by focus. The reasoning is as follows: Why didn't Frank choose to assert some higher alternatives?[61]

(123) ALT_{foc} (121b): {(≥b), (≥a⊕b), (≥ b⊕c), (≥a⊕b⊕c)}

Another source of alternatives comes from the set of exhaustified alternatives, given the selected domain (121b). The reasoning is as follows: Why didn't Frank assert the maximally informative unique answer?

(124) ALT_{exh} (121b): {(Exh b), (Exh a⊕b), (Exh b⊕c), (Exh a⊕b⊕c)}

Crucially, unlike Mendia (2016a-c)'s and Schwarz (2016a)'s two-scale analysis, I do not claim that *at least* and *only* form a Horn-scale. The set of exhaustified alternatives in (124) does not come from the substitution of *at least* with *only*; instead, they come from the exhaustivity of the *wh*-question (i.e., the CQ). This nuanced perspective is based on Westera and Brasoveanu (2014)'s insights and our discussion in section 2.2. Now, assuming **K** is an epistemic necessity operator over the speaker's knowledge/ belief, and **P** a possibility operator "the speaker considers it possible" (Gazdar 1979), the computation of ignorance inferences is similar to that under a two-scale analysis.

[61] Here, the ranking is based on the semi-lattice structure of plural individuals in Link (1983). As before, I enclose a proposition by parenthesis. Also, I use (≥ φ) to represent (*at least* φ) and (Exh φ) to represent (*only* φ).

First, we generate a quality inference and a set of primary implicatures.[62]

(125) a. Quality Inference

$K(\succeq b)$

b. Primary Implicatures

$\neg K(\text{Exh } b) \wedge$

$\neg K(\text{Exh } a\oplus b) \wedge \neg K(\text{Exh } b\oplus c) \wedge \neg K(\text{Exh } a\oplus b\oplus c) \wedge$

$\neg K(\succeq a\oplus b) \wedge \neg K(\succeq b\oplus c) \wedge \neg K(\succeq a\oplus b\oplus c)$

The quality inference is generated because of the default assumption that the speaker believes what he asserts. The set of primary implicatures is generated: because the speaker didn't assert one of the stronger alternatives, it must be that he didn't have sufficient evidence to claim so (i.e., he didn't believe those alternatives to be true).

(126) Implicature Base: quality inference & primary implicatures

$K (\succeq b) \wedge$ (125b)

As observed by Mendia (2016b,c), in a partially-ordered structure like the semi-lattice structure of plural individuals, the space of logical possibilities is **exhasutifed** by the following three conjuncts, indicated by **P1**, **P2** and **P3**, mnemonic of *possibilities*. Specifically, negating **any two** of the three epistemic possibilities (e.g., P1 and P2) would entail the truth of the third one (e.g., P3).

(127) $\underbrace{K (\succeq b)}_{\text{Assertion}} \wedge \underbrace{\neg K(\text{Exh } b)}_{\textbf{P1}} \wedge \underbrace{\neg K(\succeq a\oplus b)}_{\textbf{P2}} \wedge \underbrace{\neg K(\succeq b\oplus c)}_{\textbf{P3}}$

Crucially, asserting that *at least Bill left* is **NOT** at odds with knowing that *Bill was not the only person who left*, expressed as $K \neg (\text{Exh } b)$, though it requires ignorance

[62] Note that K and P are defined interchangeably: $K(p) \equiv \neg P\neg (p)$ and $P(p) = \neg K\neg(p)$.

about the identity of Bill's companions in our model. Therefore, (127) can be further represented as (128).[63]

(128) $K(\succeq b) \wedge \neg K(Exh\ b) \wedge \underline{K\neg (Exh\ b)} \wedge \neg K(\succeq a\oplus b) \wedge \neg K(\succeq b\oplus c)$

In (128), knowing that $K \neg (Exh\ b)$ (the underlined part) does NOT settle the question as to which one of $K(\succeq a\oplus b)$ or $K(\succeq a\oplus b)$ might be true. Therefore, two possibility inferences are entailed: **(a)** $P(\succeq a\oplus b) \equiv \neg K\neg (\succeq a\oplus b)$; **(b)** $P(\succeq b\oplus c) \equiv \neg K\neg (\succeq b\oplus c)$. By conjoining the relevant primary implicatures and the two possibility inferences, the two ignorance inferences are generated in (129).

(129) Ignorance Inferences from Frank's answer: *At least* [*Bill*]$_F$ *left*

a. $\neg K(\succeq a\oplus b) \wedge \neg K\neg (\succeq a\oplus b)$

b. $\neg K(\succeq b\oplus c) \wedge \neg K\neg (\succeq b\oplus c)$

(129) amounts to saying **(a)** that the speaker is ignorant about whether Adam and Bill left and **(b)** that the speaker is ignorant about whether Bill and Chris left.[64] Taken together, Frank's assertion *at least Bill left* is compatible with his certainty that Bill left, but crucially conveys ignorance about whether Adam also left and the ignorance about whether Chris also left.

It is worth noting that the current analysis is similar to Mendia (2016a-c)'s and Schwarz (2016a)'s two-scale analysis in three crucial aspects: **(a)** the calculation of ignorance inferences given by SMs requires two sources of alternatives; **(b)** the core of ignorance inferences is the *symmetric* nature of the alternatives; **(c)** the nature of

[63] In other words, in a partial ordering structure, the speaker ***can*** be certain/ non-ignorant about the exhaustive interpretation of the prejacent. This forms a contrast to a total ordering structure, where the speaker ***must*** be ignorant about the exhaustive interpretation of the prejacent. This is one key insight in Mendia (2016b, c). I refer readers to his papers for details.

[64] As in Geurts (2010), an ignorance inference that the speaker is ignorant about a proposition *p* can be understood as follows: *the speaker does not believe p to be true* and *the speaker does not believe p to be false*. The signature of an ignorance inference can be represented as follows: $\neg Bel_s\, p \wedge \neg Bel_s\, \neg p$.

ignorance inferences given by SMs is quantity-based conversational implicatures. However, the current analysis crucially differs from the previous two-scale analyses in that no stipulation on the Horn Scale-mates of *at least* is needed. Among those two-scale analyses, Schwarz (2016a: section 3.2) argues that *only* (but not other elements like *exactly*, *more than* or *at most*) forms a Horn-scale with *at least* and presents a syntactic argument: *only* has the same syntactic distribution as *at least*.

(130) a. Al **at least/ only** hired two cooks.

b. Al is **at least/ only** allowed to hire two cooks.

I believe that Schwarz's syntactic argument is fairly reasonable because having the same syntactic distribution is a precondition for the substitution of Horn-scale mates (see also Katzir 2007). However, Schwarz's syntactic argument holds for English but not necessarily for other languages. For example, Buccola and Haida (2017) points out that the German equivalent of *at least* (i.e., *mindestens* 'at least') has wider distribution than the German equivalent of only (i.e., *nur* 'only').

(131) *Ann hat zwei Hunde besessen,* ***mindestens/ *nur****.*

Ann has two dogs owned at least only

'Ann has at least/ only two dogs.'

Crucially, German is not alone in this respect. Chinese is another language that makes the same point, as in (132). If the calculation of ignorance inferences crucially relies on the substitution of the Horn Scale-mate of *at least*, the failure of the substitution would wrongly predict that no ignorance inference arises in (131) and (132).

(132) *Liubei xie-le* ***zhishao/ *zhiyou*** [*san*]$_F$-*ben-xiaoshuo*.

Liubei write-ASP at least only three-CL-novel

'Liubei wrote at least/ only three novels.'

The current analysis avoids these empirical problems because the set of exhasutified alternatives does not come from the substitution of *at least* with *only*, but from the exhaustivity of the *wh*-question (the CQ). Seen in this light, the current analysis not only addresses the question of *how* an ignorance inference arises with *at least*, but also connects it with the question of *why* an ignorance inference rises with *at least*. The two questions are not independent of each other; the answer to both questions is traced back to the idea that epistemic *at least* addresses the issue of informativity: ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.[65]

Finally, I briefly point out how *at least* is parallel to *disjunction* and *epistemic indefinites* under my analysis. First, *at least*, like the latter two, gives ignorance inferences. However, ignorance inferences given by *at least* may be cancelled because of the sensitivity to the maxim of quantity and other contextual factors (see Fox 2014 for disjunction). In contrast, those given by epistemic indefinites seem obligatory. Second, *at least* has an anti-specific domain. Traditionally, this is encoded by a non-strict comparison relation, which has been understood as a disjunction relation or recast as an anti-specificity presupposition. Under the analysis in section 2.4, the non-strict comparison relation is not a semantic primitive; it is derived from focus presuppositions combining with the superlative component of *at least*.

2.6 Conclusion

In this chapter, I have taken English *at least* as a case study and presented a unified analysis of the two meanings generally shown by SMs across many languages: an

[65] One important insight of Schwarz (2016a) is to import Fox (2007)'s method of Innocent Exclusion as a consistency check, into a Gricean calculation of ignorance inferences. Although the current mechanism does not make use of Innocent Exclusion, I believe that the analysis here can be adapted in the manner represented in Schwarz (2016a) to incorporate his insights.

epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. In the analysis, I have shown **(a)** what the uniform semantic representation of *at least* is; **(b)** how the EPI-CON ambiguity arises from one unified semantic entry combining with different pragmatic factors such as informativity and evaluativity; **(c)** how the three common properties of the two meanings follow from the current analysis: the focus-sensitivity, the compatibility with different scales, the two scalar effects and their discrepancy; **(d)** why the concessive meaning has a "settle-for-less" flavor and is unavailable in the prenominal position. Crucially, the two properties of CON are not independent of each other; instead, both can be traced back to the evaluative nature of concessive *at least*. *Ceteris paribus*, the current analysis is expected to hold for the cross-linguistic counterparts of *at least* demonstrating the ambiguity (see section 2.1.1 for a sample list).

The core ingredients for the unified analysis of the ambiguity are listed below.

(133) **A propositional version of *at least***

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$$

(134) **A non-propositional version (by the Geach rule)**

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\ st>}\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

(135) **A non-propositional version (by the backward Geach rule)**

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\lambda P_{<\eta,\ st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) < \mu_c(P_w(\beta))]]$$

(136) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

(137) **CON and the issue of evaluativity**

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is true; while it is not the best situation, it is not the worst situation either.

(138) **Scalarity** (the semantic core of EPI and CON)

The set of focus alternatives (the set of answers addressing the CQ) is ordered along a contextually given scale.

Chapter 3 *At least* in discourse

This chapter takes a close look at discourse properties of concessive *at least* and epistemic *at least*. In the spirit of the unified account presented in chapter 2, this chapter presents a formal analysis and illustrates how concessive *at least* and epistemic *at least* are pragmatic variants. The central idea is that although concessive *at least* and epistemic *at least* share one single semantic representation (see chapter 2), they have different discourse profiles. Adopting the discourse model (i.e., the conversational scoreboard, incorporating the insights from Lewis 1979) presented by Farkas and Bruce (2010) and further developed in Malamud and Stephenson (2015) and Beltrama (2018), it is argued the discrepancy in their discourse profiles crucially comes from where propositions are added in the conversational scoreboard. More specifically, assertions with concessive *at least* add associated propositions to the speaker's present commitments, while assertions with epistemic *at least* add associated propositions to the speaker's projected commitments. Furthermore, in this chapter, a number of issues are discussed in detail: What is put forward in the discourse under assertions with concessive/ epistemic *at least*? What is placed on the Table under assertions with concessive/ epistemic *at least*? What are the participants' discourse commitments under assertions with concessive/ epistemic *at least*? What is updated/ added to the discourse under assertions with concessive/ epistemic *at least*? How are assertions with concessive/ epistemic *at least* different from (factual) assertions? Finally, the connection between the unified semantic representation of *at least* (proposed in chapter 2) and their different discourse profiles under the two readings (ignorance vs. concession) is also discussed in this chapter.

This chapter is structured as follows. Section 3.1 first introduces a distinction in propositional content between what is at-issue and what is not-at-issue, and then discusses discourse properties of concessive *at least* and epistemic *at least*, with respect to their (not-)at-issue content. Section 3.2 presents a formal analysis of concessive *at least* and epistemic *at least*, capturing their different discourse profiles in terms of the conversational scoreboard. Section 3.3 concludes the chapter.

3.1 The basic facts

This section is devoted to the information status of concessive *at least* and epistemic *at least*. In particular, section 3.1.1 first introduces the distinction between at-issue and not-at-issue information content, and then sections 3.1.2 − 3.1.3 present empirical facts showing that **(a)** under both readings, the information content of the prejacent is always at-issue; **(b)** under both readings, the information content of the lower alternatives is at-issue; **(c)** under both readings, the information content of the ranking between the prejacent and its alternatives seems to be not-at-issue; **(d)** under the concessive reading, the information content of the higher alternatives can be at-issue or not-at-issue; while under the ignorance reading, it is at-issue; **(e)** the speaker's discourse commitments are different under the two readings: under the concessive reading, the speaker is committed to both the truth of the prejacent and the falsity of the higher alternatives; in contrast, under the ignorance reading, the speaker is *tentatively* committed to the truth of the prejacent or that of the higher alternatives.

3.1.1 At-issue vs. not-at-issue content

Recent studies on appositves, evidentials, and expressives among other expressions, have motivate the view that it is necessary to distinguish meaning contributions further than our traditional categories of what is asserted, what is presupposed and what is implicated (e.g., Potts 2005, 2007, McCready 2008, 2010, Simons et al. 2010, Murray 2010, 2014, AnderBois et al. 2010, 2015, Tonhauser 2012, Koev 2012, 2013, Tonhauser et al. 2013, Syrett and Koev 2015, among many others). In particular, many of these analyses maintain a distinction in propositional content between what is at-issue and what is not-at-issue. For example, Simons et al. (2010) suggests the following characterization of the notion of at-issueness, within the framework of Question-Under-Discussion (QUD).

(1) The characterization of at-issueness (Simons et al. 2010: 323)

a. A proposition *p* is **at-issue** iff the speaker intends to address the QUD via ?*p*.

b. An intention to address the QUD via ?*p* is **felicitous** only if:

i. ?*p* is relevant to the QUD, and

ii. the speaker can reasonably expect the addressee to recognize this intention.

The definition of at-issue content in (1) suggests the following distinction between at-issueness and non-at-issueness: A proposition expressed by a constituent is at-issue if it contributes to the ordinary semantics of the clause in which it is located, and entails that some possible answer to the QUD is false; otherwise the proposition is not at-issue. Along this line of thought, Tonhauser (2012) further suggests three discourse properties of at-issue content: **(a)** at-issue content can be directly assented or dissented with; **(b)** at-issue content addresses the question-under-discussion; **(c)** at-issue content determines the relevant set of alternatives. Given these discourse properties of at-issue content, the following two tests have been applied to diagnose whether a given (propositional) content is at-issue or not-at-issue in the discourse (Simons et al. 2010, AnderBois et al. 2010, 2015, Murray 2010, 2014, Tonhauser 2012, Koev 2012, 2013).

(2) Direct Response Test

Only at-issue content can be directly targeted by the addressee, e.g., by replies like “Yes”, “No”, “That’s not true”, etc., in subsequent discourse.

(3) Answerability Test

Only at-issue content can be employed by interlocutors to answer questions.

Below, I briefly illustrate how these two tests distinguish not-at-issue content from at-issue content. Let’s first consider the direct response test. In (4), speaker A’s utterance containing a clause-medial appositive (i.e., *who was talking to Mary a minute ago*) contributes to both at-issue meaning and not-at-issue meaning, as in (5). Crucially, at-issue meaning is directly challengeable (as indicated in speaker B’s reply), while not-at-issue meaning is not (as indicated in speaker B’’s reply). In this line, the example with expressives (i.e., *that bastard*) in (6) makes the same point: speaker A’s utterance conveys a meaning that Bill is a bastard; crucially, this meaning is not directly challengeable, in contrast to the at-issue meaning that Bill stole my car.

(4) Clause-medial appositives (adapted from Koev 2012: (6))

A: John, who was talking to Mary a minute ago, has gone home.

B: No, he hasn't. He is still at the party.

B’: #No, he wasn't. He was talking to Stacy.

(5) a. At-issue meaning: John has gone home.

b. Not-at-issue meaning: John was walking to Mary a minute ago.

(6) Expressives (adapted from Potts 2007: 168)

A: That bastard Bill stole my car.

B: No, he didn't. It was John that stole it.

B': #No, he isn't (a bastard). Bill is a very nice person.

(7) a. At-issue meaning: Bill stole my car.

b. Not-at-issue meaning: Bill is a bastard.

Next, let's consider the answerability test. In (8), speaker B's utterance containing a clause-medial appositive (i.e., *who had prostate cancer*) contributes to both at-issue meaning and not-at-issue meaning, as in (9). Crucially, only at-issue meaning can be used as an answer to the question-under-discussion: even though the semantic content of the appositive does answer the question, speaker B's response (in contrast to speaker B''s) is infelicitous in (8). Put differently, the test indicates that the semantic content of the appositive is not-at-issue and thus cannot be used to answer questions.

(8) Clause-medial appositives (borrowed from Koev 2012: (11))

A: What disease did Tammy's husband have?

B: #Tammy's husband, who had prostate cancer, was treated at the Dominican Hospital.

B': Tammy's husband had prostate cancer.

(9) a. At-issue meaning: Tammy's husband was treated at the Dominican Hospital.

b. Not-at-issue meaning: Tammy's husband had prostate cancer.

Although the examples illustrated here for the two diagnoses involve only expressives and appositives, expressions contributing to not-a-issue content go far beyond those two types; see e.g., Murray (2010, 2014) on evidentials, and Simons et al. (2010), Tonhauser (2012), Tonhauser et al.(2013) on how and why projective content projects in general and how the projection behavior is connected with information structure.

Summing up, in this section, we have seen that **(a)** not-at-issue content cannot be directly assented or dissented with, in contrast to at-issue content; **(b)** not-at-issue content cannot be used to answer the QUDs, contrasting with at-issue content. Although at-issue content is roughly what is proffered (contributed by a regular factual assertion), the group of expressions contributing to not-at-issue content is heterogeneous: it includes appositives, expressives, and among other expressions. In the next two sections, we discuss how concessive *at least* and epistemic *at least* fare with the two tests on the distinction between at-issue and not-at-issue content.

3.1.2 The information status of concessive *at least*

First of all, let us observe that under concessive *at least*, the propositional content of the prejacent and that of its higher alternatives are both at-issue. Suppose that Adam, Bill and Chris are the three relevant individuals in the discourse. Consider (10).

(10) A: Who did John invite? Did John invite Adam, Bill and Chris?

B: No, John didn't invite Adam, Bill and Chris.

But, **at least** he invited [Adam and Bill]$_F$.

In (10), speaker A explicitly raises an issue concerning whether the content of the higher alternative (i.e., John invited Adam, Bill and Chris) is true and speaker B's assertion serves as a proposal to settle the raised issue. These discourse moves indicate that the content of the higher alternative is at-issue (by the answerability test). In the same vein, the three possible responses (to speaker B's assertion) in (11) − (13) further illustrate that not only the propositional content of the higher alternative, but also that of the prejacent are at-issue (by the direct response test).

(11) C: That's true!

(12) C: No, that's not true! John only invited Adam.

(13) C: No, that's not true! John invited all of them.

In (11), speaker C **assents** to speaker B's assertion with respect to the content that the higher alternative (i.e., John invited Adam, Bill and Chris) is false and that the prejacent (i.e., John invited Adam and Bill) is true. This indicates that the propositional content of the prejacent and that of its higher alternative are both at-issue. In (12), speaker C **assents** to speaker B regarding the content that the higher alternative (i.e., John invited Adam, Bill and Chris) is false, but **dissents** with speaker B from the content that the prejacent is true. This again indicates that the propositional content of the prejacent and that of its higher alternative are both at-issue. Crucially, notice that the lower alternative (i.e., John only invited Adam) is involved as part of speaker C's response addressing speaker A's question. This indicates that the propositional content of the lower alternative is also at-issue. Finally, in (13), speaker C **dissents** with speaker B on the content that the higher alternative is false. This indicates that the propositional content of the higher alternative is at-issue.

In short, the fact that the three responses from speaker C in (11) − (13) are all felicitous to speaker B's assertion in (10) indicate that the propositional content of the prejacent and those of its (higher/ lower) alternatives are all at-issue, because they can be directly assented or dissented between interlocutors in subsequent discourse.

Next, let us observe that while the propositional content of the prejacent remains at-issue, the propositional content of the higher alternative can be not-at-issue. Suppose that Adam, Bill and Chris are the three relevant individuals in the discourse. Consider (14), where the polar question targets the content of the prejacent.

(14) Context: Speaker A wants to know who John invited for his party last night.

She is particularly interested in whether John invited Adam and Bill;

But she does not know that John did not invite Chris.

A: Who did John invite? Did John invite Adam and Bill?

B: Yeah, **at least** he invited [Adam and Bill]$_F$.

A: **Hey, wait a minute!** John didn't invite Chris/ all of them?

I didn't know that John only invited Adam and Bill.

In (14), speaker A explicitly raises an issue concerning whether the content of the prejacent (i.e., John invited Adam and Bill) is true and speaker B's assertion serves as a proposal to settle the raised issue. These discourse moves indicate that the content of the prejacent is at-issue (by the answerability test). Crucially, speaker A is entitled to use the phrase "hey, wait a minute" and conveys his surprise at the fact that the propositional content of the higher alternative (i.e., John invited Adam, Bill and Chris) is false. The so-called "hey, wait a minute" test is first discussed in Shannon (1976) and later introduced in von Fintel (2004) as a diagnostic of speaker presupposition. Below, (15) illustrates the fact that at-issue content cannot enter into the frame "Hey, wait minute. I didn't know…", in contrast to not-at-issue content (e.g., presuppositions among others).

(15) Mary's aunt is visiting today. (from Pearson 2010: (1))

a. #**Hey, wait a minute!** I didn't know Mary's aunt is visiting today.

b. **Hey, wait a minute!** I didn't know Mary has an aunt.

(16) The (not-)at-issue content in (15)

a. Assertion (at-issue): Mary's aunt is visiting today.

b. Presupposition (not-at-issue): Mary has an aunt.

Given examples like (15), the felicitous use of "Hey, wait minute" by speaker A in (14) indicates that the information content that the higher alternative is false is not-at-issue. Crucially, examples like (14) further suggest that the requirement of the concessive reading that the higher alternatives are known to be false in the discourse should be **speaker-oriented**, rather than interlocutors-based (i.e., part of the common ground).

Below, (17) demonstrates that the information content that the higher alternative (i.e., John invited Adam, Bill and Chris) is false **can** be part of the common ground for the interlocutors in a given discourse. Note that like the conversation in (14), the polar question by speaker A in (17) targets the content of the prejacent. (18) − (20) illustrate three possible continuations (by speaker C) to speaker B's assertion. Notice that (18) and (19) are felicitous continuations, while (20) is not.

(17) Context: Speaker A wants to know who John invited for his party last night. All the three speakers A, B and C know that John didn't invite Chris. Speaker A is interested in whether the other two people (i.e., Adam and Bill) are invited.

A: Who did John invite? Did John invite Adam and Bill?

B: Yeah, **at least** he invited [Adam and Bill]$_F$.

(18) C: That's true!

(19) C: No, that's not true! John only invited Adam.

(20) C: #No, that's not true! John invited all of them.

Given the context, it is part of the common ground that the propositional content of the higher alternative (i.e., John invited Adam, Bill and Chris) is false. More specifically, the interlocutors are committed to the fact that the higher alternative is false. In (18), speaker C **assents** with speaker B with respect to the content that the prejacent (i.e., John invited Adam and Bill) is true. This indicates that the propositional content of the prejcent is at-issue. In (19), speaker C **dissents** with speaker B on the content that the prejacent is true. Notice that the lower alternative (i.e., John only invited Adam) is involved as part of speaker C's response addressing speaker A's question. This indicates that besides the prejacent, the content of the lower alternative is also at-issue. Finally, the assertion in (20) is infelicitous because it is self-contradictory: given the common ground, speaker C has committed herself to the fact that the content of the higher alternative (i.e., John invited Adam, Bill and Chris) is false, but she continues to assert that the content of the higher alternative is true.

At this point, it is worth noting that the information content regarding the ranking between the prejacent and its alternatives seems to be not-at-issue, given that it cannot be directly assented or dissented across the conversations we have seen above. For example, (18) cannot target a meaning like "John's inviting Adam, Bill and Chris is better than/ is ranked above his inviting (only) Adam and Bill". Thus, I tentatively conclude that the ranking information is not-at-issue, while the content of the prejacent is at-issue and that of its higher alternatives can be at-issue or not-at-issue.

To sum up, we have seen in this section that under concessive *at least*, the propositional content of the prejacent is always at-issue, but the propositional content of the higher alternative can be at-issue or not-at-issue, depending on the discourse. Put differently, under the assertion with concessive *at least*, the speaker is committed to both the truth of the prejacent and the falsity of the higher alternative; however, the discourse commitment to the falsity of the higher alternative can be a result of the assertion (see (10)) or part of the common ground (see (14) and (17)). Finally, the content of the lower alternative is at-issue because it addresses the QUD (see (12) and (19)); the ranking between the prejacent and its alternatives is not-at-issue because it cannot be directly assented or dissented. The next section discusses epistemic *at least*.

3.1.3 The information status of epistemic *at least*

To begin with, let us observe that under epistemic *at least*, the propositional content of the prejacent and that of its higher alternatives are both at-issue. Suppose that Adam, Bill and Chris are the three relevant individuals in the discourse. Consider (21).

(21) A: Who did John invite?

B: John invited **at least** [Adam and Bill]$_F$.

In (21), speaker A explicitly raises an issue concerning who John invited; assuming that the *wh*-question imposes an existential presupposition that John has invited someone (e.g., Dayal 2016), the domain of speaker A's question consists of a set of propositional alternatives in the structure of a semi-lattice. Next, speaker B's assertion serves as a proposal to settle the raised issue. Crucially, the issue raised by speaker A is not completely resolved under speaker B's assertion, because there are two possibilities remaining open: either John invited all the three individuals (Adam, Bill and Chris), or John (only) invited Adam and Bill. In this sense, speaker B's utterance with *at least* provides only a partial answer. (22) presents an update of the domain.

(22) a. The domain of speaker A's question:

$$\left\{\begin{array}{l} \lambda w.\text{John invited}_w \text{ Adam, Bill and Chris;} \\ \lambda w.\text{John invited}_w \text{ Adam and Bill; } \lambda w.\text{John invited}_w \text{ Bill and Chris;} \\ \lambda w.\text{John invited}_w \text{ Adam and Chris; } \lambda w.\text{John invited}_w \text{ Adam;} \\ \lambda w.\text{John invited}_w \text{ Bill; } \lambda w.\text{John invited}_w \text{ Chris;} \end{array}\right\}$$

b. The domain updated after speaker B's assertion:

$\{\lambda w.\text{John invited}_w \text{ Adam, Bill and Chris; } \lambda w.\text{John invited}_w \text{ Adam and Bill}\}$

Recall that Tonhauser (2012) suggests three discourse properties of at-issue content: **(a)** at-issue content can be directly assented or dissented with; **(b)** at-issue content addresses the question-under-discussion; **(c)** at-issue content determines the relevant set of alternatives. According to properties (b) − (c) and (22), the alternatives in the domain on which epistemic *at least* is operating, namely the prejacent and its higher/ lower alternatives, all contribute to at-issue content (by the answerability test). Moreover, their discourse property of at-issueness is further confirmed by the fact that the alternatives can be directly assented or dissented, as illustrated in (23) and (24).

(23) C: That's true!

(24) C: No, that's not true! John invited only Adam.

In (23), speaker C **assents** with speaker's assertion with respect to the content that either the prejacent or its higher alternative is true. In (24), speaker C **dissents** with speaker B with respect to the content that the prejacent or its higher alternative is true. Crucially, the lower alternative (i.e., John invited (only) Adam) is involved as part of speaker C's response addressing speaker A's question. This indicates that the propositional content of the lower alternative is also at-issue. Taken together, these discourse moves indicate that the propositional content of the prejacent and those of its higher/ lower alternatives are all at-issue (by the direct response test).

So far, we have seen that the information content of the alternatives on which epistemic *at least* is operating are all at-issue. Now, let us consider what the speaker's discourse commitments are under epistemic *at least*. First, the two responses in (25) and (26), where the prejacent or the higher alternative is targeted, are degraded or not justified. In (25), speaker C challenges speaker B on the content that the prejacent is true, while asserting that the content of the higher alternative is true. In (26), speaker

C challenges speaker B on the content that the higher alternative is true, while asserting that the content of the prejacent is true.

(25) C: ?No, that's not true! John invited all of them/ Adam, Bill and Chris.

(26) C: ?No, that's not true! John invited (only) [Adam and Bill]$_F$.

Second, when epistemic *at least* is absent in speaker B's assertion, speaker C's challenge is akin to a contradiction. Consider (27).

(27) A: Who did John invite?

B: John invited [Adam and Bill]$_F$.

C: #No, that's not true! John invited [Adam and Bill]$_F$.

The contrast on the degradedness of the objection between (25)/ (26) and (27) suggests that under the assertion with epistemic *at least*, speaker B does not *fully* commit herself to the necessary truth of the prejacent and that of the higher alternative. Instead, what speaker B is committed to seems to be the *possibility* that the prejacent is true and the *possibility* that the higher alternative is true *in subsequent discourse*. I argue that this is why epistemic *at least* has some flavor of epistemic modals. More specifically, the modal flavor does not arise at the level of lexical semantics, but at the level of pragmatics concerning the speaker's commitments in the discourse. Seen in this light, the task then is how to capture this modal flavor without hard-wiring a modal component into the meaning of SMs.[1] Along this line of thought, the use of epistemic *at least* is infelicitous when the speaker **knows** that the content of the

[1] The fact that SMs have a flavor of epistemic modals has led a number of researchers to consider SMs as expressions involving a covert epistemic modal in one way or another: Geurts and Nouwen (2007), Nouwen (2010), cf. Spychalska (2018). However, the modal approach has faced many serious empirical challenges: one of them is the fact that SMs can be embedded under deontic modals. This is surprising and puzzling, because epistemic modals resist being embedded under deontic modals across languages (Hacquard 2006 and references therein). For discussion of the modal approach, see Coppock and Brochhagen (2013: section 4) and Mihoc (2018: section 3); see also Kennedy (2015) for a detailed discussion of Nouwen (2010) where a covert epistemic modal is inserted as a last resort.

prejacent is true or the content of the higher alternative is true, as in (28) and (29).

(28) Context: Speaker B knows that John has won a silver medal in the race.

A: What medal did John win in the race?

B: #John won **at least** a $[\text{silver}]_F$ medal.

(29) Context: Speaker B knows that John has won a gold medal in the race.

A: What medal did John win in the race?

B: #John won **at least** a $[\text{silver}]_F$ medal

In (28), the use of *at least* is infelicitous, presumably because the speaker could have been more informative (using an alternative utterance *John won a silver medal*); put differently, the possibility that John won a gold medal is unavailable in the discourse. Similarly, the use of *at least* is infelicitous in (29), presumably because the speaker could have been more informative (uttering *John won a gold medal*); again, the possibility that John won a silver medal is unavailable in the discourse. Taken together, what (25)/ (26) and (28)/ (29) show is that the use of epistemic *at least* is felicitous, only when the speaker is **ignorant** about whether the prejacent is true and whether its higher alternative is true (if the maxim of quantity is active in the discourse).[2] Crucially, this speaker ignorance allows the *possibility* that the prejacent is true and the *possibility* that the higher alternative to both project in subsequent discourse.[3]

[2] Recall that in chapter 2 (section 2.2), we have seen cases where the speaker may not be ignorant when she uses epistemic *at least*. Crucially, in those cases, the maxim of quantity is *deactivated* in the discourse, e.g., the scenario of TV show repeated below; thus no ignorance inferences arise.

(i) Context: In a game, my friend has to guess the number of marbles that I have hidden. I know how many I have hidden and she knows that I have that information. I provide the clue below:

I have at least five marbles.
~> no ignorance about the number of marbles that I have

[3] The same observation applies to disjunction, where in subsequent discourse, the content of each disjunct must be (epistemically) possible to the speaker.

(i) Context: Speaker B knows that John read Hamlet yesterday.

It has been noted in previous studies that SMs are compatible with partial ignorance (Mendia 2016c, Schwarz 2016, among others).[4] Specifically, the speaker may not be completely ignorant when she uses epistemic *at least*, as shown below.

(30) A: How many apples did John buy yesterday?

B: John bought **at least** [three]$_F$ apples.

But I know that he didn't buy {six/ seven/ eight…} apples.

(30) indicates that when using epistemic *at least*, the speaker may know that some of the higher alternatives are false. I believe that the compatibility of SMs with partial ignorance does not conflict with the conclusion drawn from examples like (28)/ (29). In particular, when there are multiple higher alternatives (as in the numeral case above), the speaker may know that some of them, but crucially not all of them, are false in the discourse. In contrast, however, when there is only one higher alternative, as in the case of plurality scales (see (21)) or lexical scales (see (28) and (29)) above, the speaker would be ignorant about whether the higher alternative is true. Taken together, I conclude that in any case, when using epistemic *at least*, the speaker does not fully commit herself to the necessary truth of the prejacent or its higher alternative(s); instead, what she is committed to is the **projection** of the possibility

---

A: What did John read yesterday?
B: #John read Hamlet or Macbeth.

(ii) Context: Speaker B knows that John read Macbeth yesterday.
A: What did John read yesterday?
B: #John read Hamlet or Macbeth.

[4] The same compatibility with partial ignorance is observed with *at most*.

(i) A: How many apples did John buy yesterday?
B: John bought at most [five]$_F$ apples. But I know that he bought at least two apples.

In this respect, SMs form a sharp contrast with disjunction, which necessarily lead to *total ignorance* (see Mendia 2016c and Schwarz 2016 for discussion).

(ii) A: How many apples did John buy yesterday?
B: John bought exactly three apples or more than three apples.
#But I know that he didn't buy {six/ seven/ eight…} apples.

that the prejacent is true and the possibility that the higher alternative/ some of the higher alternatives is/ are true **in subsequent discourse**.

Finally, like the case of concessive *at least*, the information content regarding the ranking between the prejacent and its alternatives seems to be not-at-issue under epistemic *at least*, as well. For example, (23) cannot target a meaning like "John's inviting Adam, Bill and Chris is better than/ is ranked above his inviting (only) Adam and Bill". Therefore, I tentatively conclude that the ranking information is not-at-issue under epistemic *at least*, as in the case of concessive *at least*.

To sum up, in this section, we have seen that under epistemic *at least*, the propositional content of the prejacent and its higher/ lower alternatives are all at-issue, and the ranking information between the prejacent and its alternatives is not-at-issue. Finally, the use of epistemic *at least* is felicitous, only when the speaker is ignorant about whether the prejacent is true and whether its higher alternative is true (if the maxim of quantity is active in the discourse). Crucially, this speaker ignorance allows the *possibility* that the prejacent is true and the *possibility* that the higher alternative to both project in subsequent discourse. I consider that the projection of the two possibilities in subsequent discourse is the source of the modal flavor associated with SMs. However, I don't analyze SMs as modal expressions. Therefore, the task is then **(a)** how to capture the modal flavor without hardwiring a covert epistemic modal into the semantics of SMs; **(b)** how to capture the intuition that the speaker does not *fully* commit herself to the necessary truth of either the prejacent or its higher alternatives, and what the speaker is committed to are the possibility that the prejacent is true and the possibility that the higher alternative is true in subsequent discourse.

3.1.4 An interim summary

Below, (31) and (32) summarizes the discourse profile of concessive *at least* and epistemic *at least* for the (not-)at-issueness and the speaker's discourse commitments.

(31) **Concessive *at least***

a. The prejacent is always at-issue, while the higher alternatives can be at-issue or not-at-issue. The lower alternatives are at-issue.

b. The ranking information is not-at-issue.

c. The speaker is committed to both the falsity of the higher alternatives and the truth of the prejaent.

(32) **Epistemic *at least***

a. The prejacent and the higher/ lower alternatives are all at-issue.

b. The ranking information is not-at-issue.

c. The speaker does not *fully* commit herself to the necessary truth of either the prejacent or the higher alternatives. Instead, what the speaker is committed to is the *possibility* that the prejacent is true and the *possibility* that the higher alternative is true in subsequent discourse.

At this point, let us briefly discuss how the two different discourse profiles of *at least* is connected with its unified semantics (in chapter 2). Notice that concessive *at least* and epistemic *at least* pattern together in three respects: **(a)** the prejacent is at-issue; **(b)** the lower alternatives are at-issue; **(c)** the ranking information is not-at-issue. These three discourse properties are not completely unexpected under the unified semantic representation of *at least*. The propositional entry is repeated in (33).

(33) **A propositional version of *at least***

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$$

In particular, according to (33), the core semantic contribution of *at least* under both readings is to exclude the lower alternatives and assert that one element in the set consisting of the prejacent and its higher alternatives is true. Notice that although the contribution of *at least* relies on the existence of the ranking, the information content of the ranking itself does not address the QUD and is thus expected to be not-at-issue. In contrast, the contribution of *at least* excluding the lower alternatives and conveying that either the prejacent or its higher alternative is true (partially) resolves the QUD. The content of the prejacent and its lower alternatives are thus expected to be at-issue.

Although concessive *at least* and epistemic *at least* share the three discourse properties, the two readings crucially differ in two respects: **(a)** whether the higher alternatives are known to be false in the discourse (an important insight from Biezma 2013's pragmatic analysis); **(b)** what the speaker's discourse commitments are when she makes an assertion with *at least*. These two pragmatic points of variation are precisely the locus of how the two readings are pragmatic variants under one single semantic entry of *at least*. Crucially, the proposed semantics of *at least* leave room for the pragmatics of its two readings. For the first pragmatic point of variation, under concessive *at least*, the higher alternatives must be **known to be false** in the discourse; while they must be **left open** to be true/ false in the discourse under epistemic *at least*. The proposed semantics of *at least* is in principle compatible with these two pragmatic situations, because the truth-conditions require that either the prejacent or its higher alternative(s) be true and the truth-conditions are met in both pragmatic situations. For the second pragmatic point of variation, the speaker is committed to both the falsity of the higher alternatives and the truth of the prejacent under concessive *at least*, while under epistemic *at least* the speaker is committed to the projection of the possibility that the prejacent is true and the possibility that the higher alternative is true in subsequent discourse. The proposed semantics is neutral with respect to the speaker's discourse commitments in either pragmatic situation.

In sum, we have seen that concessive *at least* and epistemic *at least* have two different discourse profiles. The next section provides a formal model capturing the discourse profile of concessive *at least* and epistemic *at least*.

3.2 *At least* in the conversational scoreboard

This section proceeds as follows. Section 3.2.1 provides pragmatics preliminaries and introduces the discourse model (in terms of the conversational scoreboard, with the insights from Lewis 1979) presented by Farkas and Bruce (2010) and further developed in Malamud and Stephenson (2015) and Beltrama (2018). Sections 3.2.2 and 3.2.3 present a formal analysis of concessive *at least* and epistemic *at least*, capturing their different discourse profiles in terms of the conversational scoreboard.

3.2.1 Pragmatic background: the conversational scoreboard

In this dissertation, I adopt the idea that discourse is structured around one or more Question-Under-Discussion (QUD; Roberts 1996/ 2012, Büring 2003, Beaver and Clark 2008, among others; see chapter 2 on the relation between QUDs and information focus). Briefly put, a QUD is a (possibly implicit) question that amounts to a goal, at a stage of discourse: cooperative interlocutors attempt to collectively resolve the current QUD(s). In a QUD-based model, two types of core discourse moves can be made by a speaker; she can either attempt to resolve some QUD by providing an answer, or raising an issue by positing a (sub)question that could be used as a new QUD by discourse participants. For purposes of this dissertation and simplification, I ignore other types of discourse moves. Furthermore, I follow Biezma and Rawlins (2017a, b) and others in assuming that questioning happens against the background of a Stalnakerian common ground/ context set and questions raise issues that are not settled in the context set; in contrast, answers narrow the context set and eliminate possibilities. An important idea behind the QUD-based viewpoint of discourse is the notion of *strategy*: a strategy is (roughly) a discourse path toward resolving some QUD by providing a partial/ complete answer or by asking a subquestion. Finally, discourse moves must be *relevant*. In this dissertation, I assume Roberts (1996/ 2012)'s characterization of the notion of relevance, as shown below.

(34) Relevance (adapted from Roberts 1996/ 2012: (15))

A move *M* is *Relevant* to a question *q* iff *M* either introduces an (at least) partial answer to *q* in context c_M (*M* is an assertion) or is part of a strategy to answer *q* (*M* is a question).

I further assume with Roberts (1996/ 2012) that the notion of partial answers and complete answers are defined in terms of contextual entailment. The definition below is a modified version taken from Biezma and Rawlins (2017b: (48)).

(35) Partial answers and complete answers

a. A partial answer to a question *q* is a proposition which contextually entails the evaluation – either true or false – of at least one element of the alternative set characterized by *q*.

b. A complete answer is a proposition which contextually entails an evaluation for each element of the alternative set characterized by *q*.

c. p contextually entails q in a context c just in case $(p \cap cs_c) \subseteq (q \cap cs_c)$, where cs_c is the Stalnakerian context set in context c.

d. p contextually entails the evaluation of q in context c iff either p contextually entails q, or p contextually entails $\neg q$.

According to the definitions above, a declarative response will be relevant if in a context, it decides (either positively or negatively) any alternative in the current QUD.

With these pragmatic preliminaries, let's turn to the discourse model presented by Farkas and Bruce (2010) (henceforth F&B).[5] A conversational state is represented in F&B's system by means of the following four main components.

(36) a. **The participants' discourse commitments** (DC_X): for each participant X, X's public discourse commitments.

b. **Table**: a stack of issues (the top issue first), where issues are represented as sets of propositions. Issues can remain on the Table only when they have been raised by previous moves and have not been resolved yet (i.e., still "under discussion").

c. **Common Ground** (CG): the set of propositions that haven already been publicly committed by all discourse participants (cf. Stalnaker 1978).

d. **Projected CG** (*CG; F&B's Projected Set): the set of *possible* CGs that give resolution(s) of the top issue on the Table in the expected next stage of the conversation (i.e., in the felicitous continuations of the conversation).

---

[5] Farkas and Bruce (2010)'s model is adopted here because I believe that it is more reader-friendly and straightforward. The current analysis can be translated into an alternative discourse representation where a context is considered as a tuple consisting of various discourse components (e.g., commitment sets, the Table, CG, Projected CG, etc.) and various discourse moves are defined (e.g., push, pop, top, etc.); see Biezma and Rawlins (2017a) and Beltrama (2018) for such discourse representation.

A key feature of F&B's system is that discourse moves, such as assertions and questions, are distinguished by where their associated propositions are added in the conversational scoreboard. For example, if speaker A asserts a proposition φ, then φ is added to DC_A, $\{\varphi\}$ is added to the top of the Table, and φ is added to the projected CG (i.e., CG*). If speaker B accepts the assertion (or has no objection to it), then $\{\varphi\}$ is removed from the Table and added to the CG.[6] The scoreboard in (37) illustrates how factual assertions are computed in F&B's system.[7]

(37) (CG$_i$ represents the input CG)

A asserts: John left.

	Before A's assertion	After A's assertion	After B accepts A's assertion
DC_A	{ }	{John left}	{ }
DC_B	{ }	{ }	{ }
Table	< >	<{John left}>	< >
CG	CG$_i$	CG$_i$	CG$_i$ ∩ {John left}
CG*	CG$_i$	CG$_i$ ∩ {John left}	CG$_i$ ∩ {John left}

As shown above, once speaker B accepts the assertion, the new CG is obtained by intersecting the input CG and the proposition {John left} (i.e., restricting CG$_i$ to those worlds where the proposition that John left is true; in other words, removing those worlds where John did not leave). On this view, crucially, assertions do not directly update the CG; instead, they are seen as proposals to update the input CG, available for a variety of reactions from other discourse participants.

[6] I follow the convention from F&B that when a proposition φ is added to the CG, it is also simultaneously removed from any discourse participant's commitment sets. This avoids redundancy, because the CG represents the public commitments of every discourse participant in the conversation.

[7] I follow Malamud and Stephenson (2015)'s simplification of F&B's representation of the Table. Under F&B's original representation, items placed on the Table are pairs consisting of the syntactic representation of the utterance and its denotation. Only the denotation is represented here.

Now, let's consider another case discussed in F&B: polar questions. In contrast to the assertion, the corresponding polar question creates projected CGs containing φ and those containing $\neg\varphi$. The scoreboard in (38) illustrates how polar questions are computed in F&B's system.

(38) (CG_i represents the input CG)

A: Has John left?

B: Yes.

	Before A's move	After A's move	After B's answer	After A accepts B's answer
DC_A	{ }	{ }	{ }	{ }
DC_B	{ }	{ }	{John has left}	{ }
Table	< >	<{John has left, John has not left}>	<{John has left}>	< >
CG	CG_i	CG_i	CG_i	$CG_i \cap$ {John left}
CG*	CG_i	{$CG_i \cap$ {John left}, $CG_i \cap$ {John has not left}}	$CG_i \cap$ {John left}	$CG_i \cap$ {John left}

In F&B's system, a polar question is distinguished from an assertion in three respects. First, when a polar question is posited, what is being placed on the Table is a set consisting of the proposition φ and its complement $\neg\varphi$. Second, asking a polar question creates two possible projected CGs: one in which φ is added and one in which $\neg\varphi$ is added. Third, asking a polar question does not change the speaker's discourse commitment sets.

Although F&B's system has its merit of modeling basic discourse moves such as (factual) assertions and questions, as discussed in Malamud and Stephenson (2015) and Beltrama (2018), the original system is insufficient (or not directly applicable) to model more fine-grained types of content (e.g., the interlocutors' private doxastic states; Farkas and Bruce 2010: 89) or various types of assertions (e.g., subjective assertions with a judge parameter, and assertions with tentative commitments). In the case at hand, it is not clear how to model the fact that under epistemic *at least*, the speaker does not committ herself to the necessary truth of the prejacent or the higher alternative. Notice that in F&B's system, the four components are not completely independent of each other. For example, as Malamud and Stephenson (2015: 286) has pointed out: "the commitment sets and the Table completely determine the other elements of the scoreboard: the CG consists of propositions that both (all) participants are committed to, while the projected CG consists of these joint commitments updated with all possible resolutions to the issues on the Table". In order to model cases where the speaker may not be fully committed to the propositional content that she asserts (i.e., assertions with tentative commitments), Malamud and Stephenson (2015: 288) suggest adding "Projected Commitments" into F&B's original system.

(39) **The participants' projected commitments** (DC_X*): for each participant X, X's discourse commitments in the net expected stage of the conversation (i.e., the felicitous continuations of the conversation).

Malamud and Stephenson (2015) posit sets of "projected commitments" of the speaker and the hearer(s). According to them, the modified system is able to model discourse moves giving tentative commitments (i.e., by adding propositions to the speaker's projected commitment sets) or discourse moves offering the speaker's best guess of other participants' commitments (i.e., by adding propositions to other participants' projected commitment sets).[8] An important insight behind Malamud and

[8] Note that there is an asymmetry between regular present commitments (DC_X) and projected commitments (DC_X*): a discourse move may add propositions to either the speaker' or the hearer's

Stephenson (2015)'s idea of projected commitments is that adding propositions to the speaker's projected commitments leads to an inference of tentativeness. More specifically, given that the speaker is always in full control of her own commitment sets, if the speaker chooses to add propositions φ to her projected commitments but not her present commitments, then the hearer is entitled to infer that the speaker has some reason to delay making her present commitments. With no other obvious pragmatic reasons, typically, the hearer can infer that *the speaker thinks* φ*, but she is uncertain about* φ. This licenses an inference of tentativeness.

I follow Malamud and Stephenson (2015) in adding "projected commitments" into the conversational scoreboard. However, for purposes of this dissertation, the hearer's projected commitments will not concern us. Instead, what matter to us will be the speaker's projected commitments. As we will see shortly, I suggest that the tentativeness is the pragmatic source of the modal flavor associated with epistemic *at least*. To anticipate, my proposal in a nutshell is that although concessive *at least* and epistemic *at least* share one single semantic representation, they have different discourse profiles and the discrepancy in their discourse profiles crucially comes from where propositions are added in the conversational scoreboard. More specifically, assertions with concessive *at least* add propositions to the speaker's present commitments, while assertions with epistemic *at least* add propositions to the speaker's projected commitments.

The next section presents a formal analysis of concessive *at least* capturing its discourse profile and illustrates how assertions with concessive *at least* are computed in terms of the conversational scoreboard.

---

projected commitment sets, but it can only add propositions to the speaker's (but crucially not the hearer's) regular present commitments.

3.2.2 Concessive *at least*

Below, (40) repeats the discourse profile of concessive *at least* that I attempt to capture in this dissertation.

(40) **Concessive *at least***

a. The prejacent is always at-issue, while the higher alternatives can be at-issue or not-at-issue. The lower alternatives are at-issue.

b. The ranking information is not-at-issue.

c. The speaker is committed to both the falsity of the higher alternatives and the truth of the prejaent.

Recall that we have discussed two cases of concessive *at least* with respect to the (not-)at-issueness. In one case, the propositional content of the prejacent and that of the higher alternative are both at-issue. In the other case, while the propositional content of the prejacent remains at-issue, that of the higher alternative is not-at-issue. Crucially, in both cases, the ranking information is not-at-issue and the speaker is committed to the falsity of the higher alternative and the truth of the prejacent.

Let's first consider the case where the content of the prejacent and that of the higher alternative are both at-issue. The relevant example is repeated below.

(41) A: Who did John invite? Did John invite Adam, Bill and Chris?

B: No, John didn't invite Adam, Bill and Chris.

But, **at least** he invited [Adam and Bill]$_F$.

Notice that to address the QUD (the issue raised by speaker A whether John invited Adam, Bill and Chris), speaker B first makes an assertion that John didn't invite Adam, Bill and Chris, and then makes another assertion with concessive *at least* that

John invited Adam and Bill. The former assertion amounts to a proposal to update the CG with the content that the higher alternative is false, and the latter assertion a proposal to update the CG with the content that the prejacent is true. With this in mind, the conversational scoreboard in (42) illustrates how assertions with concessive *at least* are computed when the content of the prejacent and the higher alternative is at-issue.[9]

(42) (CG_i represents the input CG;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts $\neg q$ and then asserts p with concessive *at least*

	Before the move	After the move
DC_A	{ }	$\{\{\neg q\}, \{p\}\}$
DC_A*	{ }	{ }
DC_B	{ }	{ }
Table	< >	$<\{\neg q\}, \{p\}>$
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$(CG_i \cap \{q \succ_c p\}) \cap \{\neg q\} \cap \{p\}$

In (42), the speaker makes two proposals to update the CG: $\neg q$ and p. Therefore, two propositions (i.e., $\neg q$ and p) are added to the speaker's present commitments and two items (i.e., $\{\neg q\}$ and $\{p\}$) are placed on the Table. Assume that not-at-issue content directly updates the input CG without being subject to the acceptance of other discourse participants (e.g., AnderBois et al. 2010, 2015 on clause-medial appositives

[9] Given that the meaning of concessive *at least* in general does not depend on the presence of *but*, for purposes of illustration, the contribution of *but* is ignored here. Note that this simplification is only an ideal situation; the absence of *but* certainly leads to some awkwardness in the current example.

(i) A: Who did John invite? Did John invite Adam, Bill and Chris?
B: #No, John didn't invite Adam, Bill and Chris. At least, he invited [Adam and Bill]$_F$.

and Murray 2014 on evidentials), the information content of the ranking (i.e., $q \succ_c p$) is directly added into the CG (i.e., $\mathrm{CG}_i \cap \{q \succ_c p\}$). Finally, the projected CG is obtained by intersecting the input CG (with the added not-at-issue content) and the two propositions (i.e., $(\mathrm{CG}_i \cap \{q \succ_c p\}) \cap \{\neg q\} \cap \{p\}$). Given the conversational scoreboard in (42), the concessive meaning comes from the fact that q is higher ranked than p in the context c (with some additional pragmatic attachment of preference), but q is false and p is true. Moreover, it captures our intuition that under concessive *at least*, the speaker is committed to both the falsity of the higher alternative and the truth of the prejacent, because both $\{\neg q\}$ and $\{p\}$ are in the speaker's present commitments.

Now, let's turn to the case where the content of the prejacent is at-issue, but that of the higher alternative is not-at-issue. The relevant example is repeated below.

(43) Context: Speaker A wants to know who John invited for his party last night. All the three speakers A, B and C know that John didn't invite Chris. Speaker A is interested in whether the other two people (i.e., Adam and Bill) are invited.

A: Who did John invite? Did John invite Adam and Bill?

B: Yes, **at least** he invited [Adam and Bill]$_F$.

In contrast to the previous example, to address the QUD (the issue raised by speaker A whether John invited Adam and Bill), speaker B makes only one assertion with concessive *at least* that John invited Adam and Bill. The conversational scoreboard in (44) illustrates how assertions with concessive *at least* are computed when the content of the prejacent is at-issue but that of the higher alternative is not-at-issue.

(44) (CG_i represents the input CG and $\{\neg q\}$ is part of CG_i;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts p with concessive *at least*

	Before the move	After the move
DC_A	{ }	$\{p\}$
DC_A*	{ }	{ }
DC_B	{ }	{ }
Table	< >	<$\{p\}$>
CG	CG_i	$\mathrm{CG}_i \cap \{q \succ_c p\}$
CG*	CG_i	$(\mathrm{CG}_i \cap \{q \succ_c p\}) \cap \{p\}$

In (44), the speaker makes a proposal to update the CG: p. Therefore, the proposition p is added to the speaker's present commitments and $\{p\}$) is placed on the Table. The information content of the ranking (i.e., $q \succ_c p$) is not-at-issue and is thus directly added into the CG (i.e., $\mathrm{CG}_i \cap \{q \succ_c p\}$). The projected CG is obtained by intersecting the input CG (with the added not-at-issue content) and the proposition p (i.e., $(\mathrm{CG}_i \cap \{q \succ_c p\}) \cap \{p\}$). Notice that $\{\neg q\}$ is already part of CG_i. This is why the content of the higher alternative q is not-at-issue. Finally, given the conversational scoreboard in (44), the concessive meaning arises from the fact that q is higher ranked than p in the context c (with some additional pragmatic attachment of preference), but q is false and p is true. (44) also captures our intuition that under concessive *at least*, the speaker is committed to both the falsity of the higher alternative and the truth of the prejacent, because $\{\neg q\}$ is part of the input CG and $\{p\}$ is in the speaker's present commitments.

It is worth noting that under the current analysis, the concessive meaning comes from the meaning combination of scalarity ($q \succ_c p$) and two propositions ($\neg q$ and p). Crucially, this predicts that in the absence of *at least*, a simple conjunction of two propositions ($\neg q$ and p) may or may not come with a concessive meaning, depending on whether the scalar component ($q \succ_c p$) is available in the discourse. I believe that this prediction is correct and borne out by examples like (45).

(45) Context: Adam, Bill and Chris are the relevant individuals in the discourse.

A: Who did John invite? Did John invite all the three people?

B: No, John didn't invite all of them;

But he invited two of them, Adam and Bill.

In (45), although concessive *at least* is absent, the sentence may still receive a concessive interpretation if there is a preference ranking, provided by the discourse, on the number of people receiving invitation (e.g., the more, the better). Crucially, in the absence of such ranking information, speaker B's assertion in (45) may simply convey the fact that John didn't invite everyone, and he invited only Adam and Bill, with no concessive flavor.

Before leaving this section, I would like to briefly discuss the relation between concessive *at least* and the structure of QUDs. One issue that has not been addressed so far concerns why a polar question, targeting either the content of the prejacent or that of the higher alternative, usually needs to be posited as the immediate QUD in the discourse. For example, Veneeta Dayal (p.c.) points out that with no explicit polar question posited, the use of concessive *at least* seems infelicitous in (46).

(46) Context: Adam, Bill and Chris are the relevant individuals in the discourse.

A: Who did John invite?

B: #(**At least**) John (**at least**) invited [Adam and Bill]$_F$.

The use of concessive *at least* in (46) is infelicitous, regardless of its position. There is no doubt that speaker B's assertion with concessive *at least* is relevant to the QUD (the issue raised by speaker A), because it decides some alternatives in the context positively and negatively: **(a)** it is not the case that John invited Adam, Bill and Chris (i.e., $\neg q$); **(b)** John invited Adam and Bill (i.e., p). Therefore, the discourse move taken by Speaker B should be licensed, but it is not. Why is this so? What is wrong with speaker B's move? The current analysis suggests that the awkwardness arises because speaker B's assertion with concessive *at least* is over-informative. Recall that under concessive *at least*, the speaker is committed to the information content that the higher alternative is false (i.e., $\neg q$) and that the prejacent is true (i.e., p). Moreover, the former information content can be at-issue (when the speaker explicitly make an assertion addressing it; see (41)) or not-at-issue (as part of the CG; see (43)). In (46), neither speaker B explicitly mentions that she is committed to $\neg q$, nor speaker A explicitly requests for the information about whether q is true. Crucially, however, by using concessive *at least* without first asserting $\neg q$, speaker B takes for granted that speaker A would recognize her commitment to $\neg q$. This leaves multiple tasks for speaker A simultaneously and too much information comes at once. Suppose that speaker A does not know whether John invited all the three individuals (i.e., whether q is true). Upon hearing speaker B's assertion with concessive *at least*, speaker A is then forced to accommodate several pieces of information: **(a)** speaker B is committed to $\neg q$; **(b)** speaker B is committed to p; **(c)** speaker B presupposes $\neg q$ when she makes the assertion with concessive *at least*; **(d)** $\neg q$ provides only a (negative) partial answer (i.e., the alternative q is decided negatively). A natural puzzle that arises for speaker A may be why speaker B particularly chose to address the issue concerning whether q is true. Along this line of thought, when speaker B explicitly mentions that she is committed to $\neg q$, the awkwardness is removed in (47), even though no explicit polar question is posited by speaker A concerning whether q is true.

(47) A: Who did John invite?

B: John didn't invite all the three people (Adam, Bill and Chris);

But, **at least** he invited [Adam and Bill]$_F$.

Now, suppose that in (46), speaker A shares with speaker B the knowledge that John didn't invite all the three people (i.e., $\neg q$). Given that the denotation of a question consists of a set of possible true answers, this means that the alternative "λw.John invited$_w$ Adam, Bill and Chris" would be completely missing in the domain of speaker A's question. In other words, the domain of speaker A's question would be the set of propositional alternatives in the structure of a semi-lattice, with only the alternative involving the invitation of all the three people removed: { λw.John invited$_w$ Adam and Bill; λw.John invited$_w$ Adam and Chris; λw.John invited$_w$ Bill and Chris; λw.John invited$_w$ Adam; λw.John invited$_w$ Bill, λw.John invited$_w$ Chris}. The intuition here is that once some answers are known to be false by the questioner, they should be no longer available in the set of possible true answers (i.e., the domain of the question). Now, because the alternative "λw.John invited$_w$ Adam, Bill and Chris" is missing in the domain, speaker B's assertion with concessive *at least* becomes infelicitous.

In short, in examples like (46) where no explicit (or implicit) polar question particularly targets the content of the higher alternative or the prejacent, the speaker's assertion with concessive *at least* is infelicitous, regardless of whether the speaker shares with other discourse participants the knowledge that the content of the higher alternative is false.

Finally, let me briefly point out another issue concerning whether the prejacent of concessive *at least* always provides discourse new information. Roger Schwarzschild (p.c.) provides (48) and wonders whether the content of the prejacent under concessive *at least* must be known to the interlocutors in the discourse. Notice that the proposition that *Jack gave A a book* entails the proposition that *Jack gave A something*; in this sense, the prejacent of the concessive *at least* in (48), at first glance, does not provide any new information (updating the CG/ shrinking the context set).

(48) A: Jack only gave me a book for my birthday.

B: Well, **at least** he gave you [something]$_F$.

Given the current analysis, assertions with concessive *at least* are like factual assertions in updating the speaker's commitment sets and serving as a proposal to update the CG (see (42) and (44)). Seen in this light, the content of the prejacent should be at-issue and assertions with concessive *at least* should add new information to the discourse. So, what is going on in (48)? To begin with, notice that whether the information content is at-issue/ not-at-issue and whether the information content is discourse new/ old are two different categories in information structure. More specifically, the two notions are non-identical and may overlap, as shown below.

(49) a. At-issue content contributes to new information (cf. Stalnaker 1978).

b. Not-at-issue content may contribute to either new information (e.g., appositives and evidentials; see AnderBois et al. 2010, 2015, Murray 2014, Koev 2012, 2013, among others) or old information (e.g., presuppositions; see Simons et al. 2010, Tonhauser 2012, Tonhauser et al. 2013, among others)

Given the empirical landscape in (49), there are (at least) two possible responses to examples like (48). One analytical possibility is to relax the claim that the prejacent of concessive *at least* must be at-issue and thus allows the prejacent to sometimes provide new information and sometimes provide old information. Another analytical possibility is to maintain the claim that the prejacent of concessive *at least* must be at-issue. This line of thought is inspired by Veneeta Dayal (p.c.)'s suggestion that concessive *at least* in (48) may involve a shift of the scale. Seen in this light, it may well be the case that the use of concessive *at least* in (48) still contributes to new information because of the shift of the relevant scale, as illustrated in (50)

(50) A shift of the relevant scale in (49)

a. The scale envisioned by speaker A:

a car $\succ$ a Macbook $\succ$ a book

b. The scale envisioned by speaker B:

the stuff Jack desired the most $\succ$ something $\succ$ nothing

Crucially, because the relevant scales are shifted, the CG in (48) is updated by speaker B's assertion with concessive *at least* with respect to the alternatives and the ranking. Note that although the relevant scales have been shifted, the QUD does not change in the cross-speaker conversation: it still concerns what Jack received for his birthday.

To sum up, in this section, I have illustrated how the discourse profile of concessive *at least* is captured with respect to its (not-)at-issue content and how assertions with concessive *at least* are computed in the discourse model presented by F&B. The next section presents a formal analysis of epistemic *at least*.

3.2.3 Epistemic *at least*

Below, (51) repeats the discourse profile of epistemic *at least* that I attempt to capture in this dissertation.

(51) **Epistemic *at least***

a. The prejacent and the higher/ lower alternatives are all at-issue.

b. The ranking information is not-at-issue.

c. The speaker does not *fully* commit herself to the necessary truth of either the prejacent or the higher alternatives. Instead, what the speaker is committed to is the *possibility* that the prejacent is true and the *possibility* that the higher alternative is true in subsequent discourse.

Recall that as discussed in section 3.1.3, the use of epistemic *at least* is felicitous only when the speaker is ignorant about whether the prejacent is true and whether its higher alternative is true (if the maxim of quantity is active in the discourse). Crucially, this speaker ignorance allows the *possibility* that the prejacent is true and the *possibility* that the higher alternative to both project in subsequent discourse. This projection behavior of the two possibilities in subsequent discourse is considered the source of the modal flavor associated with SMs. Therefore, the main task of this section is to illustrate **(a)** how to capture the modal flavor of epistemic *at least* (more generally, SMs) at the level of discourse, without hardwiring a covert epistemic modal into their lexical semantics; **(b)** how to capture the intuition that the speaker does not *fully* commit herself to the necessary truth of either the prejacent or the higher alternatives, and what the speaker is committed to are the possibility that the prejacent is true and the possibility that the higher alternative is true in subsequent discourse.

A canonical example of epistemic *at least* is repeated below.

(52) A: Who did John invite?

B: John invited **at least** [Adam and Bill]$_F$.

Notice that to address the QUD (the issue raised by speaker A concerning the individuals invited by John), speaker B makes an assertion with epistemic *at least* that John invited Adam and Bill. Crucially, the speaker's assertion provides only a partial answer, because two possibilities remain open in the discourse: (a) John invited Adam, Bill and Chris (i.e., the higher alternative); (b) John invited (only) Adam and Bill (i.e., the prejacent). With this in mind, the conversational scoreboard in (53) illustrates how assertions with epistemic *at least* are computed.

(53) (CG_i represents the input CG;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts p with epistemic *at least*

	Before the move	After the move
DC_A	{ }	{ }
DC_A*	{ }	$\{\{q\}, \{p\}\}$
DC_B	{ }	{ }
Table	< >	<$\{q\}, \{p\}$>
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$\{(CG_i \cap \{q \succ_c p\}) \cap \{q\}, (CG_i \cap \{q \succ_c p\}) \cap \{p\}\}$

In (53), the speaker makes two tentative proposals to update the CG: q and p. Therefore, two items (i.e., $\{q\}$ and $\{p\}$) are placed on the Table. However, two propositions (i.e., q and p) are added to the speaker's projected commitments, but crucially not to the speaker's present commitments. Assume that not-at-issue content directly updates the input CG, the information content of the ranking (i.e., $q \succ_c p$) is directly added into the CG (i.e., $CG_i \cap \{q \succ_c p\}$). Finally, the assertion with epistemic *at least* creates two possible projected CGs: one in which $\{q\}$ is added is obtained by intersecting the input CG (with the added not-at-issue content) and the proposition q (i.e., $(CG_i \cap \{q \succ_c p\}) \cap \{q\}$); one in which $\{p\}$ is added is obtained by intersecting the input CG (with the added not-at-issue content) and the proposition p (i.e., $(CG_i \cap \{q \succ_c p\}) \cap \{p\}$). Given the conversational scoreboard in (53), the modal flavor of epistemic *at least* comes from the fact that both propositions p and q (the prejacnet and its higher alternative) are projected in subsequent discourse, more specifically, in two different projected CGs. Furthermore, it also captures the intuition that under epistemic *at least*, the speaker does not *fully* commit herself to the necessary truth of either the prejacent or the higher alternatives (i.e., two items $\{q\}$ and $\{p\}$ are placed in DC_A*); instead, what the speaker is committed to are the possibility that the prejacent is true and the possibility that the higher alternative is true in subsequent discourse (i.e., $\{(\text{CG}_i \cap \{q \succ_c p\}) \cap \{q\}\}, (\text{CG}_i \cap \{q \succ_c p\}) \cap \{p\}\})$.

It is worth emphasizing that according to (53), assertions with epistemic *at least* are both **informative** and **inquisitive**, namely, they have mixed properties of both assertions and questions. On the one hand, they are informative because they are like (factual) assertions in **(a)** updating the speaker's commitment sets and **(b)** serving as a proposal to update the CG (subject to other discourse participants' acceptance). On the other hand, they are inquisitive because they are like (polar) questions in creating multiple possible projected CGs.

To appreciate the mixed properties of those assertions with epistemic *at least*, let's briefly recall how assertions and polar questions are modeled in F&B's system. For (factual) assertions, if speaker A asserts a proposition φ, then φ is added to DC_A, $\{\varphi\}$ is added to the top of the Table, and φ is added to the projected CG (i.e., CG*). If speaker B accepts the assertion (or has no objection to it), then $\{\varphi\}$ is removed from the Table and added to the CG. In contrast, a polar question is distinguished from an assertion in three respects. First, when a polar question is posited, what is being placed on the Table is a set consisting of the proposition φ and its complement $\neg\varphi$. Second, asking a polar question creates two possible projected CGs: one in which φ is added and one in which $\neg\varphi$ is added. Third, asking a polar question does not change the speaker's discourse commitment sets.

Before leaving this section, I would like to highlight several important features of the current analysis: **(a)** the modal flavor associated with epistemic *at least* is not at the level of lexical semantics (e.g., a covert epistemic modal), but lies in the level of discourse; **(b)** assertions with epistemic *at least* add associated propositions to the speaker's projected commitments, crucially not to the speaker's present commitments. This delay in making a commitment (resulting in an inference of tentativeness/ uncertainty) constitutes one pragmatic source of the modal flavor; **(c)** under an assertion with epistemic *at least*, multiple possible projected CGs are generated in the discourse, which constitutes another pragmatic source of the modal flavor; **(d)** assertions with epistemic *at least* are both informative and inquisitive.

3.3 Conclusion

In this chapter, I have shown how different discourse profiles of concessive *at least* and epistemic *at least* with respect to their (not-)at-issue content are captured; I have also illustrated how assertions with concessive *at least*/ epistemic *at least* are computed in the conversational scoreboard, a discourse model (incorporating the insights from Lewis 1979) presented by Farkas and Bruce (2010) and further developed in Malamud and Stephenson (2015) and Beltrama (2018). One central idea underlying my proposal is that although concessive *at least* and epistemic *at least* share one single semantic representation (proposed in chapter 2), they have different discourse profiles and the discrepancy in their discourse profiles crucially comes from where propositions are added in the conversational scoreboard. More specifically, assertions with concessive *at least* add associated propositions to the speaker's present commitments, while assertions with epistemic *at least* add associated propositions to the speaker's projected commitments. Overall, the presented discussion and the developed analysis of concessive *at least* and epistemic *at least* in this chapter reveals an intrinsic relation between (not-)at-issue content, information structure and QUDs.

To recap, some crucial pieces of the empirical findings and the theoretical analysis are listed below.

First, (54) presents the discourse profile of concessive *at least* and (55) the discourse profile of epistemic *at least*.

(54) **Concessive *at least***

a. The prejacent is always at-issue, while the higher alternatives can be at-issue or not-at-issue. The lower alternatives are at-issue.

b. The ranking information is not-at-issue.

c. The speaker is committed to both the falsity of the higher alternatives and the truth of the prejaent.

(55) **Epistemic *at least***

a. The prejacent and the higher/ lower alternatives are all at-issue.

b. The ranking information is not-at-issue.

c. The speaker does not *fully* commit herself to the necessary truth of either the prejacent or the higher alternatives. Instead, what the speaker is committed to is the *possibility* that the prejacent is true and the *possibility* that the higher alternative is true in subsequent discourse.

Second, (56) and (57) illustrate how the discourse profile of concessive *at least* is captured and how assertions with concessive *at least* are computed in the conversational scoreboard; (58) illustrates the case of epistemic *at least*.

(56) (CG_i represents the input CG;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts $\neg q$ and then asserts p with concessive *at least*

	Before the move	After the move
DC_A	$\{ \}$	$\{\{\neg q\}, \{p\}\}$
DC_A*	$\{ \}$	$\{ \}$
DC_B	$\{ \}$	$\{ \}$
Table	$< >$	$<\{\neg q\}, \{p\}>$
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$(CG_i \cap \{q \succ_c p\}) \cap \{\neg q\} \cap \{p\}$

(57) (CG_i represents the input CG and $\{\neg q\}$ is part of CG_i;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts p with concessive *at least*

	Before the move	After the move
DC_A	$\{ \}$	$\{p\}$
DC_A*	$\{ \}$	$\{ \}$
DC_B	$\{ \}$	$\{ \}$
Table	$< >$	$<\{p\}>$
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$(CG_i \cap \{q \succ_c p\}) \cap \{p\}$

(58) (CG$_i$ represents the input CG;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts p with epistemic *at least*

	Before the move	After the move
DC_A	{ }	{ }
DC_A*	{ }	$\{\{q\}, \{p\}\}$
DC_B	{ }	{ }
Table	< >	$<\{q\}, \{p\}>$
CG	CG$_i$	$\mathrm{CG}_i \cap \{q \succ_c p\}$
CG*	CG$_i$	$\{(\mathrm{CG}_i \cap \{q \succ_c p\}) \cap \{q\}\}, (\mathrm{CG}_i \cap \{q \succ_c p\}) \cap \{p\}\}$

Chapter 4 An extension to *at most*

This chapter extends the unified analysis of *at least* proposed in chapter 2, by taking English *at most* as a case study and proposing a uniform semantic representation of it. In particular, it is shown that *at most*, like *at least*, delivers the EPI-CON ambiguity and the two meanings are pragmatic variants resulting from one unified semantic entry interacting with different pragmatic factors such as informativity and evaluativity. Furthermore, three contrasts between *at most* and *at least* are discussed: First, *at most* shows a mirror image of *at least* with respect to the discrepancy between the two scalar effects TSE and BSE. Second, the concessive reading of *at most* looks like an epiphenomenon, while that of *at least* does not. Third, concessive *at most* gives rise to a "settle-for-less" flavor slightly different than that given by concessive *at least*. Crucially, these three contrasts are not arbitrary, but systematic: the three contrasts all result from different semantic bounding properties of *at most* and *at least*.

This chapter is structured as follows. Section 4.1 discusses epistemic *at most* and shows that *at most* under EPI, resembling *at least*, also demonstrates the three properties: **(a)** the focus-sensitivity; **(b)** the compatibility with various scales; **(c)** the two scalar effects TSE and BSE. Moreover, like the case of *at least*, the ignorance inference given by *at most* is also pragmatic, rather than semantic, in its nature. Section 4.2 looks at concessive *at most* and argues that unlike the case of *at least*, the prejacent is not necessarily entailed under CON. In particular, this interpretational difference results from the (semantic) bounding property of *at most*: the pragmatic condition of CON and the semantics of *at most* converge on excluding the relevant higher alternatives. Section 4.3 spells out the core ingredients of my proposal and presents a unified semantics of *at most*. Section 4.4 presents a derivation of ignorance inferences given by *at most*. Section 4.5 briefly discusses the issue concerning the null individual in the case of SMs. Section 4.6 concludes this chapter.

4.1 Epistemic *at most*

To begin with, it has been observed in the literature that English *at most* is focus-sensitive(e.g., Krifka 1999, Coppock and Brochhagen 2013, Kennedy 2015, Mendia 2016a-c). The position of the focus associate makes explicit the piece of information that the speaker's ignorance is about, as illustrated below.

(1) a. Adam **at most** invited an $[\text{associate}]_F$ professor to lunch.

b. Adam **at most** invited an associate professor to $[\text{lunch}]_F$.

Assume an academic ranking: a full professor $\succ$ an associate professor $\succ$an assistant professor; (1a) implicates that the speaker is ignorant about whether an associate professor or an assistant professor is invited. In contrast, assume a contextual ranking: dinner $\succ$lunch $\succ$breakfast; (1b) implicates that the speaker is ignorant about whether the associate professor is invited to lunch or breakfast.

Next, it has been observed in previous studies that English *at most*, like *at least*, is also compatible with various scales (e.g., Coppock and Brochhagen 2013, Kennedy 2015, Mendia 2016a-c), as demonstrated below.

(2) a. **Numeral Scales** (a contextual ranking: 4 $\succ$3 $\succ$2)

John **at most** wrote $[\text{three}]_F$ novels.

b. **Plurality Scales** (a contextual ranking: adam⊕bill⊕chris $\succ$adam⊕bill $\succ$adam)

John **at most** hired [Adam and Bill]$_F$.

c. **Lexical Scales** (a contextual ranking: gold medal≻silver medal≻bronze medal)

John **at most** won a [silver]$_F$ medal.

d. **Pragmatic Scales** (a contextual ranking: cherries ≻apples ≻bananas)

John **at most** bought [apples]$_F$.

As discussed in chapter 2, a novel observation here is that there is a discrepancy between scales based on semantic strength vs. pragmatic strength. By manipulating the context, it is easy enough to reverse the ordering between the alternatives in the case of pragmatic scales or lexical scales.[1] In contrast, it does not seem possible to reverse the ordering in those scales based on semantic strength, such as numerical scales or plurality scales, even with some contextual effort.

(3) Context: Adam, Bill and John are planning to buy some fruit for their party tonight. There are three types of fruit available to them: cherries, apples and bananas. However, they are poor and do not have enough money to buy everything. For them, bananas are optimal because they are the cheapest; apples are less optimal but acceptable because they are still cheaper than cherries.

The contextual ranking (in terms of price): bananas ≻apples ≻cherries

(4) Context: John is planning to hire some people. There are three applicants in the discourse: Adam, Bill and Chris. But the budget is limited. If three people are all hired, John needs to pay a great amount of money for their salary. If only two people (say, Adam and Bill) are hired, the situation is better, but John still pays more than he does in hiring just one person. The best situation for John is simply

[1] For lexical scales, although context manipulations are not impossible, they are indeed harder because the ordering is based on our common world knowledge. Instances of lexical scale are such as *gold medal ≻silver medal ≻bronze medal*, and *full professor ≻associate professor ≻assistant professor*.

to hire only one person (say, Adam) while getting all the work done.

The **intended** contextual ranking:

only adam $\succ$only adam&bill $\succ$only adam&bill&chris

In the context (3), the utterance with *at most* in (2d) is understood to convey that John bought apples or cherries (given the contextual ranking: bananas $\succ$apples $\succ$cherries). This means that the original ranking (cherries $\succ$apples $\succ$bananas) in (2d) is now **reversed**. In contrast, the utterance with *at most* in (2b) cannot be understood as saying that John hired only Adam and Bill, or hired only Adam, even with the contextual massage in (4). This indicates that the original ranking (adam&bill&chris $\succ$adam&bill $\succ$adam) in (2b) **cannot** be reversed. The same observation applies to numeral scales. I leave it for readers to verify the case of numerical scales.[2]

In addition to the focus-sensitivity and the diversity of scales, a novel observation is that *at most*, like *at least*, demonstrates the two scalar effects: the top-of-the-scale effect (TSE) and the bottom-of-the-scale effect (BSE). Specifically, TSE demands that the associate cannot be the element at the top of the scale while BSE requires that the associate cannot be the element at the bottom of the scale. Consider the scenario (5). Neither (7) nor (8) are felicitous as a continuation in Bill's response (6) to Adam's question.

(5) Scenario: Adam, Bill and Chris are playing dice. In each round, whoever gets a bigger number wins; scores are not cumulated. A dice has six numbers on it: <u>***Six*** is the upper bound and ***one*** the lower bound on the possible results</u>. Chris threw the dice but Adam missed the result. During his turn, Adam asks about the result.

[2] This discrepancy between scales also holds in Chinese. A detailed discussion of Chinese data is deferred until Chapter 4, where I take Chinese SMs as a case study and present a detailed decompositional analysis of SMs.

(6) Adam: What number did Chris get?

Bill: I don't know…

(7) #Chris **at most** got $[\text{one}]_F$. [**Bottom-of-the-Scale Effect** (**BSE**)]

(8) #Chris **at most** got $[\text{six}]_F$. [**Top-of-the-Scale Effect** (**TSE**)]

Intuitively, given the scenario, (8) is infelicitous because the utterance is contextually uninformative: it is already in the common ground that the number *six* is the upper bound on the six possible results. Crucially, in the same scenario, (7) is also deviant. In contrast to (8), my informants consider the sentence (7) plainly unassertable in the given scenario.[3] Such a contrast raises many questions concerning the two scalar effects: Why should such contrast between (7) and (8) exist? What exactly is the nature of the two scalar effects? How are the two scalar effects connected to the semantics of *at most*? Intriguingly, the contrast between TSE and BSE in the case of *at most* is a mirror image of that in the case of *at least*. The relevant examples of *at least* are repeated in (9) and (10). The Table in (11) shows a comparison between *at least* and *at most* regarding the two scalar effects.[4] In particular, given the analysis of *at least* presented in chapter 2, we expect not only that TSE and BSE are of different nature, but also that how and why the two scalar effects arise are intrinsically connected to the semantics of *at most*.

[3] Veneeta Dayal (p.c.) correctly points out that in English, the sentence *Chris at most got* $[\textit{one}]_F$ can be felicitous if it is understood to be an indirect evidential inference. For example, imagine that Adam, Bill and Chris have played dice for many rounds. Chris had very bad luck and he has never got more than one. In this scenario, Bill can justify his response in saying *Chris at most got* $[\textit{one}]_F$ while pointing at the score board with Chris's previous results. At this point, I have no explanation for why the BSE of *at most* can be improved under such an indirect evidential inference. But notice that the corresponding "good luck" scenario does not help the TSE of *at least*. The contrast here suggests that the pairs of SMs may not be fully parallel in their semantic or pragmatic contributions.

[4] To anticipate, I argue that the two scalar effects BSE and TSE are of different nature and they are intrinsically connected to the semantics of SMs. In the case of *at least*, BSE arises from discourse uninformativity while TSE from semantic vacuity. In contrast, in the case of *at most*, TSE arises from discourse uninformativity while BSE from semantic vacuity. In both cases, only the infelicity resulting from discourse uninformativity can be *pragmatically repaired*, when the speaker intentionally flouts the maxim of quantity. I will return to this point in section 4.3.

(9) #Chris **at least** got $[\text{one}]_F$. [**Bottom-of-the-Scale Effect** (**BSE**)]

(10) #Chris **at least** got $[\text{six}]_F$. [**Top-of-the-Scale Effect** (**TSE**)]

(11) Division of labor between semantics and pragmatics of SMs

	TSE (the top-of-the-scale effect)	**BSE (the bottom-of-the-scale effect)**
at least	# (see (10))	△ (see (9))
at most	△ (see (8))	# (see (7))

# indicates semantic vacuity, and △ discourse uninformativity

Finally, I would like to show that patterning with *at least*, the ignorance inference given by *at most* is pragmatic (rather than semantic) and sensitive to contextual factors. First of all, the relevant examples of *at least* discussed in Mendia (2016a-c) can be reproduced for *at most*, as shown in (12) − (14) below.

(12) **Cancellability**

Context: Bill has two kids. Yesterday he saw a sign at a supermarket: "Discounts for parents. To qualify you must have at most three kids." Bill reasoned as follows.

I qualify: I have two kids, so I do have **at most** three kids.

(13) **Reinforceability**

Bill has **at most** three kids, I don't know how many exactly.

(14) Context: In a game, my friend has to guess the number of marbles that I have hidden. <u>I know how many I have hidden and she knows that I have that information</u>. I provide the following clue:

I have **at most** five marbles.

~> no ignorance about the number of marbles that I have

(12) and (13) show that the ignorance inference is cancellable and reinforceable, which are the two hallmarks of conversational implicatures (Grice 1989). (14) shows that when the maxim of quantity is deactivated in the context, the ignorance inference does not arise. This, again, confirms the pragmatic nature of the ignorance inference.

Second, as discussed in Westera and Brasoveanu (2014), whether the ignorance inference arises depends primarily on whether a precise answer is requested or not (i.e., the question-under-discussions (QUDs) in the sense of Roberts 1996/ 2012). The examples below illustrate their point for *at most*.

(15) Ignorance Inference

A: Exactly how many students took Experimental Pragmatics?

B: **At most** ten students took Experimental Pragmatics.

(16) No Ignorance Inference

A: Did at most ten students take Experimental Pragmatics?

B: Yes, **at most** ten students took Experimental Pragmatics.

In fact/ To be precise, only five students took Experimental Pragmatics.

Third, patterning with *at least*, the ignorance inference conveyed by *at most* is justified in (17) when a *wh*-question is interpreted exhaustively (i.e., a precise answer is requested), while it is not justified in (18) when a *wh*-question is interpreted non-exhaustively (as indicated by the partiality marker: *for example*).

(17) A: Who did John invite?

B: John invited **at most** [Adam and Bill]$_F$. Felicitous: Ignorance Inference

(18) A: Who did John invite, **for example**?

B: #John invited **at most** [Adam and Bill]$_F$. Infelicitous: Ignorance Inference

Suppose that there are three individuals relevant in the discourse: Adam, Bill and Chris. The ignorance inference about who exactly John invited is justified in (17) because the *wh*-question is requesting for a precise answer on the individuals that John invited and the use of *at most* signals the failure of providing the maximally informative unique answer. In contrast, the infelicity of (18) intuitively comes from the fact that the speaker B's response is **over-informative**. Informally put, any non-exhaustive answer would suffice, but the speaker B is trying to signal that there is one maximally informative unique answer and he fails to provide that particular answer.[5] Thus, the ignorance inference is not justified in (18).

Given our discussion of the ignorance inference given by *at most*, a generalization regarding EPI suggests itself.

(19) **Informativity and Speaker Ignorance**

a. Ignorance inferences arise in responses to *wh*-questions but not (necessarily) to polar questions.

b. Ignorance inferences are justified when *wh*-questions are interpreted exhaustively, but not when they are interpreted non-exhaustively.

Like the case of epistemic *at least*, epistemic *at most* addresses the issue of informativity: ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer. As we will see shortly, in

[5] The notion of "the maximally informative unique answer" can be defined in terms of the relation between the answerhood operator ANS and the denotation of the *wh*-question (abbreviated as Q) below, as proposed in Dayal (1996, 2016).

(i) $\text{ANS}_{\text{Dayal}}(Q)(w)$ is defined iff $\exists p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$.
When defined, $\text{ANS}_{\text{Dayal}}(Q)(w) = 1$ iff $\iota p[w \in p \in Q \wedge \forall q[w \in q \in Q \rightarrow p \subseteq q]]$

In words: the answerhood operator $\text{ANS}_{\text{Dayal}}$ is defined if and only if there is one proposition p in the denotation of the question Q (which is a set of possible answers/ propositions) such that p is true and p entails all the true propositions q in Q; When defined, $\text{ANS}_{\text{Dayal}}(Q)(w)$ is true if and only if the unique proposition p in Q is true (i.e., the uniqueness) and p entails all the true propositions q in Q (i.e., the maximal informativity). Readers are referred to Dayal (2016: chapter 2) for a detailed discussion on the existential presupposition and the uniqueness requirement of *wh*-questions.

contrast, concessive *at most* addresses a different issue: the issue of evaluativity.

To sum up, we have seen several parallels between epistemic *at least* and epistemic *at most* in this section: **(a)** focus-sensitivity; **(b)** the diversity of scales and their discrepancy; **(c)** the two scalar effects and their discrepancy. Moreover, ignorance inferences given by both items are pragmatic in nature and sensitive to contextual factors. The next section discusses concessive *at most*. In particular, unlike concessive *at least*, the prejacent is not necessarily entailed under concessive *at most*. This lack of entailment makes the concessive reading of *at most* look like an epiphenomenon (i.e., not easily detected in some cases).

4.2 Concessive *at most*

Given our discussion of *at least* in chapter 2 and epistemic *at most* in section 4.1, we expect that *at most* similarly has a concessive reading. Now, a valid question to ask is: Does *at most* have a concessive reading? If yes, what does concessive *at most* look like? If no, why? Previous analyses of concessive *at least* do not discuss concessive *at most* (e.g., Kay 1992, N&R and Biezma 2013). Empirically, in some cases, it is indeed unclear what a concessive reading of *at most* should look like. For example, (20) illustrates sentences with *at most* associated with various scales.

(20) a. **Numeral Scales** (a contextual ranking: $4 \succ 3 \succ 2$)

John **at most** wrote [three]$_F$ novels.

b. **Plurality Scales** (a contextual ranking: adam⊕bill⊕chris ≻ adam⊕bill ≻ adam)

John **at most** hired [Adam and Bill]$_F$.

c. **Lexical Scales** (a contextual ranking: gold medal ≻ silver medal ≻ bronze medal)

John **at most** won a [silver]$_F$ medal.

d. **Pragmatic Scales** (a contextual ranking: cherries ≻apples ≻bananas)

John **at most** bought $[\text{apples}]_F$.

Note that all the four sentences in (20) can be interpreted with an evaluative flavor. Take (20c) and (20d) for example. (20c) can convey that given the context, the best medal John can win is a silver medal. (20d) can convey that given the context, the best thing John can buy is apples. Crucially, regardless of whether those sentences are interpreted with evaluativity or not, the prejacent is not (necessarily) entailed in (20). However, in other cases, we do observe a concessive reading of *at most* and the prejacent is entailed. (21) is an example borrowed from Cohen and Krifka (2014: 75).

(21) This is a bad hotel; **at most**, it's centrally located.

Two remarks are in order. First, the sentence is interpreted with an evaluative flavor: a set of relevant properties concerning the hotel is evaluated against the speaker's goals and interlocutors' interests in the discourse. Second, the prejacent is entailed: (21) entails that the hotel is centrally-located.

In addition to (21), similar examples can be constructed under the scenario in Biezma (2013)'s discussion of concessive *at least*. (22) presents the relevant conversation and (23) illustrates the relevant dimensions of evaluativity.

(22) Tom dated with someone he met online. He got home to find his friend Jim.

Jim: How was your date?

Tom: It was bad; **at most**, she was smart.

(23) Great: She was tall, smart and beautiful

Good: She was tall and smart, or She was smart and beautiful, or She was tall and beautiful

Bad: She was tall, or She was smart, or She was beautiful

Very Bad: She was not tall and She was not smart and She was not beautiful

Examples like (21) and (22) indicate that *at most* does have a concessive reading. In contrast to the epistemic reading (see section 4.1), the speaker is NOT ignorant when he/ she uses concessive *at most*. More specifically, neither is the speaker in (21) ignorant about whether the hotel is centrally-located, nor is the speaker in (22) ignorant about whether the girl is smart. Thus, in this dissertation, I argue that *at most,* like *at least*, does have a concessive reading. Seen from this perspective, the question now is not so much whether *at most* has a concessive reading. Instead, the question is why the concessive reading of *at most* is not detected as easily as that of *at least*. As we will see shortly in section 4.3 where I spell out my proposal, the reason why the concessive reading of *at most* looks like an epiphenomenon of its epistemic reading is because the prejacent is NOT necessarily entailed under CON. This property of *at most* in turn results from the fact that the semantic bounding property of *at most* and the pragmatic condition of CON converges on excluding the higher alternatives. Finally, like the case of concessive *at least*, I consider that concessive *at most* addresses the issue of evaluativity, as defined below.

(24) **Evaluativity and Speaker Concession**

a. The "settle-for-less" flavor arises when **(i)** the set of answers is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse; and **(ii)** the relevant higher alternatives are known to be false.

b. The "settle-for-less" flavor conveys speaker concession: given the evaluation, the asserted content is the best situation (we can get/ we got).[6]

---

[6] Note that the ordering can be flipped so that under concessive *at most*, the prejacen is worst situation. Imagine that there are three individuals Adam, Bill and Chris. They are good friends. They are planning on their trip and Bill is notorious for having many bad habits. Chris does not want share the room with Bill. Bill's snoring is the most unbearable habit to Chris. Adam tries to persuade Chris:

At this point, it is worth noting that concessive *at most* has a "settle-for-less" flavor that is slightly different from concessive *at least*. Descriptively, concessive *at most* conveys that the prejacent is the best situation (we can get/ we got), while concessive *at least* indicates that the prejacent is not the best situation but it is not the worst situation either. Again, as we will see shortly in section 4.3, this nuanced difference in the "settle-for-less" flavor results from the different semantic bounding properties of *at least* and *at most*.

Before closing this section, (25) presents a summary of the relevant facts that this dissertation is intended to capture.

(25) a. **The EPI-CON ambiguity**: Cross-linguistically, SMs in general demonstrate an ambiguity in giving an ignorance inference and a concessive inference.

b. **Focus-sensitivity**: the semantic contribution of SMs under both meanings (EPI and CON) depends on the position of their focus associate.

c. **Scale types and their discrepancy**: SMs under both meanings (EPI and CON) are compatible with various scales (based on semantic strength or pragmatic strength). However, in contrast to lexical scales and pragmatic scales, the ordering between focus alternatives in numerical scales and plurality scales cannot be reversed.

d. **Two scalar effects (TSE and BSE) and their discrepancy**: SMs under both meanings (EPI and CON) demonstrate two scalar effects, but there is a contrast in the type of infelicity that arises. For English *at most* (and Chinese *zuiduo*, etc), TSE may be pragmatically repaired while BSE may not.

(i) At most, Bill snores. Sharing room can save us a lot of money. Plus, you can have my earplugs. They have the latest technology and I promise: with them, you won't hear anything at night.

e. **Three Contrasts between *at least* and *at most***: First, *at most* shows a mirror image of *at least* with respect to the discrepancy between the two scalar effects TSE and BSE. Second, the concessive reading of *at most* looks like an epiphenomenon, while that of *at least* does not. Third, concessive *at most* gives rise to a "settle-for-less" flavor slightly different from that given by concessive *at least*.

4.3 The proposal: a unified semantics of *at most*

This section spells out my analysis of the EPI-CON ambiguity shown by English *at most*. In a nutshell, the idea is that **(a)** one uniform semantic representation of *at most* can be maintained; **(b)** the two readings arise from the semantic core interacting with different pragmatic factors such as informativity (see (19)) and evaluativity (see (24)).

This section proceeds as follows. Section 4.3.1 offers the semantic entry of *at most*. Section 4.3.2 shows how the semantic core combining with different pragmatic ingredients leads to EPI and CON. Section 4.3.3 explains why the two scalar effects arise and demonstrates how they are connected to the semantics of *at most*.

4.3.1 The semantics of *at most*

Let us recall that the two meanings share three common properties: the focus-sensitivity, the compatibility with various scales and the two scalar effects TSE and BSE. Seen in this light, I propose that the semantic core of the two meanings is scalarity. In particular, I suggest that scalarity can be understood as (26).

(26) **Scalarity** (the semantic core of EPI and CON)

The set of focus alternatives (the set of answers addressing the CQ) is ordered along a contextually given scale.

Furthermore, I suggest that a uniform semantic representation of the two meanings should encode scalarity. Given these considerations, I propose that English *at most* has the following semantic representation.[7, 8]

(27) **A propositional version of *at most***

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]^{9}$$

Several remarks are in order. First, C represents the contextual restriction as in Rooth's representation of focus. Second, μ_c is defined as a measure function (of type $<\eta, d>$; η in principle could be any type), mapping the focus alternatives to their corresponding positions along a contextually-valued scale. Third, the ordering between alternatives is represented in terms of a (strict) comparison relation between the prejacent α and its alternatives along a contextually-given scale: $\mu_c(\alpha) > \mu_c(\beta)$. Fourth, a superlative component, $\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)$, is encoded in the semantic representation of *at most* and imposes an additional restriction on the answer space: all the alternatives that are non-identical to the prejacent are ranked below the prejacent; this amounts to excluding the higher alternatives from the domain C. Fifth, the domain C further restricted by the superlative now denotes a set consisting of the prejacent (obtained by the focus presuppositions) and its relevant lower alternatives (obtained by the superlative component). Putting these pieces together, the semantics of *at most* in (27) asserts that there is one proposition γ in the domain consisting of the

---

[7] Other things being equal, the proposed semantic representation of English *at most* in principle applies to its cross-linguistic counterparts showing the EPI-CON ambiguity.

[8] A fully decompositional analysis of SMs is presented in chapter 4, illustrating how this semantic representation is connected to the morphological pieces of *at most*. For now, I focus on how this semantic entry interacting with different pragmatic factors results in the EPI-CON ambiguity.

[9] I thank Roger Schwarzschild for drawing my attention to the issue of presuppositions. This semantic representation is also inspired by his original suggestion adding the component $\exists\gamma[\gamma\in C \wedge \gamma_w]$ in the case of *at least* (under the discourse approach discussed in chapter 2).

prejacent and its relevant lower alternatives such that the proposition γ is true. Crucially, the semantic representation in (27) captures not only the parallel between SMs and disjunction: both *at most* and *at least* yield disjunctive statements without encoding a disjunction in the semantics, because an existential claim over a set amounts to a disjunctive statement of the elements in that set; but also the bounding property of *at most*: the prejacent is the upper bound among the set of focus alternatives ordered along a contextually-valued scale. In particular, differing from the case of *at least*, the semantic bounding property of *at most* converges on the pragmatic condition of CON that the relevant higher alternatives are (known to be) false in the discourse. As we will see shortly, this semantic-pragmatic difference is crucial for understanding why the concessive reading of *at most*, in contrast to that of *at least*, is not easily detected in some cases. Finally, to cover cases where *at most* is syntactically adjoined to constituents that are not propositional, I assume that the following two entries can be obtained by type-shifting (see also Coppock and Beaver 2013 for similar treatment of English exclusive particles).

(28) a. **A non-propositional version (by the Geach rule)**

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

b. **A non-propositional version (by the backward Geach rule)**

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

Crucially, *at most*, aligning with *at least*, does not semantically encode a non-strict comparison under the current analysis. That is, the non-strict comparison relation (discussed in chapter 2) is again illusive for *at most*: it is not a semantic primitive, but derived from focus presuppositions and the superlative component.

Finally, it is worth noting that the semantic core (27) already predicts two common properties of the two meanings: the focus-sensitivity and the compatibility with various scales. More specifically, the focus-sensitivity follows because the unified entry operates on a set of focus alternatives and imposes further restriction on the answer space (i.e., excluding the relevant higher alternatives). The compatibility with various scales follows because the unified entry requires a contextually-valued scale and the set of focus alternatives is ordered along that scale.[10] The next section discusses **(a)** how the EPI-CON ambiguity arises from the unified semantic entry interacting with different pragmatic factors: informativity vs. evaluativity; **(b)** why the concessive reading of *at most*, in contrast to that of *at least*, is not easily detected.

4.3.2 Explaining the EPI-CON ambiguity

Let's consider the case of EPI first. In section 4.1, we have seen that the ignorance inference is pragmatic and arises when a precise answer is requested in a given discourse. This is exactly what the unified entry in (27) leads us to expect. First, the speaker ignorance is not lexicalized in the semantics of *at most*. This means that the ignorance inference in question does not come from the semantics. Second, the ignorance inference arises from pragmatics because the contribution of *at most* provides a partial answer to the CQ in a discourse where the precise answer is

[10] The discrepancy between scales based on semantic strength vs. pragmatic strength will be explained in chapter 5, where the formal property of the measure function μ_c is discussed. To anticipate, the leading idea is that the measure function μ_c presents a structure-preserving mapping between the focus alternatives and their positions along a contextually-given scale. More specifically, when the set of alternatives has its own internal structure such as plurality scales (partial ordering) or numerical scales (total ordering), the ordering between alternatives is structurally-preserved and cannot be altered even with contextual manipulations.

requested. The example below illustrates the point. Below, Sup abbreviates the contribution of the superlative component in the entry (see (27) and (28)).

(29) Context: John won a bronze medal. Emily knows that John won a medal but she doesn't know what kind of medal John has won. She asks John's friend Frank.

Emily: What medal did John win?

Frank: John won **at most** a $[\text{silver}]_F$ medal.

(30) a. LF: $[[_{DP}$ **at most**$(C)[_{DP}[_{DP}$ a $[\text{silver}]_F$ medal$]{\sim}C]$ λz [John won z]]

b. C = {a gold medal, a silver medal, a bronze medal}

c. Sup = {a silver medal, a bronze medal}

d. $C \cap$ Sup = {a silver medal, a bronze medal}

e. The prejacent: a silver medal

(31) $[\![(30a)]\!]^{w,\,c} = 1$

$$\text{iff } \exists\gamma[\gamma\in C \wedge \gamma_w(\lambda z.\text{John won}_w\, z) \wedge$$
$$\forall\beta[\beta\in C \wedge \beta \neq [\![\text{a silver medal}]\!]^{w,\,c}$$
$$\rightarrow \mu_c([\![\text{a silver medal}]\!]^{w,\,c}(\lambda z.\text{John won}_w\, z)) > \mu_c(\beta(\lambda z.\text{John won}_w\, z))]]$$

Assume that Frank obeys the Gricean maxims (Grice 1989) and understands that Emily's question is requesting information about the medal John won; Frank's answer apparently fails to provide that maximally informative unique answer: John won a bronze medal. Given the semantic entry, Frank's utterance conveys that there is one element in the set represented by $C \cap$ Sup such that John won that element. Put differently, by using *at most*, Frank's utterance conveys that John won a silver medal or John won a bronze medal. An ignorance inference arises to justify Frank's failure of proving the unique answer. Crucially, given that Frank obeyed the Gricean maxims, if he had known the unique answer, he would have uttered it. In this line of reasoning, the unified semantic entry leads to ignorance inferences only under certain contexts where a precise answer is requested (as discussed in section 4.1). Taken together, I propose that SMs under EPI is addressing the issue of informativity, as defined below.

(32) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

Next, let's consider the case of CON. As discussed in chapter 2, CON has a "settle-for-less" flavor (N&R's observation) and requires two pragmatic ingredients: **(a)** the relevant higher alternatives are contextually known to be false; **(b)** the set of alternatives is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse (Biezma 2013). As we have seen in section 4.2, although N&R and Biezma (2013) discuss only the conceive *at least*, those properties are similarly observed in concessive *at most*. For instance, imagine a scenario that Emily has been a good friend of Frank. Frank has never dated someone. He just had a date with a girl he met online. After the date, Frank calls Emily and they have a discussion about the date. (33) presents the relevant conversation and (35) the relevant evaluative dimensions. (35) illustrates the computation of concessive *at most*.

(33) Emily: How was your date?

Frank: It was bad actually; **at most**, the girl was talkative.

(34) The relevant dimensions of evaluating Frank's date

Good:	the girl was beautiful, funny and talkative,
Ok:	the girl was beautiful and funny, or the girl was funny and talkative, or the girl was funny and talkative
Bad:	the girl was beautiful, the girl was funny, the girl was talkative

Very bad: the girl had none of the properties

(35) a. LF: $[_{IP}$ **at most**$(C)[_{IP}[_{IP}$ the girl [was tall]$_F]{\sim}C]]$

b. C = (34)

c. Sup = {the girl was talkative, the girl had none of the properties}

d. $C \cap$ Sup = {the girl was talkative, the girl had none of the properties}

e. The prejacent = the girl was talkative

(36) $[\![(35a)]\!]^{w,c} = 1$

iff $\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq [\![\text{the girl was talkative}]\!]^{w,c}$

$\rightarrow \mu_c([\![\text{the girl was talkative}]\!]^{w,c}) > \mu_c(\beta)]]$

In (35), the unified entry of *at most* in (27) is applied: there is one element in the set represented by $C \cap$ Sup such that the element is true. Crucially, in the case of CON, the two pragmatic requirements are fulfilled: **(a)** the relevant higher alternatives are contextually known to be false; **(b)** the set of alternatives is evaluated and ranked against the speaker's goals and the interlocutors' interests in a given discourse. The first condition ensures that the speaker Frank knows that the relevant higher alternatives are false: the girl did not have all the three properties, and the girl did not have any two of the three properties. Moreover, this pragmatic condition converges on the semantic bounding property of *at most*: the prejacent is the upper bound among the set of focus alternatives. By using *at most*, the speaker Frank signals the fact that because the higher alternatives are excluded, the prejacent is the best alternative that is true in the set represented by $C \cap$ Sup. The "settle-for-less" flavor arises from the fact the prejacent is the best true alternative in the domain represented by $C \cap$ Sup, despite some relevant alternative ranked higher than the prejacent with respect to the interlocutors' interests in the discourse. Therefore, I propose that *at most* under CON is addressing the issue of evaluativity, as defined in (37) below.

(37) **CON and the issue of evaluativity** (the case of *at most*)

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is the best situation.

At this point, it is instructive to compare *at most* with *at least*, repeated as (38).

(38) **CON and the issue of evaluativity** (the case of *at least*)

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is true; while it is not the best situation, it is not the worst situation either.

Crucially, the different "settle-for-less" flavors of concessive *at most* and concessive *at least* can be traced back to their semantic bounding properties: semantically, *at most* makes the prejacent **the upper bound** by excluding the relevant higher alternatives, while *at least* makes the prejacent **the lower bound** by excluding the relevant lower alternatives. For concessive *at most*, the semantic bounding property and the pragmatics of CON converge on requiring the relevant higher alternatives to be false. This convergence leaves open whether the relevant lower alternatives are false. Importantly, this is why the concessive reading of *at most* is not easily detected: the prejacent is not necessarily entailed under both EPI and CON. For concessive *at least*, by contrast, the semantic bounding property excludes the relevant lower alternatives while the pragmatics of CON requires the relevant higher alternatives to be false. This yields the fact that the prejacent is entailed under CON. This makes the concessive reading of *at least* more observable: the prejacent is entailed under CON, but not necessarily entailed under EPI.

Before leaving this section, it is worth emphasizing that given the current unified analysis, the semantic core underlying the EPI-CON ambiguity is still the notion of scalarity (defined in (26)) along with the bounding property of *at most*: under both readings, the prejacent is set up as the upper bound among the set of focus alternatives (the set of answers addressing the CQ). Depending on how this semantic core combines with different pragmatic factors such as informativity and evaluativity, an ignorance inference or a concessive interpretation may arise.

The next section illustrates how the third common property, the two scalar effects (TSE and BSE) and their discrepancy, follows from the current analysis. The contrast between *at least* and *at most* regarding the two scalar effects is also discussed.

4.3.3 Deriving TSE and BSE

Recall that *at most* demonstrates two scalar effects. The top-of-the-scale effect (TSE) demands that the associate cannot be the element at the top of the scale and the bottom-of-the-scale effect (BSE) that the associate cannot be the element at the bottom of the scale. The two utterances with *at most* (39) and (40) are infelicitous in a dice-playing scenario where it is known that a dice has six numbers and that the number *six* is the upper bound and the number *one* the lower bound on the possible results. Moreover, according to native speakers' judgments, there is a discrepancy: for *at most*, BSE seems *more infelicitous* than TSE.

(39) #Chris **at most** got $[\text{six}]_F$. [**Top-of-the-Scale Effect** (**TSE**)]

(40) #Chris **at most** got $[\text{one}]_F$. [**Bottom-of-the-Scale Effect** (**BSE**)]

Are these two scalar effects qualitatively the same? Why and how do they arise? What do they tell us about the semantics of *at most*? Before answering these questions, recall that the two utterances with *at least* (41) and (42) are also infelicitous in the same dice-playing scenario, as discussed in chapter 2. According to native speakers' judgments, there is a discrepancy: for *at least*, TSE seems *more infelicitous* than BSE.

(41) #Chris **at least** got $[\text{six}]_F$. [**Top-of-the-Scale Effect** (**TSE**)]

(42) #Chris **at least** got $[\text{one}]_F$. [**Bottom-of-the-Scale Effect** (**BSE**)]

The explanation offered in chapter 2 is that for *at least*, TSE results from semantic vacuity while BSE from discourse uninformativity. Moreover, this discrepancy between the two scalar effects is predictable from the semantics of *at least*. Crucially, the contrast observed in the case of *at least*, we note, is a mirror image of that in the case of *at most*, as illustrated in (43).

(43) Division of labor between semantics and pragmatics of SMs (= (11))

	TSE (the top-of-the-scale effect)	**BSE (the bottom-of-the-scale effect)**
at least	# (see (42))	△ (see (41))
at most	△ (see (39))	# (see (40))

# indicates semantic vacuity, and △ discourse uninformativity

In what follows, I show that the proposed semantics of *at most* not only predicts the two scalar effects but also predicts them to be different in nature. In particular, in the case of *at most*, BSE arises from **semantic vacuity** and is semantic in nature; in contrast, TSE arises from **discourse uninformativity** and is pragmatic in nature. Thus, only TSE can be *pragmatically repaired* by certain conversational strategies.

Let's consider BSE first. Informally put, the use of *at most* is vacuous when no relevant lower alternatives exist in the first place because the associate (the number *one*) is the upper bound. Formally, (44) illustrates the relevant pieces of the computation. In particular, **semantic vacuity** arises because the contribution of the superlative component Sup is vacuous. Recall that Sup requires all the alternatives non-identical to the prejacent to be ranked below the prejacent (i.e., $\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta))$. When no lower alternatives exist in the first place, the contribution of Sup becomes vacuous. This is illustrated by the representation that $C \cap$ Sup is a singleton set of the element that is precisely identical to the prejacent: *Chris got one*.

(44) a. The LF of (40): $[_{IP}$ **at most**(C)$[_{IP}\,[_{IP}$ Chris got $[\text{one}]_F]{\sim}C]]$

b. C = {Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}

c. Sup = {Chris got one}

d. $C \cap$ Sup = {Chris got one}

e. The prejacent = Chris got one

Thus, BSE arises as a consequence of violating a general constraint against vacuous quantification in natural language: no semantic operators can be used vacuously.

Next, let's consider TSE. I propose that the utterance (39) is infelicitous because it is **contextually uninformative**: it is already part of the common ground that only six results are possible and the number *six* is the upper bound. Below, (45) illustrates the relevant pieces of the computation. In particular, discourse uninformaitvity is illustrated by the representation that the domain $C \cap$ Sup is exactly the same set as C: {*Chris got one*, *Chris got two*, *Chris got three*, *Chris got four*, *Chris got five*, *Chris got six*}. What this means is that an assertion with *at most* does not substantially remove any possible results from the original discourse (in more dynamic terms, the assertion with *at most*, if accepted, does not *update* the common ground).

(45) a. The LF of (39): $[_{IP}$ **at most**(C)$[_{IP}\,[_{IP}$ Chris got $[\text{six}]_F]{\sim}C]]$

b. C = {Chris got one, Chris got two, Chris got three, Chris got four, Chris got five, Chris got six}

c. Sup = {Chris got one, Chris got two, Chris got three, Chris got four,

Chris got five, Chris got six}

d. $C \cap$ Sup = {Chris got one, Chris got two, Chris got three, Chris got four,

Chris got five, Chris got six}

e. The prejacent = Chris got six

Thus, TSE arises as a consequence of violating the maxim of quantity: be as informative as required. Crucially, seen in this light, the utterance (39) becomes *felicitous* once it is understood in a way that the speaker is joking or being sarcastic about Chris: that is, the speaker is being *intentionally uncooperative* and *flouting* the maxim of quantity (Grice 1989).

In short, the proposed semantics of *at most* predicts TSE and BSE to be of different nature. In the case of *at most*, BSE results from **semantic vacuity** while TSE from **discourse uninformativity**. This is evidenced by the fact that TSE, but not BSE, can be *pragmatically repaired* by certain conversational strategies.

Before closing this section, it is worth pointing out that the mirror image between *at most* and *at least* regarding the two scalar effects is not arbitrary, but predictable from their semantic bounding properties: semantically, *at most* makes the prejacent **the upper bound** by excluding the relevant higher alternatives, while *at least* makes the prejacent **the lower bound** by excluding the relevant lower alternatives. First, when there are no relevant lower alternatives (i.e., the prejacent is the bottommost element), the contribution of *at most* becomes vacuous (*Chris got at most one* is equivalent to *Chris got one*). This is why the BSE of *at most* results from semantic vacuity. Next, when there are no relevant higher alternatives (i.e., the prejacent is the topmost element), the contribution of *at least* becomes vacuous (*Chris got at least six* is equivalent to *Chris got six*). This is why the TSE of *at least* results from semantic vacuity. Finally, their bounding properties also help us understand why the TSE of *at most* and the BSE of *at least* result from discourse uninformativity: the semantic contribution of *at most* and *at least* in these cases is pragmatically uninformative.

To sum up, in this section, we have seen **(a)** what the uniform semantic representation of *at most* is; **(b)** how the EPI-CON ambiguity and the three common properties of the two meanings follow from the current analysis. Moreover, some contrasts between *at most* and *at least* are discussed. In particular, it is shown **(a)** that the different "settle-for-less" flavors of concessive *at most* and concessive *at least*, **(b)** that the phantom-like concessive reading of *at most* and **(c)** that the mirror image of *at most* and *at least* regarding the discrepancy between the two scalar effects, are all predictable from their bounding properties. The next section shows how the ignorance inference given by *at most* is derived, based on the idea that epistemic *at most* addresses the issue of informativity.

4.4 Deriving ignorance inferences

Recall that in section 4.1, we have seen that epistemic *at most* gives rise to an ignorance inference only when a precise answer is requested, for instance, when a *wh*-question serving as the CQ is interpreted exhaustively. Based on the relevant facts, I propose that epistemic *at most* addresses the issue of informativity.

(46) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

In what follows, I present a derivation of ignorance inferences based on the idea in (46), while incorporating the insights from Mendia (2016a-c)'s two scale analysis (see also Schwarz 2016a).[11] To begin with, let's consider a toy context in which epistemic *at most* gives rise to an ignorance inference. Suppose that there are three relevant individuals in the discourse: Adam, Bill and Chris. Emily knows that someone left but she doesn't know who. Consider the question-answer pair in (47) and their corresponding domains shown in (48).

(47) a. Emily: Who left?

b. Frank: **At most** $[\text{Adam and Bill}]_F$ left.

(48) a. The domain of (47a): {a, b, c, a⊕b, b⊕c, a⊕c, a⊕b⊕c}[12]

b. The domain of (47b): {a, b, a⊕b,}

Given the discourse and Emily's questions in (49a), the power set of the three individuals (Adam, Bill and Chris) are all relevant. However, after Frank's response in (49b), only three individuals remain in the domain: {a, b, a⊕b}. Now, given Frank's response, the hearer is invited to reason about the alternatives in the domain selected by Frank's answer. Following Mendia (2016a-c)'s and Schwarz (2016a)'s two-scale analysis, I assume that the calculation of ignorance inferences given by SMs requires two sources of alternatives. One source of alternatives comes from the set of alternatives induced by focus. The reasoning is as follows: Why didn't Frank choose to assert some lower alternatives?[13]

(49) ALT_{foc} (47b): $\{(\preceq a), (\preceq b), (\preceq a \oplus b)\}$

[11] The so-called "two-scale" analysis (a term coined by Schwarz 2016a) assumes that ignorance inferences associated with SMs are calculated, based on two sources of alternatives: one comes from focus and the other from the Horn-scale mate(s) with SMs. Some candidates for the Horn-scale mate with *at most* have been proposed in previous studies such as *exactly* and *at least*. Variants of the "two-scale" analysis include Mayr (2013), Kennedy (2015), Mendia (2016a-c) and Schwarz (2016a).

[12] As in chapter 2, I use the lowercase *a*, *b*, *c* to represent the three individuals Adam, Bill and Chris respectively. In addition, I assume Link (1983)'s semi-lattice structure of plural individuals.

[13] Here, the ranking is based on the semi-lattice structure of plural individuals in Link (1983). As before, I enclose a proposition by parenthesis. Also, I use ($\succeq \varphi$) to represent (*at least* φ) and (Exh φ) to represent (*only* φ).

Another source of alternatives comes from the set of exhaustified alternatives, given the selected domain (49b). The reasoning is as follows: Why didn't Frank assert the maximally informative unique answer?

(50) ALT_{exh} (47b): {(Exh a), (Exh b), (Exh a⊕b)}

Crucially, unlike Mendia (2016a-c)'s two-scale analysis, I do not claim that *at most* and *only* form a Horn-scale. The set of exhaustified alternatives in (50) does not come from the substitution of *at most* with *only*; instead, they come from the exhaustivity of the *wh*-question (i.e., the CQ). This nuanced perspective is based on Westera and Brasoveanu (2014)'s insights and our discussion in section 2.2. Now, assuming **K** is an epistemic necessity operator over the speaker's knowledge/ belief, and **P** a possibility operator "the speaker considers it possible" (Gazdar 1979), the computation of ignorance inferences is similar to that under a two-scale analysis. First, we generate a quality inference and a set of primary implicatures.[14]

(51) a. Quality Inference

$K(\preceq a \oplus b)$

b. Primary Implicatures

$\neg K(\text{Exh a}) \wedge \neg K(\text{Exh b}) \wedge$

$\neg K(\text{Exh a}\oplus\text{b}) \wedge$

$\neg K(\preceq \text{a}) \wedge \neg K(\preceq \text{b})$

The quality inference is generated because of the default assumption that the speaker believes what he asserted. The set of primary implicatures is generated: because the speaker didn't assert one of the domain alternatives, it must be that he didn't have sufficient evidence to claim so (i.e., he didn't believe those alternatives to be true).

[14] Note that K and P are defined interchangeably: $K(p) \equiv \neg P \neg (p)$ and $P(p) = \neg K \neg (p)$.

(52) Implicature Base: quality inference & primary implicatures

$K(\preceq a \oplus b) \wedge$ (53b)

The space of logical possibilities in (52) is **exhasutifed** by the following three conjuncts, indicated by **P1**, **P2** and **P3**, mnemonic of *possibilities*. Specifically, negating **any one** of the three possibilities (e.g., P1) would entail the truth of the others (e.g., P2 and P3).

(53) $\underset{\text{Assertion}}{K(\preceq a \oplus b)} \wedge \underset{\textbf{P1}}{\neg K(\text{Exh } a)} \wedge \underset{\textbf{P2}}{\neg K(\text{Exh } b)} \wedge \underset{\textbf{P3}}{\neg K(\text{Exh } a \oplus b)}$

Crucially, the three conjuncts are **symmetric**: neither of them can be negated. For example, when only two individuals Adam and Bill are under consideration, asserting that *at most Adam and Bill left* **is** at odds with knowing that *Adam was not the only person who left*, expressed as $K \neg (\text{Exh } a)$. Similar reasoning applies to the other two conjuncts. Therefore, (54) is obtained.

(54) $\neg K \neg (\text{Exh } a) \wedge \neg K \neg (\text{Exh } b) \wedge \neg K \neg (\text{Exh } a \oplus b)$

By conjoining the relevant primary implicatures (e.g., P1, P2 and P3) and (54), three ignorance inferences are generated, as shown in (55).

(55) Ignorance Inferences from Frank's answer: *At most* [*Adam and Bill*]$_F$ *left*

a. $\neg K(\text{Exh } a) \wedge \neg K \neg (\text{Exh } a)$

b. $\neg K(\text{Exh } b) \wedge \neg K \neg (\text{Exh } b)$

c. $\neg K(\text{Exh } a \oplus b) \wedge \neg K \neg (\text{Exh } a \oplus b)$

(55) amounts to saying that the speaker is ignorant about **(a)** whether Adam and Bill

left; **(b)** whether Adam left; **(c)** whether Bill left.[15] Taken together, Frank's assertion *at most Adam and Bill left* conveys his ignorance about whether only Adam left, whether only Bill left, and whether both Adam and Bill left.

It is worth noting that the current analysis is similar to other variants of the so-called "two-scale" analysis (e.g., Mendia 2016a-c) in three crucial aspects: **(a)** the calculation of ignorance inferences given by SMs requires two sources of alternatives; **(b)** the core of ignorance inferences is the *symmetric* nature of the alternatives; **(c)** the nature of ignorance inferences given by SMs is quantity-based conversational implicatures. However, the current analysis crucially differs from previous two-scale analyses in that no stipulation on the Horn Scale-mates of *at most* is needed. Therefore, the current analysis avoids those empirical problems from the non-parallel syntactic distribution between the counterpart of *at most* and that of *only* in other languages, because the set of exhasutified alternatives does not directly come from the substitution of *at most* with *only*, but comes from the exhaustivity of the *wh*-question (the CQ). Seen in this light, the current analysis not only addresses the question of *how* an ignorance inference arises with *at most*, but also connects it with the question of *why* an ignorance inference rises with *at most*. The answer to both questions is rooted in the idea that epistemic *at most* addresses the issue of informativity: ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.[16]

---

[15] As in Geurts (2010), an ignorance inference that the speaker is ignorant about a proposition *p* can be understood as follows: *the speaker does not believe p to be true* and *the speaker does not believe p to be false*. The signature of an ignorance inference can be represented as follows: $\neg Bel_s\, p \wedge \neg Bel_s\, \neg p$.

[16] One important insight of Schwarz (2016a) is to import Fox (2007)'s method of Innocent Exclusion as a consistency check, into a Gricean calculation of ignorance inferences. Although the current mechanism does not make use of Innocent Exclusion, I am positive that the analysis here can be adapted in the manner represented in Schwarz (2016a) to incorporate his insights.

4.5 The zero element in the domain

In this section, I would like to briefly address an issue that I have been sidestepping in my illustration of *at most*: the zero element. For example, when (56) is uttered out of blue, the truth-conditions of (56a) is compatible with the situation where no student left. Similarly, (56b) is judged true even if it turns out that John hired no one. The same point can be made for (56c) and (56d): the former is still judged true even though John did not win any medal and the latter is still judged true even though John bought nothing.

(56) a. **Numeral Scales**

John **at most** wrote [three]$_F$ novels.

b. **Plurality Scales**

John **at most** hired [Adam and Bill]$_F$.

c. **Lexical Scales**

John **at most** won a [silver]$_F$ medal.

d. **Pragmatic Scales**

John **at most** bought [apples]$_F$.

In my previous illustrations of *at most*, I sidestep the issue concerning the lack of existential entailment by manipulating the speaker's knowledge in the context and thus precluding the zero element in the domain in the first place. For example, in (56a) the speaker knows that John has written some number of novels; in (56b) the speakers knows that John has hired some people; in (56c) the speaker knows that John has won a medal; and in (56d) John has bought something. Notice that the issue concerning the lack of existential entailment is not unique to *at most*; in fact, it is a general feature of negative quantifiers such as *less than three*, as shown in (57).

(57) Less than three students left.

Patterning with *at most*, when (57) is uttered out of blue; it is compatible with (i.e., judged true) the situation that no student left.

There are at least two ways to approaching this issue. The first approach to this issue is that one may attempt to define a notion of *null individual* and incorporates the null individual into the ontology (see e.g., Landman 2004, Buccola and Spector 2016: section 8.3 for some tentative proposals). This semantic approach is fairly legitimate though it requires some revolutionary revision of our current theories of plurality. Similar issues concerning the null individual is also raised in recent studies on the semantics of the word *zero* (see Bylinina and Nouwen 2018). In this respect, a related issue concerns whether numerals are separate from the domain of individuals.

Given that *at most* is a focus-sensitive operator across different scales (i.e., not unique to numerical scales) and its quantificational domain is anaphoric to the denotation of the current question in the discourse, we may try to pursue a pragmatic approach and address the issue by looking into question-answer congruence in the discourse. In particular, we can pursue the idea that depending on the content of the answerhood operator, the zero element is not always included in the domain (i.e., in the denotation of *wh*-questions). Below, (58) illustrates a case where the answer *no one left* is not in the domain.[17]

(58) Context: there are three relevant individuals Adam, Bill and Chris.

a. Who left?

[17] Readers are referred to Dayal (2016: chapter 2) for a comprehensive overview of different theories of questions/ answers and relevant discussion of the content of answerhood operators.

$$\text{b. } [\![58a]\!]^{w,\,c} = \left\{\begin{array}{l} \text{Adam, Bill, Chris,} \\ \text{Adam}\oplus\text{Bill, Adam}\oplus\text{Chris, Bill}\oplus\text{Chris} \\ \text{Adam}\oplus\text{Bill}\oplus\text{Chris} \end{array}\right\}$$

Suppose that someone responds to the question in (58a) by the utterance *at most Adam and Bill*. The relevant computation is illustrated in (59). In particular (59b) presents the LF; (59c) demonstrates the focus presuppositions imposed by the squiggle operator ~ and the bolded part indicates that the prejacent has to be an element in the domain; (59d) illustrates the question-answer congruence and the anaphoricity of the domain variable C; (59e) shows the semantic contribution of the superlative component in *at most*: it keeps the two lower alternatives {Adam, Bill}; (59f) indicates the prejacent; (59g) shows that the domain $C \cap$ Sup is obtained via focus presuppositions and Sup: {Adam, Bill, Adam⊕Bill}. Finally, the semantics of *at most* (see section 4.3.1) conveys that there is one element in the domain $C \cap$ Sup such that the element left.

(59) a. **At most** [Adam and Bill]$_F$.

b. the LF of (59a): [[$_{DP}$ **At most**(C) [$_{DP}$ [$_{DP}$ Adam and Bill]$_F$ ~C]] λx [x left]]

c. α ~C is defined iff

$\boldsymbol{[\![\alpha]\!]^{o} \in C} \wedge \exists\alpha'[\alpha' \neq \alpha \wedge [\![\alpha']\!]^{o} \in C] \wedge C \subseteq [\![\alpha]\!]^{f}$

d. $[\![Q]\!]^{o} \subseteq [\![\alpha]\!]^{f}$

C = $[\![Q]\!]^{o}$ = {Adam, Bill, Chris, Adam⊕Bill, Adam⊕Chris, Bill⊕Chris, Adam⊕Bill⊕Chris}

e. Sup = {Adam, Bill}

f. The prejacent: Adam⊕Bill

g. $C \cap$ Sup = {Adam, Bill, Adam⊕Bill} via focus presuppositions and Sup

(58) and (59) above illustrate a case where the zero element is not included in the domain. When the zero element is included in the domain, a slightly different representation is obtained. Below, (60) illustrates the relevant pieces of computation.

(60) a. **At most** [Adam and Bill]$_F$.

b. the LF of (60a): [[$_{DP}$ **At most**(*C*) [$_{DP}$ [$_{DP}$ Adam and Bill]$_F$ ~*C*]] λ*x* [*x* left]]

c. α ~*C* is defined iff

$[\![\boldsymbol{\alpha}]\!]^{\mathbf{o}} \in \boldsymbol{C} \wedge \exists\alpha'[\alpha'\neq\alpha \wedge [\![\alpha']\!]^{o} \in C] \wedge C \subseteq [\![\alpha]\!]^{f}$

d. $[\![Q]\!]^{o} \subseteq [\![\alpha]\!]^{f}$

C = $[\![Q]\!]^{o}$ = {**no one**, Adam, Bill, Chris, Adam⊕Bill, Adam⊕Chris, Bill⊕Chris, Adam⊕Bill⊕Chris}

e. Sup = {**no one**, Adam, Bill}

f. The prejacent: Adam⊕Bill

g. *C* ∩ Sup = {**no one**, Adam, Bill, Adam⊕Bill}

(obtained via focus presuppositions and Sup)

The crucial difference between (59) and (60) is whether the zero element "no one" is included in the domain. Taken together, what (58), (59) and (60) above show us is that whether the zero element is included in the domain actually depends on the content of the answerhood operator and the semantics of the current question (e.g., *wh*-questions) in the discourse. Crucially, the semantics of *at most*, under the current analysis, is compatible with both cases.

Before closing this section, it is worth noting that the current analysis not only predicts that *at most* is compatible with the zero element, but also derives the fact that in contrast to *at most*, *at least* is **not** compatible with the zero element and thus has an existential entailment. This contrast in the compatibility with the zero element again results from the different semantic bounding properties of *at least* and *at most*.

Specifically, in the case of *at least*, the contribution of Sup keeps the relevant higher alternatives, thus always excluding the lower alternatives (i.e. including the zero element), regardless of whether the zero element is actually included in the domain. The only exception that the zero element is **not** excluded is when the zero element itself is the prejacent. But empirical facts suggest its awkwardness, as shown below.

(61) A: Who left?

B: **At least** Adam/ one student / #nobody/ #no student/ #zero student.

4.6 Conclusion

In this chapter, I have taken English *at most* as a case study and presented a unified analysis of the two meanings generally shown by SMs across many languages: an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. In the analysis, I have shown **(a)** what the uniform semantic representation of *at most* is; **(b)** how the EPI-CON ambiguity arises from one unified semantic entry combining with different pragmatic factors such as informativity and evaluativity; **(c)** how the three properties follow from the current analysis: the focus-sensitivity, the compatibility with various scales, the two scalar effects and their discrepancy; **(d)** why three contrasts between *at most* and *at least* are observed: First, *at most* shows a mirror image of *at least* with respect to the discrepancy between the two scalar effects TSE and BSE. Second, the concessive reading of *at most* looks like an epiphenomenon, while that of *at least* does not. Third, concessive *at most* has a "settle-for-less" flavor slightly different than that given by concessive *at least*. Crucially, these contrasts are not arbitrary, but predictable. All the three contrasts result from the different semantic bounding properties of *at most* and *at least*: Under both meanings, *at most* makes the prejacent the upper bound while *at least* makes the prejacent the lower bound. The analysis here, *ceteris paribus*, is expected to hold for the counterparts of *at most* in other languages.

The core ingredients for the unified analysis are listed below.

(62) **A propositional version of *at most***

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$$

(63) **A non-propositional version (by the Geach rule)**

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\ st>}\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

(64) **A non-propositional version (by the backward Geach rule)**

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\lambda P_{<\eta,\ st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

(65) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

(66) **CON and the issue of evaluativity**

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is the best situation.

(67) **Scalarity** (the semantic core of EPI and CON)

The set of focus alternatives (the set of answers addressing the CQ) is ordered along a contextually given scale.

Chapter 5 The Anatomy of Superlative Modifiers

Superlative modifiers (SMs) pose a longstanding and intriguing morpho-semantic puzzle: Why do SMs morphologically involve a quantity adjective and the superlative morpheme? What is the role of quantity adjectives and superlative morphemes inside SMs? How are these morphological pieces of SMs connected with their semantics? In this respect, Chinese makes the situation even more puzzling: the same expressions (morphologically consisting of a quantity adjective and the superlative morpheme) *zui-duo* and *zui-shao* are used as superlative modifiers and quantity superlatives. This chapter takes Chinese SMs as a case study and presents a decompositional analysis.

This chapter proceeds as follows. Section 5.1 presents the major data of Chinese SMs that this dissertation is concerned with. Section 5.2 reviews three major approaches to quantity words. Section 5.3 briefly looks at two major approaches to superlatives in the literature: movement theory vs. in-situ theory. Section 5.4 spells out the core ingredients of my proposal and presents a decompositional analysis of Chinese *zuiduo*. Section 5.5 extends the proposed decompositional analysis to Chinese *zuishao* and addresses issues concerning the apparently antonymous relation between *zuishao* and *zuiduo*. Section 5.6 illustrates how English SMs are analyzed under the current analysis. Section 5.7 highlights several significant features of the current decompositional analysis of SMs and addresses three issues: (a) What does it mean to assign numerical values to propositions via a measure function? (b) Can the proposed analysis be extended to SMs in other languages? (c) How and why are SMs parallel to disjunction and epistemic indefinites, with respect to the ignorance interpretation? Section 5.8 briefly compares my decompositional analysis of SMs with Coppock (2016). Section 5.9 concludes this chapter.

5.1 Basic data

In this section, I introduce the major data of Chinese SMs that this dissertation is concerned with. First of all, SMs and quantity superlatives (QS) in Chinese are expressed by the same lexical items. For example, the same expression *zuiduo*, morphologically consisting of the superlative morpheme *zui* and the quantity adjective *duo* 'much', is used in both SMs and QSs. The same observation applies to the expression *zuishao*, morphologically consisting of the superlative morpheme *zui* and the quantity adjective *shao* 'little'. (1) and (2) illustrate the point.[1, 2, 3]

---

[1] Although the expression *zuiduo* morphologically corresponds to English *most*, it does not have a proportional reading in Chinese. Instead, the proportional reading is conveyed by a different expression *daduoshu* 'majority', as in (i).

(i) *Daduoshu de xuesheng dou hen renzhen*.
Majority DE student DOU very diligent
'The majority of students are diligent.'
cf. Most students are diligent.

To avoid confusion, I refer to the expressions *zuiduo* and *zuishao* without showing their literal translations like 'most' or 'least' in the text. It should be noted that cross-linguistically, it is not uncommon to find superlative expressions lacking the proportional reading. See Coppock et al. (2017) for a typological perspective on the connection between superlatives and the proportional reading.

[2] It has been established in Hackl (2000, 2009) that quantity superlatives, unlike quality superlatives, have only the relative/ comparative reading, as shown in (i) and (ii).

(i) Adam climbed the highest mountain. ambiguous
Absolute reading: Adam climbed Mount Everest.
Relative reading: Adam climbed a higher mountain than anyone else did.

(ii) Adam climbed the most mountains. unambiguous
Relative reading: Adam climbed more mountains than anyone else did.

Readers are referred to Hackl (2000, 2009) for detailed discussions and an analysis of the contrast.

[3] An interesting fact is that classifiers may be optional in quantity superlatives. This is puzzling and surprising because Chinese is a well-known classifier language where classifiers are observed to be obligatory. Crucially, the observed optionality of classifiers is not unique to quantity superlatives; the same optionality appears in quantity comparatives and quantity positives.

(i) Liubei chi-le geng-duo/ geng-shao (ke) pinguo.

(1) a. Liubei mai-le **zui-duo** (ke) pinguo.
Liubei buy-ASP SUP-many CL apple
'Liubei bought more apples than anyone else did.'

b. Liubei mai-le **zui-duo** [san]$_F$-ke pinguo.
Liubei buy-ASP SUP-many three-CL apple
'Liubei bought at most three apples.'

(2) a. Liubei mai-le **zui-shao** (ke) pinguo.
Liubei buy-ASP SUP-little CL apple
'Liubei bought fewer apples than anyone else did.'

b. Liubei mai-le **zui-shao** [san]$_F$-ke pinguo.
Liubei buy-ASP SUP-little three-CL apple
'Liubei bought at least three apples.'

In (1a), *zuiduo* conveys the relative reading of superlatives: the quantity of apples that *Liubei* bought is more than the quantity of apples that any other relevant individual did. In contrast, in (1b), *zuiduo* conveys an upper bound on the number of apples that *Liubei* bought. The same contrast holds for (2) with the expression *zuishao*, though with a reversed polarity.

Second, like English *at least/ at most* (discussed in chapter 2 and chapter 4), Chinese SMs are also focus-sensitive: the semantic contribution of *zuiduo/ zuishao* depends on its focus associate and different positions of the associate leads to

---

Liubei eat-ASP more-many more-few CL apple
'Liubei ate more/ less apples.'

(ii) Liubei chi-le hen-duo/ (?)hen-shao (ke) pinguo.
Liubei eat-ASP very-many very-few CL apple
'Liubei ate many/ few apples.'

I leave for future study a detailed investigation of issues concerning the optionality of classifiers in Chinese degree constructions.

truth-conditional differences.[4] Consider the contexts in (3) and the utterances in (4).

(3) Context A: a contextual ranking: cherries ≻apples ≻bananas

What did *Liubei* buy for our plan tonight?

Context B: a contextual ranking: make dinner ≻buy apples ≻boil water

What did *Liubei* do for our plan tonight?

(4) a. Liubei **zui-duo/** **zui-shao** mai-le $[\text{pinguo}]_F$.

Liubei SUP-many SUP-little buy-ASP apple

'Liubei at most/ at least bought apples.'

b. Liubei **zui-duo/** **zui-shao** [mai-le $\text{pinguo}]_F$.

Liubei SUP-many SUP-little buy-ASP apple

'Liubei at most/ at least bought apples.'

Due to different positions of the focus associates, (4a) and (4b) are truth-conditionally different. In (4a), a lower bound (in the case of *zuishao*)/ an upper bound (in the case of *zuiduo*) is imposed on what *Liubei* has bought. In contrast, the relevant bounding property is placed on what *Liubei* has done in (4b). Therefore, (4a) is felicitous as a continuation to the question in context A, but (4b) is not. Conversely, (4b) is felicitous as a continuation to the question in context B, but (4a) is not.

Third, Chinese SMs, being parallel to their English counterparts, are also compatible with various scales, as shown below.[5, 6]

---

[4] In English, focused constituents usually bear prosodic prominence. One may wonder whether Chinese focused constituents bear similar prosodic realizations as English. Because I didn't conduct any phonetic experiment, I don't intend to make any phonetic claim here. Nevertheless, the focus effect in Chinese can be clearly observed in the question-answer congruence between focused constituents and their discourse questions. We leave for future study the issue of how Chinese focused constituents are phonetically realized.

[5] In Chinese, when *zuiduo*/ *zuishao* occur with proper names or quantifiers in a prenominal position, the sentences are reported to be degraded for some native speakers.

(5) **Numerical Scales** (a contextual ranking: 4 ≻3 ≻2)

Liubei	**zui-duo/**	**zui-shao**	xie-le	[san]$_F$-ben-xiaoshuo.
Linbei	SUP-many	SUP-little	write-ASP	three-CL-novel

'Liubei at most/ at least wrote three novels.'

(6) **Plurality Scales** (a contextual ranking: adam&bill&chris ≻adam&bill ≻adam)

Liubei	**zui-duo/**	**zui-shao**	guyong-le	[Adam he Bill]$_F$.
Liubei	SUP-many	SUP-little	hire-ASP	Adam and Bill

'Liubei at most/ at least hired Adam and Bill.'

(7) **Lexical Scales** (a contextual ranking: gold medal ≻silver medal ≻bronze medal)

Liubei	**zui-duo/**	**zui-shao**	na-le	[yin]$_F$-pai.
Liubei	SUP-many	SUP-little	take-ASP	silver-medal

'Liubei at most/ at least got a silver medal.'

(8) **Pragmatic Scales** (a contextual ranking: cherries ≻apples ≻bananas)

(i) ? Liubei guyong-le zui-duo/ zui-shao [Adam he Bill]$_F$.
Liubei hire-ASP SUP-many SUP-little Adam and Bill
'Liubei hired at most/ at least Adam and Bill.'

(ii) ?? Liubei yaoqing-le zui-duo/ zui-shao [yi-xie-xueshen]$_F$.
Liubei invite-ASP SUP-many SUP-little one-CL-student
'Liubei invited at most/ at least some students.'

However, the sentences become perfect when *zuiduo*/ *zuishao* occur in a preverbal position.

(iii) Liubei zui-duo/ zui-shao guyong-le [Adam he Bill]$_F$.
Liubei SUP-many SUP-little hire-ASP Adam and Bill
'Liubei at most/ at least hired Adam and Bill.'

(iv) Liubei zui-duo/ zui-shao yaoqing-le [yi-xie-xueshen]$_F$.
Liubei SUP-many SUP-little invite-ASP one-CL-student
'Liubei at most/ at least invited some students.'

At this moment, I have nothing interesting to say about the contrast, other than suggesting that the contrast may be reduced to certain syntactic competition or incompatibility between the position of prenominal *zuiduo*/ *zuishao* and that of proper names/ quantifiers in Chinese. I leave this line of research for another occasion.

[6] As shown in chapter 2 and chapter 4, Chinese SMs also reveal the ambiguity between speaker ignorance and speaker concession. In this chapter, I focus on some relevant properties of Chinese SMs for a decompositional analysis, leaving aside the ambiguity for the moment. Readers are referred to chapter 2 and chapter 4, where I argue that the two meanings are pragmatic variants and can be unified by one single semantic representation for *zuishao*/ *at least* and *zuiduo*/ *at most* respectively.

Liubei **zui-duo/** **zui-shao** mai-le [pingguo]$_F$.

Liubei SUP-many SUP-little buy-ASP apple

'Liubei at most/ at least bought apples.'

Note that the numerical scale and the plurality scale are based on semantic strength (i.e., entailment relation). Therefore, writing four novels entails writing three novels, and hiring Adam and Bill entails hiring Adam and hiring Bill. In contrast, the lexical scale and the pragmatic scale are based on pragmatic strength (i.e., non-entailment relation). Thus, winning a gold medal does not entail winning a silver medal, and buying apples does not entail buying bananas.

Moreover, as discussed in chapter 2, by manipulating the context, it is easy enough to reverse the ordering between the alternatives in the case of pragmatic scales.[7] In contrast, however, it does not seem possible to reverse the ordering in the case of numerical scales or plurality scales, even with some contextual effort.

(9) Context: *Caocao*, *Liubei*, *Sunquan* are planning to buy some fruit for their party tonight. There are three types of fruit available to them: cherries, apples and bananas. However, they are poor and do not have enough money to buy everything. For them, bananas are the optimal because they are the cheapest; apples are less optimal but acceptable because they are still cheaper than cherries.

The contextual ranking (in terms of price): bananas ≻apples ≻cherries

(10) Context: *Liubei* is planning to hire some people. There are three applicants in the discourse: Adam, Bill and Chris. But the budget is limited. If three people are all hired, *Liubei* need to pay a great amount of money for their salary. If only Adam

---

[7] For lexical scales, although context manipulations are not impossible, they are indeed harder because the ordering is based on our common world knowledge. Instances of lexical scale are such as *gold medal ≻silver medal ≻bronze medal*, and *full professor ≻associate professor ≻assistant professor*.

and Bill are hired, the situation is better, but *Liubei* still pays more than he does in hiring only Adam. The best situation for *Liubei* is simply to hire just one person while getting all the work done.

The intended contextual ranking:

only adam ≻only adam&bill ≻only adam&bill&chris

Under the context (9), the utterance with *zuishao* in (8) is understood to convey that *Liubei* bought apples or bananas (given the contextual ranking: bananas ≻apples ≻cherries). This means that the original ranking (cherries ≻apples ≻bananas) in (8) is now *reversed*. In contrast, the utterance with *zuishao* in (6) cannot be understood to be that *Liubei* hired only Adam and Bill, or hired only Adam, even with the contextual massage in (10). This indicates that the original ranking (adam&bill&chris ≻adam&bill ≻adam) in (6) cannot be reversed. The same observation applies to the numeral scale. I leave it for readers to verify the case of numerical scales.

Fourth, like English *at least* and *at most*, Chinese SMs also demonstrate two scalar effects: the top-of-the-scale effect (TSE) and the bottom-of-the-scale effect (BSE). TSE requires that the associate cannot be the element at the top of the scale, while BSE that the associate cannot be the element at the bottom of the scale.

(11) Context: *Caocao*, *Liubei*, *Sunquan* are playing dice. In each round, whoever gets a bigger number wins; scores are not cumulated. A dice has six numbers on it: <u>***Six*** is the upper bound and ***one*** the lower bound on the possible results</u>. *Caocao* threw the dice but *Liubei* missed the result. During *Liubei*'s turn, he asked *Sunquan* what the result was.

(12) Liubei: Caocao shai-le shenme shuzi?
Caocao dice-ASP what number

‘What number did Cindy get?’

(13) *#Caocao* ***zui-shao*** *shai-le* [*yi*]$_F$. **BSE**

Caocao SUP-little dice-ASP one

‘Caocao at least got one.’

(14) *#Caocao* ***zui-shao*** *shai-le* [*liu*]$_F$. **TSE**

Caocao SUP-little dice-ASP six

‘Caocao at least got six.’

(15) *#Caocao* ***zui-duo*** *shai-le* [*yi*]$_F$.[8] **BSE**

Caocao SUP-much dice-ASP one

‘Caocao at most got one.’

(16) *#Caocao* ***zui-duo*** *shai-le* [*liu*]$_F$. **TSE**

Caocao SUP-much dice-ASP six

‘Caocao at most got six.’

Note that none of (13) − (16) are felicitous responses to *Liubei*’s question in (12) under the scenario (11). Furthermore, suppose *Sunquan* is cooperative, it cannot be that *Sunquan* made his responses in (13) − (16) when he saw *Caocao* rolling the dice and thus had direct evidence of the exact number that *Caocao* got.

In chapter 2 and chapter 4, based on empirical facts concerning English *at least* and *at most*, I argue that the nature of the two scalar effects is not fixed across the

---

[8] As mentioned in chapter 4, Veneeta Dayal (p.c.) correctly points out that in English, the sentence *Chris at most got [one]$_F$* can be felicitous if it is understood to be an indirect evidential inference. The same observation holds for the Chinese sentence (15). Imagine that the three people have played dice for many rounds. *Caocao* had a bad luck and he never got more than one. In this scenario, *Sunquan* can felicitously assert the sentence (15) and justify his response via a score board with *Caocao*’s previous results. At this point, I have no explanation for why the BSE of *at most*/ *zuiduo* can be improved under an indirect evidential inference. But notice that the corresponding “good luck” scenario does not help the TSE of *at least*/ *zuishao*. The contrast here suggests that the pairs of SMs may not be fully parallel in their semantic contributions.

board. More specifically, two types of infelicity are involved: semantic vacuity vs. discourse uninformativity. The same proposal is maintained here. Taken together, the following division between semantics and pragmatics for both English and Chinese SMs suggests itself. Note that (17) presents a mirror image between the pairs of SMs with respect to the division between semantics and pragmatics.

(17) Division of labor between semantics and pragmatics of SMs

	TSE (the top-of-the-scale effect)	**BSE (the bottom-of-the-scale effect)**
at least/ zuishao	# (e.g.,(14))	△ (e.g., (13))
at most/ zuiduo	△ (e.g., (16))	# (e.g., (15))

# indicates semantic vacuity, and △ discourse uninformativity

Briefly, discourse uninformativity arises when the semantic contribution of SMs and the pragmatic bounding property in a given context converge.[9] This suggests that the infelicity witnessed by (13)/ (16) and their corresponding English examples can be repaired by certain pragmatic strategies, such as the speaker's intentional flouting of the maxim of quantity. Obviously, another way to remove discourse uninformativity is

---

[9] One way to define discourse uninformativity is in terms of assertion and context set (Stalnaker 1978, 1998, 2002; among others), as sketched below.

(i) A context set C is a set of propositions that the interlocutors have publically committed to.
$C =_{def} \{w \mid w \in \cap \{p_{<s,t>} \mid$ the interlocutors have publically committed to $p\}\}$

(ii) An assertion of utterance U is informative in a discourse D if it updates the context set C.
$C[U] =_{def} \{w \in C: U^{w}=1\}$

(iii) An assertion of utterance U is uninformative in a discourse D if it doesn't update the context set C.
$C[U] =_{def} \{w \mid w \in C\}$

The idea is that discourse (un)informativity depends on whether asserting a proposition p updates the context set (i.e., remove those worlds where p is false from C). For our current purposes of the distinction, the definitions in (i)−(iii) should suffice. However, a more accurate way would be to characterize an assertion as a discourse move in *proposing* an update of the context set: an update happens only when the discourse move is accepted (e.g., Farkas and Bruce 2010, among others).

simply to create other scenarios where the corresponding utterances are informative.[10]

In contrast, semantic vacuity can be understood as a ban on the vacuous use of operators in natural languages (see Al Khatib 2013 on English *only*). This means that the infelicity witnessed by (14)/ (15) and their corresponding English examples may NOT be rescued by pragmatic strategies. I will come back to the issue of semantic vacuity in sections 5.4 − 5.5, where I decompose the semantics of Chinese SMs.

Fifth, Chinese SMs, being parallel to their English counterparts, show scope interaction with universal quantifiers over individuals (e.g., Büring 2008). The authoritative reading arises when SMs scope below the universal quantifier. In contrast, the speaker insecurity reading arises when SMs scope above the universal quantifier. The scope interaction in question is illustrated below.[11]

(18) Speaker Insecurity Reading (*zuishao* takes wide scope)

Zheli	mei-tai-diannao	**zui-shao**	dou	you	liang-GB	de	jiyiti.
Here	every-CL-computer	SUP-little	DOU	have	two-GB	DE	memory

‘Every computer here has at least 2-GB of memory.’

---

[10] Here is a minimally different scenario (inspired by a SALT reviewer’s comments) where the BSE of *zuishao/ at least* and the TSE of *zuiduo/ at most* vanish. Suppose everything in the scenario (11) remains the same, except that the scores are now cumulated. In this score-cumulative scenario, *Sunquan*’s asserting *Caocao at least got* $[one]_F$ or *Caocao at most got* $[six]_F$ can become felicitous.

(i) This round is the last round. So far, *Caocao*’s scores are 6. On this round, **he at least got one**. So, he is the winner (and we lose).

(ii) This round is the last round. So far, *Caocao*’s scores are only 6. On this round, **he at most got six**. So, he is out/ lost the game. Now, it’s only you and me! *Liubei*, roll the dice!

Crucially, (i) and (ii) are not cases of flouting the maxim of quantity. Instead, in the score-cumulative scenario, *Sunquan*’s response to *Liubei*’s question is informative. In (i), it conveys that no matter what number *Caocao* got, he is bound to win. In (ii), no matter what number *Caocao* got, he is bound to lose.

[11] In Chinese, quantifiers of individuals must occur with the particle *dou* and it has been proposed that *dou* is a cover-based distributive operator (e.g., Lin 1998) in the sense of Schwarzschild (1996). However, the precise semantic contribution of the particle *dou* is an ongoing debate in Chinese linguistics. In the last two decades, many works have been devoted to it and various proposals have been put forth in the literature. What is relevant to us here is the distributive meaning of *dou* in occurring with plural individuals. See Yang (2001), Chen (2008), Liu (2016), Tsai (2016), Xiang (2017) and references therein for discussion and different views on the semantic contribution of *dou*.

(19) Authoritative Reading (*zuishao* takes narrow scope)

Zheli mei-tai-diannao dou you **zui-shao** liang-GB de jiyiti.

Here every-CL-computer DOU have SUP-little two-GB DE memory

'Every computer here has at least 2-GB of memory.'

The scope interaction is manifested by two sentences in Chinese. This is presumably due to the scope-rigidity of Chinese (Huang 1982). The sentence in (18), where *zuishao* scopes above the particle *dou*, implies that the speaker is ignorant about the exact amount of memory that each computer has, while the amount is no less than 2GB. In contrast, the sentence in (19), where *zuishao* scopes below the particle *dou*, conveys that some computers have 2GB, some have more, but none have less. That is, the computers vary in the amount of memory and that amount has a lower bound 2GB.

Finally, the same scope interaction is also observed for *zuiduo*. In (20), where *zuiduo* scopes above the particle *dou*, the sentence implies that the speaker is ignorant about the exact amount of cash back that each credit card has, while she is certain that the amount is no more than five hundred. In contrast, the sentence in (21), where *zuiduo* scopes below the particle *dou*, conveys that some credit cards have 500 cash back, some have less, but none have more. That is, the credit cards vary in the amount of cash back and that amount has an upper bound 500 dollars.

(20) Speaker Insecurity Reading (*zuiduo* takes wide scope)

Zheli mei-zhang-xinyong-ka mei-ji **zui-duo** dou you

Here every-CL-credit-card every season SUP-much DOU have

wu-bai-yuan xianjin huikui.

five-hundred-dollar cash feedback

'Every credit card here has at most five hundred cash back every season.'

(21) Authoritative Reading (*zuiduo* takes narrow scope)

Zheli mei-zhang-xinyong-ka mei-ji dou you **zui-duo**

Here every-CL-credit-card every season DOU have SUP-much

wu-bai-yuan xianjin huikui.

five-hundred-dollar cash feedback

'Every credit card here has at most five hundred cash back every season.'

To sum up, we have seen several properties of Chinese SMs in this section. Most of them are parallel to those of English *at least*/ *at most* discussed in chapter 2 and chapter 4. Now, let us summarize the main data that any decompositional analysis of Chinese SMs must explain.

(22) a. **The morpho-semantic puzzle**: The same expressions *zuiduo* and *zuishao* are used in superlative modifiers and quantity superlatives.

b. **Focus-sensitivity**: The semantic contribution of *zuiduo* and *zuishao* depends on the position of their focus associate.

c. **Scale types and their discrepancy**: *Zuiduo* and *Zuishao* are compatible with various scales (based on semantic strength or pragmatic strength). However, in contrast to lexical scales and pragmatic scales, the ordering between the alternatives cannot be reversed in numerical scales and plurality scales.

d. **Two scalar effects (TSE and BSE) and their discrepancy**: *Zuiduo* and *Zuishao* demonstrate the two scalar effects and two types of infelicity are involved. In particular, semantic vacuity arises with TSE for *zuishao*/ English *at least* and arises with BSE for *zuiduo*/ English *at most*.

Before I spell out my proposal in sections 5.4−5.5, let us briefly look at previous wisdom on quantity adjectives and superlatives. To avoid any pre-theoretical confusion, I use a neutral term "quantity words" for elements like *many*, *much*, *few* and *little* in English, when I review different approaches in section 5.2.

5.2 Three major approaches to quantity words

In this section, I briefly review three approaches to quantity words in previous studies. One approach assigns a quantificational meaning to quantity words and analyzes them as determiners, with a focus on their prenominal use. In contrast, the other two approaches deal with different syntactic positions of quantity words in a sentence and propose a non-quantificational meaning for them; however, the two approaches differ in whether quantity words are semantically bleached, more specifically, whether they semantically encode a contextually-valued measure function over different sorts of semantic objects (e.g., individuals and events). For purposes of this dissertation, I adopt the view that quantity words like *much* encode a measure function whose dimension is contextually valued. Finally, I justify the choice in section 5.2.4.

5.2.1 Quantity words are determiners

In the theory of generalized quantifiers (Barwise & Cooper 1981, Keenan and Stavi 1986, cf. Greek 2014), quantifiers like *every* and *some* are analyzed as determiners, denoting relations between two sets of individuals (P and Q), as (23) shows.

(23) a. $[\![every]\!] = \lambda P\, \lambda Q[P \subseteq Q]$

b. $[\![some]\!] = \lambda P\, \lambda Q\, [P \cap Q \neq \varnothing]$

For example, (23) predicts that a sentence *every boy laughed* is true iff the set of boys is a subset of the set of individuals who laughed, and that a sentence *some boys* *laughed* is true iff the intersection of the set of boys and the set of individuals who laughed is not empty. An equivalent way of expressing these truth-conditions is to have overt logical quantifiers over individuals, as in (24).

(24) a. $[\![every]\!] = \lambda P_{<e,\,t>}\ \lambda Q_{<e,\,t>}.\forall x[P(x) \rightarrow Q(x)]$

b. $[\![some]\!] = \lambda P_{<e,\,t>}\ \lambda Q_{<e,\,t>}.\exists x[P(x) \wedge Q(x)]$

In the generalized quantifier theory, quantity words like *many* and *few* are analyzed in the same fashion, though with additional machinery. Specifically, the cardinality operator $|.|$ is required. (25) predicts that a sentence *many boys laughed* is true iff the cardinality of the intersection of the set of boys and the set of individuals who laughed is greater than or equal to some large number d, and that a sentence *few boys laughed* is true iff the cardinality of the intersection of the set of boys and the set of individuals who laughed is less than or equal to some large number d.

(25) a. $[\![many]\!]^c = \lambda P\ \lambda Q\ |P \cap Q| \geq d_c$, for some large number d_c

b. $[\![few]\!]^c = \lambda P\ \lambda Q\ |P \cap Q| < d_c$, for some small number d_c

A close variant of the generalized quantifier theory is the so-called "parameterized determiner" analysis (Romero 1998, 2015 and Hackl 2000). Under this view, quantity words are analyzed as determiners while parameterized with an additional degree argument, as shown in (26).[12]

---

[12] A central focus of the determiner approach concerns the ambiguity demonstrated by quantity words like *many* and *few* in their prenominal use. For example, it has been observed that (i) reveals a three-way ambiguity: a cardinal reading, a proportional reading and a reverse proportional reading. Assume that S represents the set of Scandinavians, W the set of Nobel Prize winners and d_c is some contextually determined quantity threshold, the three readings can be schematically illustrated below.

(i) Many Scandinavians have won the Nobel Prize in the literature.

Cardinal reading: $|S \cap W| \geq d_c$
Proportional reading: $|S \cap W| : |\mathrm{S}| \geq d_c$
Reverse proportional reading: $|S \cap W| : |\mathrm{W}| \geq d_c$

(26) a. $[\![many]\!]^c = \lambda d_{<d>}\ \lambda P_{<e,\,t>}\ \lambda Q_{<e,\,t>}.\ \exists x[P(x) \wedge Q(x) \wedge |x| \geq d]$

b. $[\![few]\!]^c = \lambda d_{<d>}\ \lambda P_{<e,\,t>}\ \lambda Q_{<e,\,t>}.\exists x[P(x) \wedge Q(x) \wedge |x| < d]$

This treatment, in contrast to traditional generalized quantifier theory, makes quantity words a hybrid between a quantifier and a gradable predicate. Despite the differences, it is worth noting that the two have in common that quantity words encode an existential quantification (or its set-theoretic equivalent) over individuals.

5.2.2 Quantity words are interval-based

Another approach focuses on the differential use of quantity words such as the sentence *Adam is much taller than Bill*, and suggests that quantity words should be defined in terms of intervals (i.e., sets of degrees). For example, Rett (2008, 2014) proposes that quantity words are degree modifiers (of type $<<d, t>, <d, t>>$), denoting relations between a set of degrees D (i.e., an interval) and its size d, as in (27).

(27) $[\![much]\!] = \lambda D_{<d,\,t>}\ \lambda d_{<d>}[d \text{ is the size of } D]$

Assume that the comparative morpheme *–er* is a degree generalized quantifier as in (28) and that each clausal argument of the comparative morpheme denotes a set of degrees (e.g., von Stechow 1984, Kennedy 1999, Heim 2001, among others), (29) illustrates how the compositional computation works for the sentence *Adam is taller than Bill is*.

Very briefly, the cardinal reading conveys that the number of Scandinavians who are Nobel Prize winners is large. The proportional reading conveys that the number of Scandinavians who are Nobel Prize winners is large, relative to the number of Scandinavians. The reverse proportional reading conveys that the number of Scandinavians who are Nobel Prize winners is large, relative to the number of Nobel Prize winners in general.

Previous studies have been devoted to deriving the ambiguity in a principled way. One key issue concerns whether in such derivations, the determiners remain conservative or not. Another issue concerns the role of focus in making available the reverse proportional reading. Readers are referred to Westerstahl 1985, Herberger 1997, Cohen 2001, Romero 2015 for more detailed discussions.

(28) $[\![\text{-}er]\!] = \lambda D_{<d,\,t>}\, \lambda D'_{<d,\,t>}\, \lambda d_{<d>}.D(d) \wedge \neg D(d)$

(29) a. Adam is taller than Bill is.

b. LF: *–er* ($\lambda d'$ Bill is tall to d') (λd Adam is tall to d)

c. $[\![(29)\text{a}]\!] = 1$ iff $\exists d$[Adam is d-tall $\wedge$ $\neg$(Bill is d-tall)]

(after existential closure)

With the differential *much*, the compositionality of the sentence *Adam is much taller than Bill* is illustrated in (30). Note that in (30), the truth-condition of the sentence requires the set of differential degrees to exceed a certain contextual standard. According to Rett (2008), the requirement is due to the null degree modifier EVAL, a function from a set of degrees to those which exceed a contextually-valued standard.[13]

(30) a. Adam is much taller than Bill.

b. LF: EVAL (much ({d: Adam is d-tall $\wedge$ $\neg$(Bill is d-tall)]}))

c. $[\![(30)\text{a}]\!] = 1$ iff $\exists d'$[d' is the size of {d: Adam is d-tall $\wedge\neg$(Bill is d-tall)]}

$\wedge d' > s_i$]   (after existential closure)

(31) $[\![\text{EVAL}_i]\!]^g = \lambda D_{<d,\,t>}\, \lambda d_{<d>}.\ D(d) \wedge d > s_i$, s_i represents some contextual standard

Because quantity words are now analyzed as a relation between a set of degrees and its size (of type <<*d*, *t*>, <*d*, *t*>>), rather than a degree-parameterized determiner or an adjectival predicate with cardinality measurement, we need some element that enables nouns to be optionally associated with degree-denoting expressions.[14] The

[13] According to Rett (2008), a construction is evaluative if it refers to a degree that exceeds some contextual threshold. In Rett (2008), the evaluative requirement is encoded by the null degree modifier EVAL and thus as part of the truth-conditions. However, Rett (2015) recently argues that the nature of the semantic property "evaluativity" is better thought of as an implicature, rather than part of what is said, by drawing empirical parallels between evaluativity and two types of implicatures: uninformativity-based Quantity implicatures and Manner implicatures.

[14] One possibility is to treat quantity words as gradable adjectives encoding a cardinality measure function (of type <*d*, <*e*, *t*>>). This view has been employed in Hack (2000, 2009)'s decompositional

association is done via a null operator or a corresponding type-shifting mechanism (Schwarzschild 2005, 2006, Nakanishi 2007), as shown in (32), where μ_c represents the relevant dimension of measurement, valued contextually.

(32) $[\![\text{M-OP}]\!]^c = \lambda P_{<e,\,t>}\ \lambda d_{<d>}\ \lambda x_{<e>}.[P(x) \wedge \mu_c(x) \geq d]$

M-OP relates individuals x in the extension of some predicate P to their degrees along some contextually valued dimension. The compositional computation of the sentence *many girls laughed* is illustrated in (33).

(33) a. Many girls laughed.

b. LF: EVAL (many (M-OP (girls laughed))

c. $[\![$ M-OP girls laughed$]\!]^c = \lambda d_{<d>}.\exists x[\text{girls}(x) \wedge \text{laughed}(x) \wedge \mu(x) \geq d]$

(after existential closure over the variable x)

d. $[\![$ many (M-OP girls laughed)$]\!]^c$

$= \lambda d'[d'$ is the size of $\{d$: $\exists x[\text{girls}(x) \wedge \text{laughed}(x) \wedge \mu(x) \geq d]\}]$

e. $[\![$(33)a$]\!]^c = 1$

iff $\exists d'[d'$ is the size of $\{d$: $\exists x[\text{girls}(x) \wedge \text{laughed}(x) \wedge \mu(x) \geq d]\} \wedge d' > s_i]$

(after EVAL and existential closure)

Another interval-based analysis is proposed in Solt (2009, 2015). In a series of papers, Solt suggests that quantity words should be analyzed as predicates of intervals (of type <*d*, <*d*, <*d*, *t*>>>), as shown below.

(34) $[\![\textit{many/much}]\!] = \lambda d_{<d>}\lambda I_{<d,\,t>}.I(d)$

(35) $[\![\textit{few/ little}]\!] = \lambda d_{<d>}\lambda I_{<d,\,t>}\neg I(d)$

analysis of English *more* and *most*, where a covert *many* with the meaning in (i) is involved in both cases. See Rett (2018) for an overview of different approaches to quantity words.

(i) $[\![\textit{many}]\!] = \lambda d_{<d>}\ \lambda x_{<e>}.\mu_{card}(x) \geq d$

Solt's proposal differs from Rett's in two crucial respects. First, Rett characterizes quantity words in terms of higher-order measurement, namely, the measurement of an interval (a set of degrees), while Solt analyzes them in terms of a set-inclusion relation. Second, Solt's proposal highlights the intuitive parallel between quantity words and gradable adjectives: both take degrees as their first argument. However, Quantity words differ from gradable adjectives in taking an interval as their second argument.

Despite these differences, the two proposals share many assumptions. First, both accounts (following Nakanishi 2007) extend to the differential use of quantity words in the nominal and verbal domain. Second, both accounts resort to existential closure over variables in the absence of overt binding or modification. Third, both accounts assume with Schwarzschild (2006) that the relevant dimension of measurement is contextually valued and constrained by monotonicity. Forth, both accounts employ a null operator to associate entities with degree arguments. In particular, Solt posits a null operator Meas, as in (36), which is slightly different from Rett's M-OP in (32).[15]

(36) $[\![\text{Meas}]\!]^c = \lambda x_{<e>}\, \lambda d_{<d>}.\mu_c(x) \geq d$

To sum up, under the interval-based approach, quantity words are characterized as denoting a relation between a degree and an interval (a set of degrees). The relation may be higher-order measurement (Rett's proposal) or set-inclusion (Solt's proposal). Crucially, quantity words do not encode measure functions over individuals (or events). Instead, the association of entities with degree arguments is done via a null

[15] Solt (2015: 237) introduces an additional compositional rule "Degree Argument Introduction" to allow the null operator Meas to compose in the prenominal position.

(i) Degree Argument Introduction (DAI):
If α is a branching node, $\{\beta, \gamma\}$ are the set of α's daughters, and $[\![\beta]\!] = \lambda x_{<e>}.P(x)$,
$[\![\gamma]\!] = \lambda x_{<e>}\lambda d_{<d>}.Q(d)(x)$, then $[\![\alpha]\!] = \lambda d_{<d>}\lambda x_{<e>}.\ P(x) \wedge Q(d)(x)$

measurement operator: M-OP under Rett's proposal and Meas under Solt's proposal.

5.2.3 Quantity words introduce measurement

Another non-quantificational approach to quantity words is taken by Wellwood et al. (2012) and Wellwood (2014, 2015). Based on empirical facts concerning the parallel between the nominal domain and the verbal domain with respect to measurement and monotonicity, Wellwood (2014, 2015) entertains the hypothesis that all comparative sentences – nominal, verbal and adjectival – contain instances of a single morpheme that introduces measurement. In particular, this morpheme in English is sometimes pronounced *much* and semantically contributes a structure-preserving mapping from entities, events or states to their measures along some contextually-valued dimensions. Consider the English examples below.[16]

(37) Nominal Comparatives

a. Adam drank more coffee than Bill did. √volume, √weight, *temperature

b. Adam drank as much as coffee as Bill did. √volume, √weight, *temperature

(38) Verbal Comparatives

a. Adam ran more than Bill did. √duration, √distance, *speed

b. Adam ran as much as Bill did. √duration, √distance, *speed

In (37), the dimension of measurement cannot be TEMPERATURE. In (38), the

[16] When the noun is pluralized, only cardinality is available for the dimension f measurement.

(i) a. Adam has more rocks than Bill does. cardinality, *weight
b. Adam has many rocks as Bill does. cardinality, *weight

See Bale and Barner (2009) and Wellwood (2014: chapter 5) for discussions.

dimension of measurement cannot be SPEED. Crucially, volume and weight are monotonic with respect to measurement while temperature is not (e.g., Schwarzschild 2006). A similar contrast holds between duration/ distance and speed (e.g., Krifka 1989). Building on these observations, Wellwood (2014, 2015) proposes an index-based entry of English *much*, of type <η, d>. In (39), α ranges over (at least) events and individuals, thus, η could be of type <v> (eventualities) or <e> (individuals); c is a contextually-valued variable, representing the relevant dimension of measurement.

(39) $[\![much]\!]^c = \lambda\alpha.\mu_c(\alpha)$ <η, d>

Furthermore, the measure function μ_c respects cumulative reference and is constrained by monotonicity (in the sense of Schwarzschild 2006), as defined below.

(40) Cumulative Reference

$\text{CUM}(P) =_{\text{def}} \forall x \forall y[P(x) \land P(y) \rightarrow P(x \oplus y)]$

A predicate P is cumulative if and only if whatever it holds of two things, it also holds of their sum.

(41) Monotonicity

A measure function μ: $D_{\preceq Part} \rightarrow D_{\preceq Deg}$ is **monotonic** iff

for all $\alpha, \beta \in D_{\preceq Part}$, if $\alpha \prec^{Part} \beta$, then $\mu(\alpha) \prec^{Deg} \mu(\beta)$.

Requiring that the mapping be monotonic ensures three things. First, the mapping is homomorphic to the structure of the measured domain (the input domain). Second, the mapping is structure-preserving; it is the part-whole structure that is preserved. Third, the preservation is non-trivial. These conditions, for instance, prevent WEIGHT from being a possible dimension for μ_c, when α is a running event. Crucially, this means that the dimension of measurement understood in a given context depends not only on how the measured entities are ordered, but also on what sort of entities they are.

One core property underlying the parallel between the nominal domain and the verbal one is that both mass nouns and atelic predicates show cumulative reference. Seen in this light, a monotonic restriction on measurement in the nominal domain means that for any two individuals that are properly ordered in a part-of relation, their measurements are similarly ordered. The same monotonic restriction holds for events in the verbal domain. Putting together, Wellwood's proposal for English *much* thus explains why the dimension of measurement in (37) and (38) must be monotonic (or extensive in the terminology of Krifka 1989).

Finally, (42) and (43) below are cases with non-monotonic dimensions (e.g., temperature and speed). In fact, Wellwood (2014, 2015) argues that these cases also involve a covert *much* introducing the measurement and she analyzes gradable adjectives as a one-place predicate of states.[17] According to Wellwood, sentences like (37) and (42) differ in their dimensions for comparison because of what is measured: the former involves measurement of the individual satisfier of ⟦*coffee*⟧ and by monotonicity, the dimension is restricted to WEIGHT or VOLUME; in contrast, the latter

---

[17] Wellwood (2014: chapter 4) assigns a neo-Davidsonian entry to the adjective *hot*, as in (i). Wellwood motivates her neo-Davidsonian analysis of gradable adjectives based on Fults (2006)'s observation on the empirical contrast in (ii).

(i) ⟦*hot*⟧ = λs. hot(s)

(ii) a. Al is more patient <u>with Mary on the playground</u> than Bill is.
b. Al is more patient than Bill is <u>with Mary on the playground</u>.

According to Wellwood, (iia) cannot be understood as comparing Al's degree of patience directed at Mary, spatially located in a certain way, with Bill's degree. This suggests that the modifiers combine with the gradable adjective *patient* **before** it combines with the comparative morpheme. In contrast, in (iib), the modifiers are contained within the scope of the *than*-clause alone.

Wellwood considers the modification facts as suggesting that gradable adjectives have a state variable, via the same line of reasoning for the existence of event argument in Davidson (1967). As acknowledged by Wellwood herself, the modification facts do not further suggest that gradable adjectives **lack** an individual or a degree argument. However, based on the empirical parallel between comparatives in the nominal, verbal and adjectival domain, Wellwood entertains the hypothesis that in all comparative constructions, the measurement is uniformly introduced by the morpheme *much*. Readers are referred to Wellwood (2014: chapter 4-5, 2015) for detailed discussions.

involves the measurement of the state satisfier of ⟦*hot*⟧, which leads to TEMPERATURE.

(42) a. Adam's coffee is hotter than Bill's is. *volume, *weight, √temperature

b. Adam's coffee is as hot as Bill's is. *volume, *weight, √temperature

(43) a. Adam ran faster than Bill did. *duration, *distance, √speed

b. Adam ran as fast as Bill did. *duration, *distance, √speed

For purposes of the dissertation, I will not go into the details of her analysis of gradable adjectives. What's important is Wellwood (2014, 2015)'s idea that the quantity word *much* encodes a contextually-valued measure function introducing the measurement and is constrained by monotonicity requirement. In the next section, I justify why I adopt Wellwood's view on quantity words in this dissertation.

5.2.4 The case of superlative modifiers

In this dissertation, I pursue Wellwood's view that quantity words semantically encode a measure function whose dimension is contextually valued in my decompositional analysis of SMs.[18] Below; I present two reasons justifying the choice. First, it is observed that cross-linguistically, SMs behave like adverbs, rather than determiners, in terms of their syntactic distribution.

(44) English

a. **At least/At most** John won a silver medal.

b. John **at least/at most** won a silver medal.

[18] As discussed in section 5.1, SMs are not exclusively restricted to dimensions that are structured by part-of relation (e.g., plurality scales). In particular, SMs are compatible with lexical scales and pragmatic scales where the domain does not necessarily involve a partial order (i.e., it could be a case of preorder or weak order). This minimally suggests that the quantity adjective involved in SMs should not be restricted to monotonicity requirement, unlike what we have seen in nominal comparatives and verbal comparatives. I will come back to this point in section 5.4, where I decompose Chinese SM *zuiduo*.

c. John won **at least/at most** a silver medal

(45) Chinese

a. **Zhishao/ Zuishao** Liubei mai-le san-ke-pinguo.

At least Liubei buy-ASP three-CL-apple

'At least Liubei bought three apples.'

b. Liubei **zhishao/ zuishao** mai-le san-ke-pinguo.

Liubei at least buy-ASP three-CL-apple

c. Liubei mai-le **zhishao/ zuishao** san-ke-pinguo.

Liubei buy-ASP at least three-CL-apple

The data in (44) and (45) suggest that quantity words involved in SMs should not be treated as determiners. This immediately rules out the determiner approach for our analysis of SMs, leaving us the choice between Rett's/ Solt's approach and Wellwood's approach: whether quantity words are semantically bleached or not.

Second, SMs cross-linguistically involve quantity words (with superlative morphemes in many languages) in their morphological makeup, as in (46) – (50).

(46) Chinese

a. *zui-duo* 'at most'

SUP-much/many

b. *zui-shao*

SUP-little/few 'at least'

(47) English

a. *at most*

many/ much-est

b. *at least*

little-est

(48) Italian

a. *al massimo* ‘at most’

at most/ maximum

b. *al meno* ‘at least’

at least

(49) Japanese

a. *ooku-temo* ‘at most’

many-even.if

b. *sukunaku-temo* ‘at least’

few-even.if

(50) Turkish

a. *en çok* or *en fazla* ‘at most’

SUP many/ much SUP many/ much

b. *en az* or *en az-ın-dan* ‘at least’

SUP little SUP little-3sgposs-ablative(from)

The list is not meant to be exhaustive. Similar morphological makeup occurs in Bangla, Brazilian Portuguese, Czech, French, Hindi, Korean, Magahi, Russian, and Taiwan Southern Min, etc…. These cross-linguistic facts would be surprising if the semantics of quantity words involved in SMs were simply bleached as in Rett (2008, 2014) and Solt (2009, 2015), where the relevant dimension of measurement is encoded by a null operator such as Rett's M-OP or Solt's Meas. Put differently, it would be very puzzling why cross-linguistically natural languages insists on the presence of these semantically bleached quantity words in the morphology of SMs.[19]

It should be noted that this dissertation is not at any attempt to argue against the determiner approach and/ or the interval-based approach being a proper analysis of quantity words. Its goal instead is to better understand the morpho-semantic puzzle and its cross-linguistic nature; given my purpose and the properties shown by SMs, Wellwood (2014, 2015)'s domain general approach to quantity words seems to be a better fit. Therefore, I adopt Wellwood's view that quantity words encode a measure function whose dimension is contextually given. In the next section, I briefly introduce two approaches to superlatives; this is done to provide a background for my

---

[19] Recall that an important motivation for an interval-based treatment of quantity words comes from their differential use. Solt (2011) illustrates how an interval-based analysis of *much* can be employed in superlatives (couched in Heim 1999's movement theory) in English. But, as Roger Schwarzschild (p.c.) points out, superlatives in English are generally incompatible with differential phrases, though they may receive modification from the *by*-phrase conveying the differential meaning.

(i) a. Adam is 3 cm taller than Bill.
b. *Adam is 3cm the tallest

(ii) a. Adam is taller than Bill by 3 cm.
b. Adam is the tallest by 3 cm.

I further observe that superlatives in the nominal domain and the verbal domain also receive modification from the *by*-phrase. Interestingly, nominal and verbal comparatives are compatible with the *by*-phrase, but less so with canonical differential phrases.

(iii) a. Adam drank the most coffee/ water by 3 litters.
b. Adam ran the most by 3 miles.

(iv) a. ??Adam drank 3 litters more coffee/ water than Bill did.
cf. Adam drank more coffee/ water than Bill did by 3 litters.

b. ??Adam ran 3 miles more than Bill did.
cf. Adam ran more than Bill by 3 miles.

For languages lacking the strategy of *by*-phrase such as Chinese, superlatives are plainly incompatible with differential phrases.

(v) a. *Liubei bi Caocao gao <u>san-gongfen</u>.*
Liubei than Caocao tall three-cm
'Liubei is 3cm taller than Caocao.'

b. **Liubei zui-gao <u>san-gongfen</u>.*
Liubei SUP-tall three-cm
Intended: 'Liubei is the tallest by 3cm.'

I leave open the issue whether an interval-based analysis of quantity words is ultimately compatible with superlatives and the puzzle how and why in English *by*-phrase differs from canonical differential phrases, if there is indeed a covert *much* involved in both cases.

illustration of how Chinese quantity superlatives are computed in sections 5.4 −5.5.

5.3 Two approaches to superlatives

A sentence containing a superlative expression, such as *the highest mountain* in (51), can receive different interpretations depending on how the comparison class is specified with respect to different constituents of the sentence (Heim 1985, Szabolcsi 1986, Gawron 1995, among others). When the comparison class is determined with respect to the superlative DP itself, the absolute reading arises. In contrast, the relative reading arises when the comparison class is established with respect to one of the constituents in the sentence, such as Adam.

(51) Adam climbed the highest mountain.

Absolute reading: Adam climbed the mountain that is higher than any other (relevant) mountain.

Relative reading: Adam climbed a mountain that is higher than any other (relevant) individual did.

Szabolcsi (1986) and Heim (1985, 1999) propose that the absolute-relative ambiguity of a superlative sentence is derived by allowing the superlative morpheme *–est*, with the semantics in (52), to take different scope within the clause. Under this movement approach, the ambiguity of a superlative sentence is actually a case of structural ambiguity. The computation of the relevant pieces is illustrated below, with the absolute reading in (53) and the relative reading in (54).

(52) a. $[\![\text{-}est]\!] = \lambda C_{<e,\,t>}\, \lambda G_{<d,\,et>}\, \lambda x_{<e>}.\forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d.G(x, d)) > \max(\lambda d.G(y, d))]$

b. Presuppositions: $x \in C$, $\forall y[y \in C \wedge y \neq x \rightarrow \exists d[G(y, d)]]$

(53) Absolute Reading

a. $[_{DP}$ the $[_{NP}$ [-***est*** (C)]$[_{NP}$ d-high mountain]]]

b. $[\![d\text{-high mountain}]\!] = \lambda d.\lambda x.\text{mountain}(x) \wedge \text{high}(x) \geq d$

c. $C = \{x\text{: } \exists d.\text{mountain}(x) \wedge \text{high}(x) \geq d\}$

d. $[\![\text{DP}]\!] = \iota x \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d.\text{mountain}(x) \wedge \text{high}(x) \geq d)$

$> \max(\lambda d.\ \text{mountain}(y) \wedge \text{high}(y) \geq d)]$

(54) Relative Reading

a. $[_{IP}$ Adam [-***est*** (C)] $\lambda d.\ \lambda x.$ $[_{vP}$ x climbed a d-high mountain]]

b. $C = \{x\text{: } \exists d \exists z[\text{mountain}(z) \wedge \text{high}(z) \geq d \wedge x \text{ climbed } z]\ \}$

c. $[\![\text{IP}]\!] = 1$ iff

$\forall y[y \in C \wedge y \neq \text{adam} \rightarrow \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z) \geq d \wedge$

$\text{adam climbed } z]) > \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z) \geq d \wedge y \text{ climbed } z])$

According to (53), under the absolute reading, the superlative morpheme takes scope within the DP and the comparison class C is a set of relevant mountains. In contrast, under the relative reading shown in (54), the superlative morpheme takes scope outside the DP (specifically, *-est* scopes over the VP) and the comparison C is a set of relevant mountain-climbers.

Alternatively, some researchers purse an in-situ approach (e.g., Farkas and Kiss 2000, Sharvit and Stateva 2002), where the superlative morpheme never moves out of the DP, and the relative reading is derived from domain restriction. Consider (55), where the bolded part indicates the additional contextual restriction on the value of C.

(55) Relative Reading (an in-situ approach)

a. $[_{DP}$ the $[_{NP}$ [-***est*** (C)]$[_{NP}$ d-high mountain]]]

b. $[\![d\text{-high mountain}]\!] = \lambda d.\lambda x.\text{mountain}(x) \wedge \text{high}(x) \geq d$

c. $C = \{x\text{: } \exists d \exists z.\text{mountain}(x) \wedge \text{high}(x) \geq d \wedge \boldsymbol{z} \textbf{ climbed } \boldsymbol{x}\}$

d. $[\![\text{DP}]\!] = \iota x \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d.\text{mountain}(x) \wedge \text{high}(x) \geq d)$

$> \max(\lambda d.\ \text{mountain}(y) \wedge \text{high}(y) \geq d)]$

The choice between a movement approach and an in-situ approach is an ongoing debate in the literature on superlatives. However, it may well be that both approaches are needed (see Tomaszewicz 2015 for a comparative perspective on the correlation between definiteness marking and different types of relative readings).[20] In this dissertation, I assume that both options are available for deriving the relative reading of a superlative sentence in natural language. But, for purposes of illustration, I adopt a movement approach to demonstrate the computation of Chinese quantity superlatives in sections 5.4 −5.5.

Finally, it is worth noting that superlatives are focus-sensitive (e.g., Heim 1999). For example, (56) and (57) are truth-conditionally different; on the relative reading, (56) conveys that John bought more apples on Sunday than any other day, while (57) says that John bought more apples than anyone else did.

(56) John bought the most apples on [Sunday]$_F$.

(57) [John]$_F$ bought the most apples on Sunday.

To synthesize the scope-taking property of the superlative morpheme and the

[20] A crucial assumption under the movement approach is that the definite article *the* in superlatives is optionally interpreted as an indefinite (Heim1999). This assumption has been a soft spot for the movement approach. However, couched in the framework of dynamic semantics, Bumford (2017) recently argues that the definite article *the* can be semantically decomposed into two components: one builds a set of witness that satisfies the restricting noun phrase and the other imposes the uniqueness test. The former amounts to the meaning of an indefinite in dynamic semantics. Under the relative reading, the superlative morpheme *–est* takes a parasitic scope (in the sense of Barker 2007) between the first and the second component of *the*. Bumford's analysis elegantly removes the long-standing soft spot for the movement approach. Readers are referred to his paper for details. See also Coppock and Beaver (2014) for discussion of definiteness marking in superlatives.

contribution of focus, Heim (1999) provides another possible entry as defined in (58).[21] The idea behind (58) is that focus helps set the contextual value of the domain *C*. Put differently; focus restricts the domain of the superlative operator.

(58) a. $[\![\text{-}est]\!] = \lambda C_{<<d,\, t>,\, t>}\, \lambda P_{<d,\, t>} \forall Q[Q \in C \wedge Q \neq P \rightarrow \max(\lambda d.P(d)) > \max(\lambda d.Q(d))]$

b. Presuppositions: $P \in C$, $\exists Q[Q \in C \wedge Q \neq P]$

With (58) in mind, the relevant computation of (56) is shown in (59), and the relevant computation of (57) is in (60).

(59) a. The LF of (56): [***-est*** (*C*)] [λ*d*. [$_{IP}$ John bought *d*-apples on Sunday$_F$]~C]

b. *C* = {*x*: λ*d*. John bought *d*-apples on *x*}

c. $[\![(56)]\!] = 1$ iff $\forall Q[Q \in C \wedge Q \neq (\lambda d.$ John bought *d*-apples on Sunday)

→ max(λ*d*. John bought *d*-apples in Sunday) > $\max(\lambda d.Q(d))]$

(60) a. The LF of (57):[***-est*** (*C*)] [λ*d*. [$_{IP}$ John$_F$ bought *d*-apples on Sunday]~C]

b. *C* = {*x*: λ*d*. *x* bought *d*-apples on Sunday}

c. $[\![(57)]\!] = 1$ iff $\forall Q[Q \in C \wedge Q \neq (\lambda d.$ John bought *d*-apples on Sunday)

→ max(λ*d*. John bought *d*-apples in Sunday) > $\max(\lambda d.Q(d))]$

In both cases; the superlative operator plus its domain restrictor ***-est*** (*C*) takes scope

---

[21] Within the framework of degree semantics, there are two major approaches to comparatives: the maximality approach (e.g., von Stechow 1984) and the A-not-A approach (e.g., Klein 1980, 1982). The semantics of the superlative morpheme originally formulated in Heim (1999) is couched in the A-not-A approach, as shown below.

(i) The three-place superlative operator
$[\![\text{-}est]\!] = \lambda C_{<e,\, t>}\, \lambda R_{<d,\, et>}\, \lambda x_{<e>}.\exists d[\, R(x, d) \wedge \forall y[y \in C \wedge y \neq x \rightarrow \neg R(y, d)]]$
Presuppositions: $x \in C$, $\forall y[y \in C \wedge y \neq x \rightarrow \exists d[G(y, d)]]$

(ii) The two-place superlative operator
$[\![\text{-}est]\!] = \lambda C_{<<d,\, t>,\, t>}\, \lambda P_{<d,\, t>} \exists d[\, P(d) \wedge \forall Q[Q \in C \wedge Q \neq P \rightarrow \neg Q(d)]]$
Presuppositions: $P \in C$, $\exists Q[Q \in C \wedge Q \neq P]$

For purposes of this dissertation, my presentation of Heim (1999) is couched in the maximality approach. Nothing crucial in my analysis hinges on the choice between these two approaches. Readers are referred to von Stechow (1984) for a comparison of different semantic approaches to comparatives. See also Schwarzschild (2008) for a recent discussion.

over the whole sentence. Recall that the crucial difference between (56) and (57) lies in the position of focus. Crucially, the effect of focus is captured by different contextual values of the domain *C*: in (59b), the set of degree properties vary with respect to the days John bought apples, while in (60b) the set of degree properties vary with respect to the individuals who bought apples on Sunday.

I adopt a variant of the two-place superlative operator along the spirit of the definition in (58).[22] As we will see shortly, this focus-sensitive entry crucially allows us to explain the morpho-semantic puzzle of SMs in Chinese (and potentially other languages like Turkish): exactly the same morphological components are involved in both superlative modifiers and quantity superlatives.

5.4 Decompose *zuiduo 'at most'*

In this section, I spell out my decompositional analysis of *zuiduo*. Recall that one intriguing puzzle is the fact that Chinese SMs are exclusively composed of a quantity adjective and the superlative morpheme in their morphology. Below, (61) is repeated as a summary of the relevant facts that this dissertation attempts to capture.

(61) a. The morpho-semantic puzzle: The same expressions *zuiduo* and *zuishao* are used in both superlative modifiers and quantity superlatives.

b. Focus-sensitivity: The semantic contribution of *zuiduo* and *zuishao* depends on the position of their focus associate.

c. Scale types and their discrepancy: *Zuiduo* and *Zuishao* are compatible with various scales (based on semantic strength or pragmatic strength). However, in

[22] In explicating the role of focus, the three-place superlative operator requires the movement of the focus-marked constituent to serve as its third argument. Heim (1999) discusses this point and explicitly expresses her doubt that multiple LFs actually go with the *relative prominence* on focus-marked constituents at PFs. Readers are referred to Heim (1999) and Sharvit and Stateva (2002) for discussion on the role of focus in superlatives.

contrast to lexical scale and pragmatic scale, the ordering between the alternatives cannot be altered in the numerical scale and plurality scale.

d. Two scalar effects (TSE and BSE) and their discrepancy: *Zuiduo* and *Zuishao* demonstrate the two scalar effects and two types of infelicity are involved. In particular, semantic vacuity arises with TSE for *zuishao*/ English *at least* and arises with BSE for *zuiduo*/ English *at most*.

The rest of this section is structured as follows. Section 5.4.1 spells out the core ingredients of my decompositional analysis of Chinese SMs. Section 5.4.2 demonstrates a detailed computation of Chinese data. In particular, I illustrate how the proposed analysis works for different syntactic positions of *zuiduo*: cases of propositional modification and those of non-propositional modification. Section 5.4.3 offers the computation of Chinese quantity superlatives within the movement approach, as a comparison with my decompositional analysis of *zuiduo*.

5.4.1 The proposal

My decompositional proposal of *zuiduo* has three pieces: **(a)** the internal compositionality of the superlative construction SupP, and the role of the superlative morpheme *zui* and the quantity adjective *duo* 'much' inside Chinese SM *zuiduo*; **(b)** the role of the SupP as a whole inside Chinese SMs; **(c)** a covert existential operator E-OP (structurally containing the SupP). Let's see the first piece of my proposal. Assuming Bobaljik (2012)'s Containment Hypothesis, I propose that in Chinese SMs, the superlative construction structurally embeds a comparative construction, as illustrated in (62). Note that Comp^{+}P represents the embedded covert comparative.

(62) $[_{\text{SupP}}\ \textit{zui}\ [_{\text{Comp+P}}\ \text{Comp}^{+}\ [_{\text{AdjP}}\ \textit{duo}]]]$

Regarding the semantic details, let's start in a bottom-up fashion. For the quantity adjective *duo*, I propose that it encodes a measure function, which maps the elements induced by focus to their corresponding positions along a contextually given dimension, as illustrated in (63). Note that the focus alternative α can be of propositional (type <*s*, *t*>) or non-propositional (e.g., type <*e*> and <*e*, *t*>).[23]

(63) $[\![duo]\!]^c = \lambda\alpha.\mu_c(\alpha)$ $\langle\eta, d\rangle$

Moreover, I propose that in the case of SMs, the measure function μ_c respects the monotonicity constraint, but crucially is not restricted to it (cf. Wellwood et al. 2012, Wellwood 2014, 2015). This has two consequences immediately. First, Chinese SM *zuiduo* (and their cross-linguistic counterparts) can apply to the alternatives whose domain is not structured (e.g., by the part-of relation), as in the case of lexical scale and pragmatic scale. Second, when the domain of the alternatives is structured, as in the case of numerical scale and plurality scale, the structure-preserving mapping guarantees that the output ranking between the alternatives cannot be altered.

Notice that the discrepancy between different scale types is now derived: When the set of focus alternatives has its own internal structure, through a structure-preserving mapping, the output ranking between the alternatives cannot be altered despite contextual effort (e.g., numerical scale and plurality scale). In contrast, when the set of focus alternatives is not structured by entailment relation or part-of relation, the output ranking between the alternatives is subject to contextual factors and thus is not constant across contexts (e.g., lexical scale and pragmatic scale). At this point, it is worth noting that if the ranking between the alternatives were simply

---

[23] Lin (2014) focuses on the differential use of the quantity adjective *duo* 'much' in Chinese differential comparatives. He follows Solt (2009, 2015) and assigns *duo* 'much' an interval-based meaning. As discussed in section 5.2.4, the reason why I purse Wellwood's approach (rather than an interval-based approach) is because Wellwood's view that quantity adjectives encode a domain-general measure function seems to be a better fit with the relevant facts observed for SMs across languages.

given by an assignment function, the discrepancy between different scale types would become a mystery. It is unclear why the assignment function would be sensitive to the internal structure of the alternatives but not to a given context. Seen in this light, I take the discrepancy between types of scales as an additional motivation to apply formal tools developed in studies on gradability to studies on scalarity.

(64) Monotonicity

A measure function μ: $D_{\preceq Part} \rightarrow D_{\preceq Deg}$ is **monotonic** iff

for all $\alpha, \beta \in D_{\preceq Part}$, if $\alpha \prec^{Part} \beta$, then $\mu(\alpha) \prec^{Deg} \mu(\beta)$.

Next, the comparative morpheme Comp^{+} (cf. English *-er*) takes the adjective *duo* as its first argument and returns a comparison relation between the alternatives.

(65) $[\![\text{Comp}^{+}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_{c}(\alpha) > \mu_{c}(\beta)$ $\quad <\eta, <\eta, t>>$

Finally, the semantics of the superlative morpheme *zui* is like the entry traditionally assigned to English *–est* (see section 5.3), except for the additional comparison relation and its type-flexibility. The semantics of SupP is provided in (67).

(66) $[\![zui]\!]^{c} = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

(67) $[\![(62)]\!]^{c} = \lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_{c}(\alpha) > \mu_{c}(\beta)]$ $\quad$ SupP

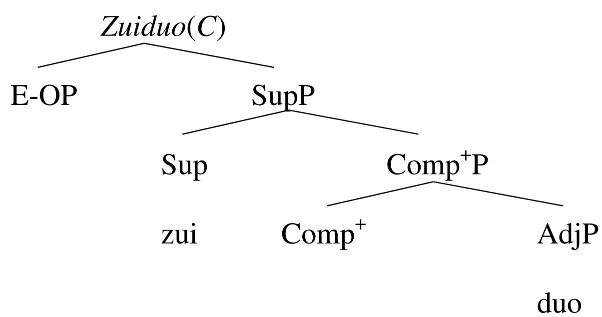
So far, we have established the first piece. The second piece of my proposal concerns the role of the superlative construction SupP as a whole inside Chinese SMs. I propose that the superlative construction SupP serves as a domain restrictor on the set of focus alternatives. For example, in the case of *zuiduo*, the superlative construction SupP requires that all the alternatives that are non-identical to the prejacent α in the domain C, be ranked **below** the prejacent along a contextually given dimension. This amounts to removing the higher alternatives from the domain.

Finally, the third piece of my proposal is that the superlative construction SupP as a whole is embedded under a covert existential operator E-OP. Below, the semantics of E-OP is defined with a propositional version in (68).

(68) $[\![\text{E-OP}]\!]^{w,\,c} = \lambda SUP_{<<st,\,t>,\,<st,\,t>>}\lambda C_{<st,\,t>}\ \lambda\alpha_{<st>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w \wedge SUP\ (C, \alpha)]$

E-OP takes SupP as its additional domain restrictor and asserts that there is one element in the domain *C* further restricted by SupP such that the element is true. A complete picture concerning the morpho-semantics of Chinese SM *zuiduo* is presented below, with the internal structure in (69) and the semantics in (70).

(69) The internal structure of Chinese SM *zuiduo* at LF



(70) $[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w \wedge \forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$

Given that the set of focus alternatives does not have to be propositional and that SMs can be adjoined to constituents of non-propositional meanings, I assume that a non-propositional version can be obtained by certain type-shifting rules such as the Geach rule (see Jacobson 1999, Coppock and Beaver 2013), as shown below.

(71) A non-propositional version (by the Geach rule)

$$[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\,st>}\lambda P_{<\eta>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w(P) \wedge \forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

(72) A non-propositional version (by the backward Geach rule)

$$[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge$$
$$\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

Before moving to the computation of Chinese data, it is worth noting that with the stage set up above, we actually have derived some core properties revealed by Chinese SMs (and their English counterparts). First, we now have a better understanding of the morpho-semantic puzzle posed by Chinese SMs and its cross-linguistic nature. Second, focus-sensitivity follows from the collaboration between focus and the measure function μ_c: the former induces a set of elements and the latter maps each of them to some position along a contextually given dimension. Third, the measure function μ_c respects the monotonicity constraint, but crucially is not restricted to it. As a consequence, depending on whether the input domain of μ_c is structured (i.e., via partial ordering or total ordering) or not, the output ranking between the alternatives may or may not be constant across different contexts. Therefore, different scale types and their discrepancy are derived. However, nothing has been said, so far, about the two scalar effects (TSE and BSE) and their discrepancy. The issue of semantic vacuity is addressed in the next section.

5.4.2 Compositions: superlative modifiers

Let us first consider the case of propositional modification. The relevant sentence is presented in (73), with its LF in (74). The computation is illustrated in (75).

(73) **Zuiduo** Liubei shi yi-wei $[\text{fu}]_F$ -jiaoshou.

SUP-much Liubei be one-CL associate-professor

'At most, Liubei is an associate professor.'

(74) LF: $[_{IP}$ **Zuiduo**(C) $[_{IP}\,[_{IP}$ Liubei is an $[\text{associate}]_F$ professor] ~C]]

(75) a. $[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge\beta \neq \alpha \rightarrow\mu_c(\alpha) > \mu_c(\beta)]]$

b. α ~C is defined iff $[\![\alpha]\!]^{o} \in C \wedge\exists\alpha'[\alpha'\neq\alpha \wedge [\![\alpha']\!]^{o} \in C] \wedge C \subseteq[\![\alpha]\!]^{f}$

c. $[\![(74)]\!]^{w,\,c} = 1$

iff $\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge\beta \neq \lambda w.$Liubei is an associate professor in w

$\rightarrow\mu_c(\lambda w.$Liubei is an associate professor in $w) > \mu_c(\beta)]]$

Because of the presuppositions introduced by the ~ squiggle operator (Rooth 1992), the prejacent is one element in the domain *C*. Furthermore, because of the domain restrictor SupP, all the elements non-identical to the prejacent are ranked below the prejacent; this amounts to removing the lower alternatives from the domain *C*. Taken together; the domain *C* further restricted by SupP is now a set consisting of the pejacent and its lower alternatives. According to (75), (73) is judged true if and only if there is one element in the domain (i.e., a set consisting of the prejacent and its lower alternatives) such that the element is true. This seems intuitively correct. Consider an academic ranking like *full professor* $\succ$*associate professor* $\succ$*assistant professor*, the sentence (73) is true only if *Liubei* is an associate professor or an assistant professor.[24]

The same analysis can be extended to the preverbal *zuiduo*, assuming the well-known VP-Internal Subject Hypothesis (Sportiche 1988). To simplify the computation, the subject is assumed to reconstruct to its base-generated position Spec, vP for interpretation.[25] The relevant sentence is presented in (76) and its LF in (77).

---

[24] As discussed in chapter 4, I argue that in the case of *zuiduo* (and English *at most*), unlike *zuishao* (and English *at least*), the prejacent is not necessarily entailed under either a concessive interpretation or an ignorance interpretation. Very briefly, recall that the concessive meaning of SMs requires two core pragmatic ingredients: (a) an evaluation of the alternatives with respect to discourse participants' goals and interests; (b) the falsity of the relevant higher alternatives. Crucially, the pragmatic requirement (b) is satisfied by the semantic contribution of *zuiduo*, which makes the prejacent the upper bound among the set of focus alternatives and thus <u>excludes the relevant higher alternatives</u>.

[25] The reconstruction of the subject for interpretation is simply to avoid unnecessary complications such as calculating the lambda-abstract created by the movement of the subject from Spec, vP to Spec,

(76) Liubei **zuiduo** shi yi-wei $[\text{fu}]_F$ -jiaoshou.

Liubei SUP-much be one-CL associate-professor

'Liubei is at most an associate professor.'

(77) LF: $[_{vP}$ **Zuiduo**(C) $[_{vP}$ $[_{vP}$ Liubei is an $[\text{associate}]_F$ professor] ~C]]

Next, let's consider the case of non-propositional modification (i.e., the prenominal case). Recall that in chapter 2 and chapter 4, in order to explain the distribution of the two meanings: speaker ignorance and speaker concession, I argue against the traditional view that, focus particles always undergo QR and thus take sentential scope regardless of their surface syntactic positions. The key ingredient in deriving the distribution lies in the idea that the syntactic position where a focus particle is merged determines its quantificational domain. Thus, when a focus particle is merged in the prenominal position, its quantificational domain is then non-propositional (e.g., a set of individuals or generalized quantifiers). This is the same line I pursue here. Now, consider the relevant sentence in (78) and its LF in (79). The computation is illustrated in (80).[26]

(78) Liubei mai-le **zui-duo** $[\text{san}]_F$-ke pinguo.

Liubei buy-ASP SUP-many three-CL apple

'Liubei bought at most three apples.'

(79) LF: $[[_{NP}$ **zuiduo**(C) $[_{NP}$ $[_{NP}$ $[\text{three}]_F$ apples] ~C]] λx [Liubei bought x]]

(80) a. $[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\, st>}\, \lambda P_{<\eta>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w(P) \wedge$

$\forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$

IP. Nothing crucial hinges on the assumption of the reconstruction.

[26] For readability, I abstract away from the issue of classifiers, kind terms and aspect. But see footnote 28 for an attempt to incorporate the contribution of classifiers and kind terms.

b. α ~C is defined iff $[\![\alpha]\!]^{o} \in C \wedge \exists\alpha'[\alpha' \neq \alpha \wedge [\![\alpha']\!]^{o} \in C] \wedge C \subseteq [\![\alpha]\!]^{f}$

c. $$[\![\text{three apples}]\!]^{f} = \left\{ \begin{array}{l} \ldots \\ \lambda Q\, \lambda w \exists z[\text{apple}(z) \wedge \mu_{\text{card}}(z) \geq 4 \wedge Q_w(z)] \\ \lambda Q\, \lambda w \exists z[\text{apple}(z) \wedge \mu_{\text{card}}(z) \geq 3 \wedge Q_w(z)] \\ \lambda Q\, \lambda w \exists z[\text{apple}(z) \wedge \mu_{\text{card}}(z) \geq 2 \wedge Q_w(z)] \\ \ldots \end{array} \right\}$$

d. $[\![(78)]\!]^{w,c} = 1$

$$\begin{array}{l} \text{iff } \exists\gamma[\gamma \in C \wedge \gamma_w(\lambda x.\text{Liubei bought}_w\, x) \\ \qquad \wedge \forall\beta[\beta \in C \wedge \beta \neq [\![\text{three apples}]\!]^{w,c} \\ \qquad\qquad \rightarrow \mu_c([\![\text{three apples}]\!]^{w,g}\, (\lambda x.\text{Liubei bought}_w\, x)) \\ \qquad\qquad\quad > \mu_c(\beta((\lambda x.\text{Liubei bought}_w\, x))]] \end{array}$$

As before, the prejacent *three apples* is an element in the domain C because of the presuppositions imposed by the ~ squiggle operator. The domain restrictor SupP excludes the higher alternatives such as four apples, five apples and so on. The domain C further restricted by SupP is now a set consisting of the prejacent and its lower alternatives. According to (80), the sentence (78) is predicted to be true if and only if there is one element in the domain such that the proposition denoted by that element composed with the relevant verbal information is true.

Before moving on to the next section, I would like to point out how the constraint semantic vacuity is connected to my decompositional analysis of *zuiduo*. Given our discussion above, it is clear that the domain restrictor SupP has played a key role in the computation of sentences containing *zuiduo*. Now, recall that the constraint against semantic vacuity imposes a ban on the vacuous use of semantic operators. In the BSE of *zuiduo* (and English *at most*), when the prejacent is itself the element at the bottom of the scale, the domain C further restricted by SupP becomes a singleton set consisting of only the prejacent itself, which corresponds to the bare form without *zuiduo*/ *at most*. More specifically, the constraint against semantic vacuity is violated because the use of *zuiduo* is vacuous in the case of BSE; that is, the BSE of *zuiduo* is a natural consequence from a general ban against vacuous quantificational claim in natural language. Therefore, the infelicity arises and is of semantic nature, in contrast to discourse uninformativity which is of pragmatic nature.

5.4.3 Compositions: quantity superlatives

For comparison and completeness, this section briefly illustrates the computation of quantity superlatives with *zuiduo* in the nominal domain and the verbal domain. Below, (81) presents the relevant sentence, with its LF in (82).[27] Note that the syntactic chunk [**zui**-Comp$^+$ **duo$_1$**]-*C* is the familiar SupP we have seen in SMs.

(81) QSs in Nominal Domain

Liubei	mai-le	**zui-duo**	(ke)	pinguo.
Liubei	buy-ASP	SUP-many	CL	apple

‘Liubei bought more apples than anyone else did.’

(82) [**zui**-Comp$^+$ **duo$_1$**]-*C* λd [$_{IP}$[$_{IP}$ [Liubei]$_F$ bought [$_{DP}\exists$ [$_{AdjP}$ *d*-[**M-OP$_2$** apples]]]]~C]

[27] In the literature on the syntax of comparative constructions, there are two major proposals for the structural position of DegP in a comparative construction, as shown in (i) and (ii).

(i) [$_{DegP}$ [$_{Deg'}$ [$_{Deg}$ [$_{AP}$]] Kennedy 1999, Grimshaw 2005, among others

(ii) [$_{AP}$ [$_{DegP}$] [$_{AP}$]] Bresnan 1973, Heim 2001, among others

In (i), the DegP is merged as the extended projection of an adjective, while the DegP is merged at the Spec, AP in (ii). Both structures are compatible with a generalized quantifier view of degree morphemes. Under the movement theory of superlatives (Heim 1999, Hackl 2009), the structure in (ii) is adopted. Readers are referred to Lechner and Corver (2017) for an overview on different structural proposals for the position of DegP and their impacts on the semantics. See also Dunbar and Wellwood (2016) for discussion on how different structural possibilities of DegP in superlatives fare with Bobaljik (2012)’s Containment Hypothesis.

Two remarks are in order. First, following Rett (2008, 2014), I assume the existence of M-OP defined in (83). Note that under Rett's original formulation, M-OP relates individuals *x* in the extension of some predicate *P* to their degrees along some contextually valued dimension. Here, I generalize her idea in a way that M-OP can be applied to both nominal domain and verbal domain. More specifically, in the case of QSs, α ranges over events and individuals, thus η could be of type $<v>$ or $<e>$.

(83) $[\![\text{M-OP}_2]\!]^c = \lambda P_{<\eta, t>}\ \lambda d_{<d>}\ \lambda \alpha_{<\eta>}.[P(\alpha) \wedge \mu_2(\alpha) \geq d]$ $\quad <<\eta, d>, <d, <\eta, t>>>$

Second, to simplify the derivation, I abstract away the denotation of classifiers and kinds (Krifka et al. 1995, Chierchia 1998) and assume that existential closure closes the individual variable.[28] Assuming exactly the same meaning pieces of quantity adjective *duo*, Comp$^+$P and the superlative morpheme *zui* (see sections 5.4.1−5.4.2), we obtain our familiar SupP again, as shown in (87).[29]

(84) $[\![duo]\!]^c = \lambda \alpha.\mu_c(\alpha)$ $\quad <\eta, d>$

---

[28] One way to incorporate the contribution of classifiers and kind terms is illustrated below (see Krifka 1995, Chierchia 1998, Yang 2001, among others):

(i) a. $[\![pinguo]\!]^w = \lambda z.{}^{\cup}\text{apple}_w(z)$

b. $[\![ke]\!] = \lambda z.\text{atom}(z)$

c. $[\![[\text{M-OP}_2\ ke\text{-}pinguo]]\!]^{w, c} = \lambda d \lambda z[{}^{\cup}\text{apple}_w(z) \wedge \text{atom}(z) \wedge \mu_2(z) \geq d]$

d. $[\![[_{\text{AdjP}}\ d\text{-}[\text{M-OP}_2\ ke\text{-}pinguo]]]\!]^{w, c} = \lambda z[{}^{\cup}\text{apple}_w(z) \wedge \text{atom}(z) \wedge \mu_2(z) \geq d]$

e. $[\![\lambda d\ [[_{\text{IP}}\ [\text{Liubei}]_F\ \text{bought}\ [_{\text{DP}}\ \exists\ [_{\text{AdjP}}\ d\text{-}[\mathbf{M\text{-}OP_2}\ \text{apples}]]]]\sim C]]\!]^{w, c}$
$= \lambda d$. Liubei bought *d* or more apples
$= \lambda d.\exists z[\ \text{bought (Liubei}, z) \wedge {}^{\cup}\text{apple}_w(z) \wedge \text{atom}(z) \wedge \mu_c(z) \geq d]$

f. The sentence (81) is true iff
$\forall y[y \in C \wedge y \neq Liubei \rightarrow \max\{d: \exists z[{}^{\cup}\text{apple}_w(z) \wedge \text{atom}(z) \wedge \text{bought}\ (Liubei, z) \wedge \mu_c(z) \geq d]\}$
$> \max\{d: \exists z[{}^{\cup}\text{apple}_w(z) \wedge \text{atom}(z) \wedge \text{bought}\ (y, z) \wedge \mu_c(z) \geq d]\}$

[29] Given that quantity adjective *duo* 'many/ much' and M-OP both encode a measure function μ and are evaluated relative to context, the number index 1 and 2 are used here to distinguish the two types of measurement. In particular, for the context *c* for QSs in the nominal domain like (82), $g_c(2)$ is a measure function μ_2 that assigns cardinalities, while $g_c(1)$ is a measure function μ_1 that measures interval sizes. As we will see shortly, for the context *c* for QSs in the verbal domain, $g_c(2)$ is a measure function μ_2 that assigns a contextually-given dimension respecting monotonicity (e.g., temporal duration or distance), and $g_c(1)$ is a measure function μ_1 that measures interval sizes.

(85) $[\![\text{Comp}^{+}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_c(\alpha) > \mu_c(\beta)$ $\quad <\eta, <\eta, t>>$

(86) $[\![zui]\!]^{c} = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

(87) $[\![[_{\text{SupP}}\ \textbf{zui}\text{-Comp}^{+}\ \textbf{duo}_{\mathbf{1}}]\text{-}C]\!]^{c} = \lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_1(\alpha) > \mu_1(\beta)]$

With the above semantic pieces in hand, the computation is illustrated in (88).

(88) a. $[\![\text{apples}]\!] = \lambda x_{<e>}.\text{apple}(x)$

b. $[\![\textbf{M-OP}_{\mathbf{2}}\ \text{apples}]\!]^{c} = \lambda d_{<d>}\ \lambda\alpha_{<e>}.[\text{apple}(\alpha) \wedge \mu_2(\alpha) \geq d]$

c. $[\![\ d\text{-}[\textbf{M-OP}_{\mathbf{2}}\ \text{apples}]]\!]^{c} = \lambda\alpha_{<e>}.[\text{apple}(\alpha) \wedge \mu_2(\alpha) \geq d]$

d. $[\![\lambda d\ [[_{\text{IP}}\ [\text{Liubei}]_{\text{F}}\ \text{bought}\ [_{\text{DP}}\ \exists\ [_{\text{AdjP}}\ d\text{-}[\textbf{M-OP}_{\mathbf{2}}\ \text{apples}]]]]\sim\text{C}]\!]^{w, c}$

$= \lambda d$. Liubei bought d or more apples

$= \lambda d.\exists z[\text{apple}(z) \wedge \text{bought}\ (Liubei, z) \wedge \mu_2(z)\geq d]$

e. $C = \{\lambda d.\ y$ bought d or more apples $|\ y \in D_e\}$

f. $[\![(82)]\!]^{w, c} = 1$ iff $\forall y[y\in C \wedge y \neq Liubei \rightarrow$

$\mu_1\{d$: Liubei bought d or more apples$\}$

$> \mu_1\{d$: y bought d or more apples$\}$

Alternatively;

$[\![(82)]\!]^{w, c} = 1$ iff $\forall y[y\in C \wedge y \neq Liubei \rightarrow$

$\mu_1\{d: \exists z[\text{apple}(z) \wedge \text{bought}\ (Liubei, z) \wedge \mu_2(z)\geq d]\}$

$> \mu_1\{d: \exists z[\text{apple}(z) \wedge \text{bought}\ (y, z) \wedge \mu_2(z)\geq d]\}$

g. In words: for all y mentioned in C, the number of apples *Liubei* bought exceeds the number of apples y bought.

(88a) is given by the semantics of the noun *pinguo* 'apples'. (88b) is derived by functional application, by applying M-OP to (88a). (88c) is obtained by the movement of the superlative phrase SupP, which leaves a degree variable saturating the degree argument of M-OP. (88d) is obtained by the lambda-abstraction, resulting from the movement of SupP. (88e) provides the contextual value of the domain *C*. (88f) is a consequence of functional application, by applying SupP (where the prejacent is of type <*d*, *t*>) to (88d). Finally, according to (88g), the sentence (81) is true if and only if the number of apples that *Liubei* bought is more than the number of apples that any other individual bought. This seems intuitively correct.

Crucially, exactly the same superlative component SupP (see (87) and sections 5.4.1−5.4.2) is involved in both QSs and SMs. In particular, in the case of QSs, the prejacent is of type <*d*, *t*> and the domain *C* is in turn of type <*d*, *t*>, *t*>. With these type-theoretical setting, the semantics of SupP is equivalent to the two-place superlative operator introduced in Heim (1999), see (58) above. As discussed in section 5.3, the superlative operator takes scope over the whole sentence (see (82)). Thus, the morpho-semantic puzzle of Chinese SMs is captured.[30]

Now, let's consider QSs in the verbal domain. Below, (89) presents the relevant sentence, along with its LF in (90). As in the nominal domain, the superlative phrase SupP undergoes movement (thus creating a degree property) and scopes over the whole sentence.

(89) QSs in Verbal Domain

(Paobu) Liubei pao **zui-duo**.

Running Liubei run SUP-much

'(As for running,) Liubei ran more than anyone else did.'

(90) [**zui**-Comp^{+} **duo$_1$**]-*C* $\lambda d\ [_{IP}[_{IP}\ \exists\ [\text{Liubei}]_F\ \lambda x\ [_{vP}\ x\ [_{VP}\ d\text{-}[\text{ran } \mathbf{M\text{-}OP_2}]\ ]]]]\sim C]$

[30] I am greatly indebted to Roger Schwarzschild for drawing my attention to the two-place superlative operator discussed in Heim (1999) and suggesting this line of analysis unifying Chinese SMs and QSs.

As in the nominal domain, I assume that an existential closure closes the event variable in the verbal domain. For current purposes, the adjunction site of M-OP in the verbal domain can be vP or VP, as long as it denotes an event predicate. For illustration, I assume that M-OP is adjoined to VP and that the subject is base-generated at Spec, vP and the composition with VP is done by Event Identification (Kratzer 1996). With these setting, the computation is shown in (91).

(91) a. $[\![\text{ran}]\!]^c = \lambda e_{<v>}.\text{ran}(e)$

b. $[\![[\text{ran } \mathbf{M\text{-}OP_2}]]\!]^c = \lambda d_{<d>}\, \lambda \alpha_{<v>}.[\text{ran}(\alpha) \wedge \mu_2(\alpha) \geq d]$

c. $[\![d\text{-}[\text{ran } \mathbf{M\text{-}OP_2}]]\!]^c = \lambda \alpha_{<v>}.[\text{ran}(\alpha) \wedge \mu_2(\alpha) \geq d]$

d. $[\![\lambda d\ [_{\text{IP}}[_{\text{IP}} \exists\ [\text{Liubei}]_{\text{F}}\, \lambda x\ [_{\text{vP}}\, x\ [_{\text{VP}}\ d\text{-}[\text{ran } \mathbf{M\text{-}OP_2}]]]]]]\text{~C}\ ]\!]^c$

$= \lambda d$. Liubei ran d or more

$= \lambda d.\exists e[\text{ran}(e) \wedge \text{agent}\ (\textit{Liubei}, e) \wedge \mu_2(e) \geq d]$

e. $C = \{\lambda d.\ y \text{ ran } d \text{ or more} \mid y \in D_e\}$

f. $[\![(90)]\!]^{w,\,c} = 1$ iff $\forall y[y \in C \wedge y \neq \textit{Liubei}$

$\rightarrow \mu_1\{d\text{: Liubei ran } d \text{ or more}\} > \mu_1\{d\text{: } y \text{ ran } d \text{ or more}\}$

Alternatively;

$[\![(90)]\!]^{w,\,c} = 1$ iff $\forall y[y \in C \wedge y \neq \textit{Liubei}$

$\rightarrow \mu_1\{d\text{: } \exists e[\text{ran}(e) \wedge \text{agent}\ (\textit{Liubei}, e) \wedge \mu_2(e) \geq d]\}$

$> \mu_1\{d\text{: } \exists e[\text{ran}(e) \wedge \text{agent}\ (y, e) \wedge \mu_2(e) \geq d]\}$

g. In words: for all y mentioned in C, the quantity that *Liubei* ran exceeds the quantity that y ran, along a contextually-given dimension (e.g., temporal duration or distance).

(91a) is given by a neo-Davidsonnian semantics of the verb *pao* 'ran'.[31] (91b) is derived by functional application, by applying M-OP to (91a). (91c) is obtained by the movement of the superlative phrase SupP, which leaves a degree variable saturating the degree argument of M-OP. (91d) is obtained by lambda-abstraction, resulting from the movement of SupP. (91e) provides the contextual value of the domain *C*. (91f) is a consequence of functional application, by applying SupP (where the prejacent is of type <*d*, *t*>) to (91d). Finally, according to (91), the sentence (89) is true if and only if the quantity of running events (along temporal duration or distance) done by *Liubei* is more than that of running events (along temporal duration or distance) done by any other individual. This seems to correctly capture the intuition. Thus, the morpho-semantic puzzle of Chinese SMs is again captured.

It is worth noting that by addressing the morpho-semantic puzzle, the current analysis of SMs and QSs not only involves the same syntactic constituent SupP, but also unifies its semantic role in establishing a comparison/ an ordering relation. Seen in this light, the morpho-semantic puzzle posed by Chinese can be further reduced to two semantic factors: (a) what semantic objects the measure function μ_c is associated with; (b) whether the relevant dimension of measurement is strictly restricted to a structured domain. Under the current analysis, regarding the first factor, QSs introduce measurement over individuals and events, while SMs establish ordering over propositions. Regarding the second factor, QSs are constrained by monotonicity requirement. SMs respect the monotonicity requirement, but crucially they are not

[31] In the literature on Chinese linguistics, it has been argued that Chinese verbs simply denote an event predicate and "arguments" (regardless of whether it is canonical or non-canonical) are all introduced by eventuality predicates (light verbs in the terminology of Lin 2001) like CAUSE and BECOME at the syntax. On this view, Chinese verbs are assigned a neo-Davidsonian semantics and thus do not (inherently) *lexicalize* any (nominal) arguments (in contrast to English verbs); Lin's proposal is intended to explain why a number of non-canonical arguments like locative phrases and instrumental phrases can be freely merged in the position of canonical nominal arguments in Chinese. Readers are referred to Lin (2001) for more details. See also Huang et al. (2009) for discussion of Lin (2001).

restricted to it. More importantly, these differences between QSs and SMs are not completely unexpected. In particular, they may be thought of as a difference between the notion of **gradability** observed in canonical comparative constructions and that of **scalarity** observed in discourse-oriented focus particles.

In the next section, I extend my decompositional proposal to Chinese *zuishao* and discuss how the apparently antonymous relation between *zuiduo* and *zuishao* is captured under my analysis.

5.5 Decompose *zuishao* 'at least'

In this section, I spell out my decompositional analysis of *zuishao*. One important issue centers on how to capture the apparently antonymous relation indicated in the morphology: *zui-duo* "SUP-much" and *zui-shao* "SUP-little". This issue of antonyms boils down to how to properly represent the morpho-semantics of *shao* 'little' and its relation to *duo* 'much'. At this point, it is worth pointing out that because the prejaent is included in both cases, strictly speaking, the pair of SMs *zuiduo* and *zuishao* is not antonymous in terms of their semantics. The question is then how to reconcile the morph-semantic mismatch: Can we provide a decompositional analysis of SM *zuishao*, while capturing the antonymous relation between the pair of quantity adjectives *duo* 'much' and *shao* 'little', and the semantic differences from SM *zuiduo*?

The rest of this section proceeds as follows. Section 5.5.1 spells out the core ingredients of my decompositional analysis of Chinese SM *zuishao*. Section 5.5.2 demonstrates a detailed computation of Chinese data. In particular, I illustrate how the proposed analysis works for different syntactic positions of *zuishao*: cases of propositional modification and those of non-propositional modification. Section 5.5.3 offers the computation of Chinese quantity superlatives with *zuishao*.

5.5.1 The proposal

My decompositional proposal of *zuishao* has three pieces: **(a)** the internal compositionality of the superlative construction SupP, and the contribution of the superlative morpheme *zui* and the quantity adjective *shao* 'little' inside Chinese SM *zuishao*; **(b)** the role of the SupP as a whole inside Chinese SMs; **(c)** a covert existential operator E-OP (structurally containing the SupP). Let's look at the first piece of my proposal. Following the decompositional analysis of English *little* and *less* along the line in Heim (2006a, b) and Büring (2007a, b), I propose that the quantity adjective *shao* 'little' contributes to two semantic components at LF: a negative feature NEG and a covert *duo* 'much' (see Solt 2009, 2015 in section 5.2). As in the case of SM *zuiduo*, I propose that in Chinese SM *zuishao*, the superlative construction structurally embeds a comparative construction (assume Bobaljik 2012's Containment Hypothesis). The covert comparative morpheme Comp^{+} combined with the negative feature NEG is reanalyzed as a covert comparative morpheme Comp^{-} (with the opposite comparison relation). The connection between Comp^{+} and Comp^{-} is reminiscent of Heim's and Büring's analyses of English *less* as a reanalyzed result from the combination of a negation contributed by adjectives with negative polarity (glossed as LITTLE in their analyses) and the comparative morpheme *–er*. The superlative construction SupP involved in Chinese SM *zuishao* is schematized in (92). The semantics of Comp^{-} (cf. English *less*) is defined in (94), which takes the adjective *duo* as its first argument and returns a comparison relation between the alternatives along a contextually-valued dimension.

(92) $[_{\text{SupP}}$ *zui* $[_{\text{Comp-P}}$ Comp^{-} $[_{\text{AdjP}}$ *duo*$]]]$

(93) NEG-Comp^{+} is reanalyzed as Comp^{-} (Heim 2006a, b; Büring 2007a,b; a.o.)

(94) $[\![\text{Comp}^{-}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_c(\alpha) < \mu_c(\beta)$ $\quad\quad\quad\quad\quad\quad <\eta, <\eta, t>>$

Next, the same superlative morpheme *zui* participating in Chinese SM *zuiduo* (see section 5.4.1) is also involved in Chinese SM *zuishao*, with its semantics repeated as (95). The semantics of SupP is provided in (96). So far, the first piece is completed.

(95) $[\![zui]\!]^{c} = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

(96) $[\![(92)]\!]^{c} = \lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$ $\quad\quad$ SupP

The second piece concerns the role of the superlative construction SupP inside Chinese SM *zuishao*. As shown in SM *zuiduo*, the superlative construction SupP serves as a domain restrictor on the set of focus alternatives. In the case of *zuishao*, the superlative construction SupP requires that for all the alternatives that are non-identical to the prejacent α in the domain C, they are ranked **above** the prejacent along a contextually given dimension. This amounts to removing the lower alternatives from the domain.

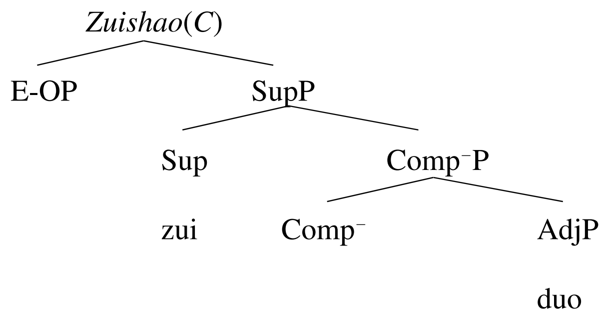
Finally, the third piece is that the superlative construction SupP is embedded under the same covert existential operator E-OP that is involved in SM *zuiduo*. The semantics of E-OP is repeated below, with a propositional version in (97).

(97) $[\![\text{E-OP}]\!]^{w, c} = \lambda SUP_{<<st, t>, <st, t>>}\lambda C_{<st, t>}\ \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge SUP\ (C, \alpha)]$

As before, E-OP takes SupP as its additional domain restrictor and asserts that there is one element in the domain C further restricted by SupP such that the element is true. The complete picture concerning the morpho-semantics of Chinese SM *zuishao* is presented below, with the structure in (98) and the semantics in (99).[32]

[32] Liu (2018) proposes that in Chinese, both comparative morpheme and *pos*-morpheme have two allomorphs: an overt form and a covert counterpart. He argues that the covert form of the comparative morpheme (and the *pos*-morpheme) is licensed only in a focus-sensitive domain (in his terminology, a

(98) The internal structure of Chinese SM *zuishao* at LF



(99) $[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w \wedge \forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

Given that the set of focus alternatives does not have to be propositional and that SMs can be adjoined to constituents with non-propositional meanings, I assume that a non-propositional version can be obtained by certain type-shifting rules such as the Geach rule (see Jacobson 1999, Coppock and Beaver 2013), as shown below.

(100) A non-propositional version (by the Geach rule)

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w(P) \wedge \forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

(101) A non-propositional version (by the backward Geach rule)

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma{\in}C \wedge P_w(\gamma) \wedge \forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) < \mu_c(P_w(\beta))]]$$

Before closing this section, it is worth noting that the current analysis of *zuishao* captures not only the apparent antonymous relation between the pair of SMs *zuiduo* and *zuishao*, but also the relevant properties shared by them. First, we have a better

domain where the bare gradable adjective is focus-anchored). My decompositional analysis of Chinese SMs *zuiduo*/ *zuishao* (where a covert comparative construction is structurally embedded under the SupP, couched in Bobaljik's Containment Hypothesis) may be compatible with a broad definition of a focus-sensitive domain, given that the covert comparative construction is inside the body of focus particles *zuiduo*/ *zuishao*. But it is less clear in the case of canonical superlatives with *zuiduo*/ *zuishao*. I leave the compatibility between my decompositional analysis and Liu (2018) for future research.

understanding of why quantity adjectives and superlative morpheme are involved in SMs across many languages and how the pair of SMs seems to be antonymous. Second, because the same measure function μ_c is involved in the pair of SMs, the explanation of focus-sensitivity and the discrepancy between scale types provided for *zuiduo* in section 5.4.1 is readily applied to *zuishao*. More specifically, focus-sensitivity follows from the conspiracy between focus and the measure function μ_c: the former induces a set of elements and the latter maps each of them to some position along a contextually given dimension. The discrepancy between scale types follows, because the measure function μ_c respects the monotonicity constraint but crucially not restricted to it. As a consequence, depending on whether the input domain of μ_c (the focus alternatives) is structured (i.e., via partial ordering or total ordering) or not, the output ranking between the alternatives may (not) be constant across different contexts. Finally, the issue of semantic vacuity (in relation to the two scalar effects TSE/ BSE and their discrepancy) is addressed in the next section.

Now, let us move to the computation of Chinese data to assure ourselves that the current analysis does drive correct truth-conditions.

5.5.2 Compositions: superlative modifiers

Let us first consider the case of propositional modification. The relevant sentence is presented in (102), with its LF in (103). The semantic computation is illustrated in (104). Note that (102) has only the concessive interpretation (i.e., the ignorance interpretation is unavailable; see chapter 2 for the distribution of the two meanings).

(102) **Zui-shao** Liubei shi yi-wei $[\text{fu}]_F$ -jiaoshou.

SUP-little Liubei be one-CL associate-professor

‘At least, Liubei is an associate professor.’

(103) LF: $[_{IP}$ **Zuishao**(C) $[_{IP}$ $[_{IP}$ Liubei is an $[\text{associate}]_F$ professor] ~C]]

(104) a. $[\![zuishao(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

b. α ~C is defined iff $[\![\alpha]\!]^{o} \in C \wedge \exists\alpha'[\alpha'\neq\alpha \wedge [\![\alpha']\!]^{o} \in C] \wedge C \subseteq [\![\alpha]\!]^{f}$

c. $[\![(103)]\!]^{w,c} = 1$

iff $\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \lambda w.$Liubei is an associate professor in w

$\rightarrow \mu_c(\lambda w.$Liubei is an associate professor in $w) < \mu_c(\beta)]]$

Because of the presuppositions introduced by the ~ squiggle operator (Rooth 1992), the prejacent is one element in the domain *C*. Furthermore, because of the domain restrictor SupP, all the elements non-identical to the prejacent are ranked above the prejacent. Put differently, the domain restrictor SupP removes the lower alternatives from the domain *C*. Taken together; the domain *C* further restricted by SupP now denotes a set consisting of the pejacent and its higher alternatives. According to (104), (102) is judged true if and only if there is one element in the domain (i.e., in the set consisting of the prejacent and its higher alternatives) such that the element is true. Recall from chapter 2 that the concessive meaning of SMs requires three core pragmatic ingredients: (a) a contextual evaluation of the focus alternatives with respect to discourse participants’ interests; (b) the relevant higher alternatives are known to be false; (c) the quantificational domain of SMs is propositional. Now, consider an academic ranking like *full professor* $\succ$*associate professor* $\succ$*assistant professor*. When these pragmatic requirements are fulfilled in a given discourse, the sentence *Liubei is at least an assistant professor* receives a concessive interpretation and is judged true if *Liubei* is an assistant professor. The fact that the prejacent is entailed under the concessive interpretation follows from the semantic contribution of *zuishao*, coupled with the pragmatic requirement that the higher alternatives are false.

The same analysis can be extended to the preverbal *zuishao*, if we assume the VP-Internal Subject Hypothesis. To simplify the computation, the subject is assumed to reconstruct to its base-generated position Spec, vP for interpretation.[33] The relevant sentence is presented in (105) and its LF in (106). Note that the preverbal case is ambiguous between an ignorance interpretation and a concessive interpretation.

(105) Liubei **zui-shao** shi yi-wei $[\text{fu}]_F$ -jiaoshou.

Liubei SUP-little be one-CL associate-professor

'Liubei is at least an associate professor.'

(106) LF: $[_{vP}$ **Zuishao**(C) $[_{vP}$ $[_{vP}$ Liubei is an $[\text{associate}]_F$ professor] ~C]]

The semantic computation here remains the same as that in the sentential case. In chapter 2, I argue that the ignorance interpretation and the concessive interpretation are pragmatic variants. Specifically, the same semantic representation of *zuiduo* and *zuishao* is involved in the computation of the two meanings. Readers are referred to chapter 2 for the details of a unified account of the ignorance-concession ambiguity.

Next, let's consider the case of non-propositional modification (i.e., the prenominal case). As in the case of SM *zuiduo*, I purse the idea that the syntactic position where a focus particle is merged determines its quantificational domain. When a focus particle is merged in the prenominal position, its quantificational domain is then non-propositional (e.g., a set of individuals or generalized quantifiers). Now, consider the relevant sentence in (107) and its LF in (108). The computation is illustrated in (109).

[33] As mentioned in section 5.4.2, the reconstruction of the subject for interpretation is simply to avoid unnecessary complications such as calculating the lambda-abstract created by the movement of the subject from Spec, vP to Spec, IP. Nothing crucial hinges on the assumption of the reconstruction.

(107) Liubei mai-le **zui-shao** $[\text{san}]_F$-ke pinguo.

Liubei buy-ASP SUP-few three-CL apple

'Liubei bought at least three apples.'

(108) LF: $[[_{\text{NP}}$ **zuishao**(C) $[_{\text{NP}}$ $[_{\text{NP}}$ $[\text{three}]_F$ apples$]$ $\sim C]]$ λx [Liubei bought x]]

(109) a. $[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma{\in}C \wedge \gamma_w(P) \wedge$

$$\forall\beta[\beta{\in}C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

b. $\alpha \sim C$ is defined iff $[\![\alpha]\!]^{o} \in C \wedge \exists\alpha'[\alpha'{\neq}\alpha \wedge [\![\alpha']\!]^{o} \in C] \wedge C \subseteq [\![\alpha]\!]^{f}$

c. $$[\![\text{three apples}]\!]^{f} = \left\{\begin{array}{l} \ldots \\ \lambda Q\ \lambda w \exists z[\text{apple}(z) \wedge \mu_{\text{card}}(z) \geq 4 \wedge Q_w(z)] \\ \lambda Q\ \lambda w \exists z[\text{apple}(z) \wedge \mu_{\text{card}}(z) \geq 3 \wedge Q_w(z)] \\ \lambda Q\ \lambda w \exists z[\text{apple}(z) \wedge \mu_{\text{card}}(z) \geq 2 \wedge Q_w(z)] \\ \ldots \end{array}\right\}$$

d. $[\![(108)]\!]^{w,\,c} = 1$

iff $\exists\gamma[\gamma{\in}C \wedge \gamma_w(\lambda x.\text{Liubei bought}_w\ x)$

$\wedge \forall\beta[\beta{\in}C \wedge \beta \neq [\![\text{three apples}]\!]^{w,\,c}$

$\rightarrow \mu_c([\![\text{three apples}]\!]^{w,\,g}\ (\lambda x.\text{Liubei bought}_w\ x))$

$< \mu_c(\beta((\lambda x.\text{Liubei bought}_w\ x))]]$

First of all, the prejacent *three apples* is presupposed to be an element in the domain C because of the presuppositions imposed by the ~ squiggle operator. Next, the domain restrictor SupP excludes those lower alternatives such as two apples, one apple and so on. The domain C further restricted by SupP is now a set consisting of the prejacent and its higher alternatives. According to (109), the sentence (107) is predicted to be true if and only if there is one element in the domain (i.e., the set consisting of the prejaent and its higher alternatives), such that the proposition denoted by that element composed with the relevant verbal information is true. This seems intuitively correct. The sentence *Liubei bought at least three apples* is judged true when *Liubei* bought exactly three apples or more than thee apples.

Before moving to the next section, I would like to point out how the constraint semantic vacuity is connected with my decompositional analysis of *zuishao*. As in the case of SM *zuiduo*, the domain restrictor SupP has played a key role in the computation of sentences containing *zuishao*. Recall that the constraint semantic vacuity imposes a ban on the vacuous use of semantic operators. In the TSE of *zuishao* (and English *at least*), when the prejacent is itself the element at the top of the scale, the domain *C* further restricted by SupP is now a singleton set consisting of <u>only the prejacent itself</u>, which corresponds to the bare form without *zuishao*/ *at least*. Therefore, the constraint semantic vacuity is violated because the use of *zuishao* is vacuous in the case of TSE. The infelicity arises and is of semantic nature, in contrast to discourse uninformativity which is of pragmatic nature. At this point, it is important to note that the current analysis correctly derives the mirror image between *zuiduo* and *zuishao* with respect to the discrepancy of the two scalar effects: the BSE of *zuiduo* and the TSE of *zuishao* are of semantic nature and arise precisely because of violating the constraint semantic vacuity.

In the next section, I illustrate the computation of quantity superlatives with *zuishao* in the nominal domain and the verbal domain.

5.5.3 Compositions: quantity superlatives

Let us first consider the case of QSs in the nominal domain. The relevant sentence is given in (110), with its LF in (111). Recall that the superlative expression *zuishao* is now structurally decomposed as the syntactic chunk [**zui**-Comp^{-} **duo$_1$**]-*C* (i.e., SupP;

see sections 5.5.1 − 5.5.2). As we have seen in section 5.4.3, the superlative phrase SupP moves (thus creating a degree property) and scopes over the whole sentence.

(110) QSs in Nominal Domain

(Zhe-ci)	Liubei	mai-le	**zui-shao**	(ke)	pinguo.
This-time	Liubei	buy-ASP	SUP-few	CL	apple

'(This time,) Liubei bought fewer apples than anyone else did.'

(111) [**zui**-Comp⁻ **duo₁**]-$C\,\lambda d$ [$_{IP}$[$_{IP}$ [Liubei]$_F$ bought [$_{DP}\exists$ [$_{AdjP}$ d-[**M-OP₂** apples]]]]~C]

The relevant semantic pieces are repeated in (112) − (116). In particular, the semantics of SupP is presented in (115). Recall that in the case of QSs, α ranges over events and individuals, thus η could be of type <v> or <e>. As before, M-OP is generalized for both nominal domain and verbal domain.

(112) $[\![duo]\!]^c = \lambda\alpha.\mu_c(\alpha)$ $\quad$ $<\eta, d>$

(113) $[\![\text{Comp}^{-}\text{P}]\!]^c = \lambda\alpha\lambda\beta.\mu_c(\alpha) < \mu_c(\beta)$ $\quad$ $<\eta, <\eta, t>>$

(114) $[\![zui]\!]^c = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

(115) $[\![[_{SupP}\ \textbf{zui}\text{-Comp}^{-}\ \textbf{duo}_\textbf{1}]\text{-}C]\!]^c = \lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_1(\alpha) < \mu_1(\beta)]$

(116) $[\![\text{M-OP}_2]\!]^c = \lambda P_{<\eta, t>}\ \lambda d_{<d>}\ \lambda\alpha_{<\eta>}.[P(\alpha) \wedge \mu_2(\alpha) \geq d]$ $\quad$ $<<\eta, d>, <d, <\eta, t>>>$

With these assumptions in place, the relevant computation is offed below. The computation of (111) is basically the same as the case of *zuiduo*, except for the comparison relation: a less-than relation encoded by Comp⁻P vs. a greater-than relation encoded by Comp⁺P.

(117) a. $[\![\text{apples}]\!] = \lambda x_{<e>}.\text{apple}(x)$

b. $[\![\textbf{M-OP}_\mathbf{2} \text{ apples}]\!]^c = \lambda d_{<d>}\ \lambda\alpha_{<e>}.[\text{apple}(\alpha) \wedge \mu_2(\alpha) \geq d]$

c. $[\![\ d\text{-}[\textbf{M-OP}_\mathbf{2} \text{ apples}]]\!]^c = \lambda\alpha_{<e>}.[\text{apple}(\alpha) \wedge \mu_2(\alpha) \geq d]$

d. $[\![\lambda d\ [[_{\text{IP}}\ [\text{Liubei}]_F \text{ bought } [_{\text{DP}}\ \exists\ [_{\text{AdjP}}\ d\text{-}[\textbf{M-OP}_\mathbf{2} \text{ apples}]]]]\text{\~{}C}]\!]^{w,\,c}$

$= \lambda d$. Liubei bought d or more apples

$= \lambda d.\exists z[\text{apple}(z) \wedge \text{bought } (Liubei, z) \wedge \mu_2(z) \geq d]$

e. $C = \{\lambda d.\ y \text{ bought } d \text{ or more apples} \mid y \in D_e\}$

f. $[\![(111)]\!]^{w,\,c} = 1$ iff $\forall y[y \in C \wedge y \neq Liubei \rightarrow$

$\mu_1\{d\text{: Liubei bought } d \text{ or more apples}\}$

$< \mu_1\{d\text{: } y \text{ bought } d \text{ or more apples}\}$

Alternatively;

$[\![(111)]\!]^{w,\,c} = 1$ iff $\forall y[y \in C \wedge y \neq Liubei \rightarrow$

$\mu_1\{d\text{: } \exists z[\text{apple}(z) \wedge \text{bought } (Liubei, z) \wedge \mu_2(z) \geq d]\}$

$< \mu_1\{d\text{: } \exists z[\text{apple}(z) \wedge \text{bought } (y, z) \wedge \mu_2(z) \geq d]\}$

g. In words: for all y mentioned in C, the number of apples *Liubei* bought is less than the number of apples y bought.

(117a) is given by the semantics of the noun *pinguo* 'apples'. (117b) is derived by functional application, by applying M-OP to (117a). (117c) is obtained by the movement of the superlative phrase SupP, which leaves a degree variable saturating the degree argument of M-OP. (117d) is obtained by lambda-abstraction, resulting from the movement of SupP. (117e) provides the contextual value of the domain *C*. (117f) is a consequence of functional application, by applying SupP (where the prejacent is of type <*d*, *t*>) to (117d). Finally, according to (117g), the sentence (110) is true if and only if the number of apples that *Liubei* bought is less than that of the apples that any other individual bought. This seems intuitively correct.

Crucially, exactly the same superlative component SupP (see (115) and sections 5.5.1−5.5.2) is again involved in both QSs and SMs. In particular, in the case of QSs, the prejacent is of type <*d*, *t*> and the domain *C* is in turn of type <<*d*, *t*>, *t*>. With these type-theoretical setting, the semantics of SupP is equivalent to the two-place superlative operator in Heim (1999), except for the opposite comparison relation, see (58) above. As discussed in section 5.3, the superlative operator scopes over the whole sentence (see (111)). Thus, the morpho-semantic puzzle of Chinese SMs is captured.

Now, let's consider QSs in the verbal domain. The relevant sentence is given in (118), with its LF in (119). Like the case in the nominal domain, the superlative phrase SupP moves (thus creating a degree property) and scopes over the whole sentence. Moreover, as we have seen in the case of *zuiduo* (section 5.4.3), an existential closure closes the event variable in the verbal domain. As before, I assume that M-OP is adjoined to VP and that the subject is base-generated at Spec, vP and the composition with VP is done by Event Identification (Kratzer 1996).

(118) QSs in Verbal Domain

(Paobu) Liubei pao **zui-shao**.

Running Liubei run SUP-little

'(As for running,) Liubei ran less than anyone else did.'

(119) [**zui**-Comp^{-} **duo$_1$**]-*C* λd [$_{IP}$[$_{IP}$ $\exists$ [Liubei]$_F$ λx [$_{vP}$ x [$_{VP}$ d-[ran **M-OP$_2$**]]]]]~C]

With these assumptions in place, the relevant computation is shown in (120).

(120) a. $[\![\text{ran}]\!]^c = \lambda e_{<v>}.\text{ran}(e)$

b. $[\![[\text{ran } \mathbf{M\text{-}OP_2}]]\!]^c = \lambda d_{<d>}\ \lambda \alpha_{<v>}.[\text{ran}(\alpha) \wedge \mu_2(\alpha) \geq d]$

c. $[\![d\text{-[ran } \mathbf{M\text{-}OP_2}]]\!]^{c} = \lambda\alpha_{<v>}.[\text{ran}(\alpha) \wedge \mu_2(\alpha) \geq d]$

d. $[\![\lambda d\ [_{IP}[_{IP}\ \exists\ [\text{Liubei}]_F\ \lambda x\ [_{vP}\ x\ [_{VP}\ d\text{-[ran } \mathbf{M\text{-}OP_2}]]]]]\text{~C}\ ]\!]^{c}$

$= \lambda d.$ Liubei ran d or more

$= \lambda d.\exists e[\text{ran}(e) \wedge \text{agent}\ (Liubei, e) \wedge \mu_2(e) \geq d]$

e. $C = \{\lambda d.\ y \text{ ran } d \text{ or more} \mid y \in D_e\}$

f. $[\![(119)]\!]^{w,c} = 1$ iff $\forall y[y \in C \wedge y \neq Liubei$

$\rightarrow \mu_1\{d\text{: Liubei ran } d \text{ or more}\} < \mu_1\{d\text{: } y \text{ ran } d \text{ or more}\}$

Alternatively;

$[\![(119)]\!]^{w,c} = 1$ iff $\forall y[y \in C \wedge y \neq Liubei$

$\rightarrow \mu_1\{d\text{: } \exists e[\text{ran}(e) \wedge \text{agent}\ (Liubei, e) \wedge \mu_2(e) \geq d]\}$

$< \mu_1\{d\text{: } \exists e[\text{ran}(e) \wedge \text{agent}\ (y, e) \wedge \mu_2(e) \geq d]\}$

g. In words: for all *y* mentioned in *C*, the quantity that *Liubei* ran is less tha the quantity that *y* ran, along a contextually-given dimension (e.g., temporal duration or distance).

(120a) is given by a neo-Davidsonnian semantics of the verb *pao* 'ran'. (120b) is derived by functional application, by applying M-OP to (120a). (120c) is obtained by the movement of the superlative phrase SupP, which leaves a degree variable saturating the degree argument of M-OP. (120d) is obtained by lambda-abstraction, resulting from the movement of SupP. (120e) provides the contextual value of the domain *C*. (120f) is a consequence of functional application, by applying SupP (where the prejacent is of type <*d*, *t*>) to (120d). Finally, according to (120), the sentence (118) is true if and only if the quantity of running events (along temporal duration or distance) done by *Liubei* is less than that of running events (along temporal duration or distance) done by any other individual. This correctly captures the intuition. This seems to correctly capture the intuition. Thus, the morpho-semantic puzzle of Chinese SMs is again captured.[34]

By now, it should be clear that the unified analysis of Chinese SMs and QSs (for both *zuiduo* and *zuishao*) put forth in these two sections 5.4 − 5.5 has incorporated the insights from previous studies on various topics: Heim's/ Büring's decompositional analysis of comparatives, Bobaljik's Containment Hypothesis, Heim's movement approach to superlatives, Rett's idea about M-OP and Wellwood's view on quantity adjectives. It is worth noting that if Bobaljik (2012)'s Containment Hypothesis is correct, an important consequence following the current decompositional analysis is that cross-linguistically, we do NOT expect to find two superlative morphemes with the opposite comparison relation. The reason is the following: If a superlative construction DOES structurally embed a comparative construction (as we assume for the structure of both Chinese SMs and QSs), the locus of different comparison relations (a less-than relation vs. a greater-than relation) should be encoded in the meaning of comparative morphemes, rather than the superlative morpheme. Therefore, across languages, we seem to only observe two comparative morphemes marking the two comparison relations (e.g., English *more* vs. *less*), but not two superlative morphemes with the opposite comparison relation.

For comparison and completeness, I would like to briefly discuss the syntactic status of DegP is in the current analysis of QSs and SMs. In the case of QSs with

[34] An issue not addressed here concerns the distinction between free association with focus and conventional association with focus. Beaver and Clark (2008) argues for a three-way distinction of focus-sensitive expressions. In particular, according to them, although both *always* and *only* in English are focus-sensitive, the latter but not the former is conventionally associated with focus. Put differently, the association with focus is obligatory for *only*, while it is optional for *always*. Similar distinction can be said between SMs and QSs; that is, SMs pattern with *only* in being conventionally associated with focus, while QSs pattern with always in being freely associated with focus. At this point, it is unclear to me how to maintain the distinction between different types of focus-sensitive expressions within the Roothian system; to my understanding, different types of focus-sensitive expressions within the Roothian focus semantics all boil down to how focus resolves the contextual value of the relevant domain restriction (cf. von Fintel 1994).

zuiduo/ *zuishao*, I follow Heim (1999)'s idea that the relative reading of superlatives results from the fact that the superlative operator moves and scopes over the sentence. However, in the current analysis, it is the syntactic constituent SupP (i.e., a DegP), but not the superlative morpheme itself that moves. This is different from Heim (2001)'s proposal that degree operators may undergo QR and thus take scope. Crucially, the scope-taking of degree operators is possible only when the DegP structurally is not the extended projection of an adjective (e.g., the DegP is merged at Spec, AP in Heim 2001; among others). This line of thought has open many structural possibilities concerning where the DegP is merged and how it is connected with an adjective at the interface between syntax and semantics (see Lechner and Corver 2017 for an overview). In contrast, the current analysis of QSs aligns with Kennedy (1999)'s and Grimshaw (2005)'s view that the DegP is the extended projection of a quantity adjective. Thus, the DegP as a whole can move and thus take scope, as we have seen. In the case of SMs with *zuiduo*/ *zuishao*, the DegP (i.e., SupP) serves as a domain restrictor leading to domain narrowing and is structurally embedded under a covert existential operator E-OP. Crucially, the SupP itself does not move to take scope. Taken together, I consider both Kennedy's/ Grimshaw's proposal and Heim's proposal for the syntax of the DegP to be possible in natural language, though I am more sympathetic with the former proposal. Finally, although the syntactic status of the DegP (our SupP) is different in SMs and QSs, under the current analysis, it is clear that the semantic core of SupP is to establish a comparison/ an ordering relation. This explains why and how both SMs and QSs involve a superlative construction.

To sum up, in this section, I have illustrated how the compositional mapping is computed in Chinese QSs with *zuishao* in both nominal domain and verbal domain. In particular, building on Herm's movement theory to superlatives, Rett's M-OP and

Wellwood's view of quantity adjectives, I have incorporated the insights from Heim's/ Büring's decompositional analysis of English *little* and *less* and Bobaljik's Containment Hypothesis, into my unified analysis of Chinese SMs and QSs (for *zuishao*). In the next section, I briefly demonstrate how the current decompositional analysis can be extended to English SMs: *at least* and *at most*.

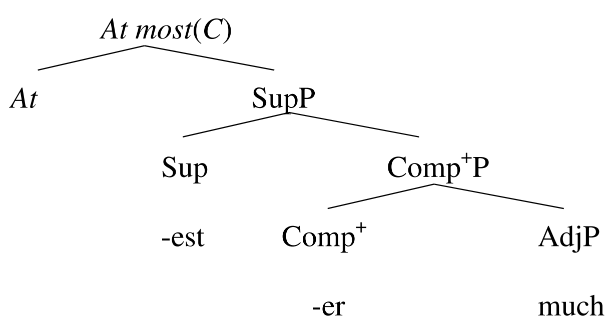
5.6 English

The key ingredient in extending the current analysis to English is the idea that the morpheme *at* in English SMs is an **overt** realization of the E-OP, as defined below.

(121) $[\![at]\!]^{w,\,c} = \lambda SUP_{<<st,\,t>,\,<st,\,t>>}\lambda C_{<st,\,t>}\ \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge SUP\ (C, \alpha)]$

Assuming again Bobaljik (2012)'s Containment Hypothesis, the internal structure of English *at most* is represented in (122).

(122) The internal structure of English SM *at most* in LF



With the same set of assumptions regarding the morpho-semantics of the SupP, including its sub-constituents such as $Comp^{+}P$ denoting a comparison relation between the prejacent and its alternatives, and *much* denoting a measure function mapping the focus elements to their positions along a contextually-given dimension, the semantics of *at most* in a propositional version is defined in (123), with its type-shifted versions in (124).

(123) $[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$

(124) a. A non-propositional version (by the Geach rule)

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\ st>}\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

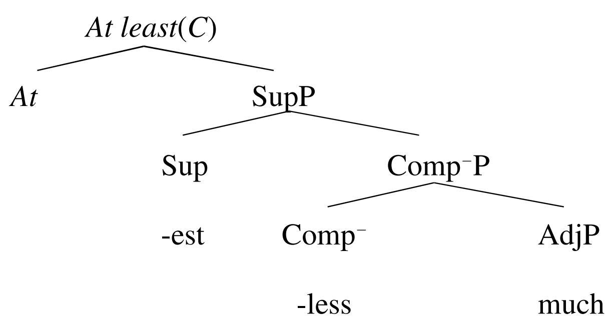
b. A non-propositional version (by the backward Geach rule)

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\lambda P_{<\eta,\ st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

It is worth noting that under the current decompositional analysis, the meaning of English *at most* and that of Chinese *zuiduo* are essentially the same, while the two languages parameterize to whether the E-OP is overtly or covertly realized.

Next, with the familiar Containment Hypothesis, the internal structure of English *at least* is represented below.

(125) The internal structure of English SM *at least* in LF



As before, we adopt the same set of assumptions regarding the morph-semantics of the SupP, especially Heim's/ Büring's view that *little* contributes a negation and a covert *much*, and that *less* is an reanalyzed result from the covert comparative morpheme –*er* combining with the negation. With these pieces in place, the semantics of *at most* in a propositional version is defined in (126), with its type-shifted versions in (127).

(126) $[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

(127) a. A non-propositional version (by the Geach rule)

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

b. A non-propositional version (by the backward Geach rule)

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

As in the case of English *at most* and Chinese *zuiduo*, under the current decompositional analysis, the meaning of English *at least* and that of Chinese *zuishao* are essentially the same, while the two languages parameterize to whether the E-OP is overtly or covertly realized.

Before closing this section, it is important to note that English may not be the only language where the E-OP is overtly realized. At this point, two other candidates are observed: the morpheme ***al*** in Italian SMs (*al massimo* 'at most' and *al meno* 'at least') and the morpheme ***au*** in French SMs (*au plus* 'at most' and *au moins* 'at least'). Further studies on the morpho-semantics of SMs are needed to verify whether the realization of the E-OP is limited to the family of Indo-European languages.

5.7 Theoretical Implications

In this section, I address three issues: **(a)** What does it mean to assign numerical values to propositions via a measure function? **(b)** Can the proposed analysis be extended to SMs in other languages? **(c)** How and why are SMs parallel to disjunction and epistemic indefinites, with respect to the ignorance interpretation?

To begin with, let us take a step back and ask ourselves: what does it mean to assign a numerical value to a proposition via a measure function? A possible candidate for such numerical assignment is probability. Probability operators have been defined as a measure function assigning numerical values to propositions. Moreover, this idea of probability operators has been applied to many areas in linguistics such as modality (Yalcin 2007, 2010, Lassiter 2017 and many of his works) and evidentials (Davis et al. 2007). In the case of SMs at hand, it is not straightforwardly clear to me how to apply the idea of probability to establish the ranking of the alternatives in connection with the ignorance interpretation and the concessive interpretation.

Alternatively, we can think of the numerical value assigned to a proposition as representing an evaluation on the proposition relative to the interlocutors' goals and interests in a given discourse. For example, we can define an interval ranging between the value one and the value minus one: [-1, 1]. The evaluative value *one* represents the best situation while the evaluative value *minus one* the worst situation. In this way, we can establish a (preference) ranking between the propositional alternatives based on the numerical values assigned to each relevant proposition (i.e., an evaluation of the relevant alternatives). As discussed in chapter 2 and chapter 4, one core pragmatic requirement of speaker concession is the evaluation of different circumstances denoted by the (propositional) focus alternatives. Under the implementation above, the connection between numerical values and the concessive interpretation becomes straightforward. In the case of the ignorance interpretation, the ranking of the propositions such as *Adam is a full professor* $\succ$*Adam is an associate professor*

$\succ$*Adam is an assistant professor* could be based on the dimension of “academic success”, “social status” or something else, depending on the discourse in which interlocutors are situated. The numerical value assigned to a proposition may simply represent its relative position along a contextually given scale, rather than the contextual evaluation. This explains Nakanish and Rullmann (2009)’s observation that the “settle-for-less” flavor shows up in the concessive interpretation, but not necessarily in the ignorance interpretation. Under the current conception, it is reduced to whether numerical values in a given context represent an evaluation of different circumstances denoted by the (propositional) focus alternatives.

Next, I would like to highlight several important features of the current decompositional analysis of Chinese SMs (see section 5.6 for an extension to English SMs). These features potentially allow the current analysis (or its overall architecture) to apply to the pair of SMs in other languages. First, the same measure function μ_c is involved in the pair of SMs. This suggests that *ceteris paribus*, the pair of SMs should be compatible with the same range of semantic objects (e.g., individuals and propositions). Second, both *zuiduo* and *zuishao* structurally contain a superlative construction SupP, leading to domain narrowing. This explains why SMs in many languages involve quantity adjectives and the superlative morpheme in their morphology. Furthermore, it also explains the semantic role of these morphemes in the pair of SMs. Third, the apparently antonymous relation between the pair of SMs is encoded within the superlative construction SupP. Crucially, the composition of the SupP closely mimics those independently proposed in the studies of superlatives. This explains why the pair of quantity adjectives *duo* ‘much’/ *shao* ‘little’ (cf. English *much* vs. *little*) is intuitively antonymous, while the pair of SMs (cf. English *at most* vs. *at least*) is less so. The decompositional analysis proposed in this chapter potentially works for Brazilian Portuguese, English, French, Italian and Turkish, where in these languages SMs morphologically employ quantity adjectives combined with the superlative morpheme. Although the current decompositional analysis of SMs has these promising features, unfortunately, it cannot be the whole story for the morpho-semantic puzzle. Specifically, I have observed that there are languages where SMs morphologically involve quantity adjectives but some other operators are used, instead of the superlative morpheme.

(128) Quantity adjectives plus even-if (e.g., Japanese and Korean)

a. *ooku-temo* 'at most' Japanese
many-even.if

b. *sukunaku-temo* 'at least'
few-even.if

(129) Quantity adjectives plus comparatives (e.g., Magahi, Hindi, Russian)

a. *jaadaa se aadaa* 'at most' Magahi
more than more

b. *kam se kam* 'at least'
less than less

These cross-linguistic facts minimally suggest that there is more than one possible mapping between morphology and semantics, leading to the meaning of SMs. Needless to say, more studies on these languages are required in the future, in order to discover how these morpho-semantic mappings work in natural language.

Finally, I would like to address the issue of how and why SMs are parallel to disjunction and epistemic indefinites with respect to the ignorance interpretation. Before we go into the parallels, let me briefly point out two differences between SMs and disjunction/ epistemic indefinites. First, although all the three types of expressions (SMs, disjunction and epistemic indefinites) can lead to ignorance inferences, the ignorance inference is obligatory in epistemic indefinites, but not in the former two cases. Put differently, the use of SMs does not necessarily lead to an ignorance inference, unlike epistemic indefinites (see Alonso-Ovalle & Menédez-Benito 2010, 2011, 2012 for a series of discussions on epistemic indefinites); whether an ignorance inference arises with SMs depends on many contextual factors (e.g., whether the maxim of quantity is deactivated in a given context, whether the unique maximally informative answer is requested in the discourse and so on) as discussed in chapter 2. Second, unlike disjunction (which necessarily lead to total ignorance), SMs are compatible with partial ignorance (see chapter 2). Despite these two differences, all the three types of expressions share the capacity of giving an ignorance inference in natural language.

In the literature on SMs, there have been roughly two views on the nature of ignorance inferences given by SMs. One view considers SMs as disjunction for the sake of implicature calculation (Büring 2008, Coppock and Brochhagen 2013, Kennedy 2015, Schwarz 2016 and among others), while the other takes SMs to be like epistemic indefinites in requiring the domain to be anti-specific (Nouwen 2015; cf. Alonso-Ovalle & Menédez-Benito 2013), a reminiscent of the anti-singleton condition imposed by the Spanish epistemic indefinite *algún* in Alonso-Ovalle & Menédez-Benito (2010).

Unlike the split in previous studies, the current decompositional analysis actually captures both intuitions for SMs. First, let us consider the similarity between SMs and epistemic indefinites. Under the current analysis, the superlative construction SupP serves as a domain restrictor leading to domain narrowing (i.e., whether it is the relevant higher/ lower alternatives that are excluded depends on the particular item that is used). Next, the focus presuppositions introduced by the squiggle operator ~ always guarantees that the prejacent is one element in the domain. Taken together, the collaboration between the SupP and focus presuppositions leads to the effect that the domain of SMs is always antis-specific (i.e., a non-singleton set): a set exclusively consisting of the prejacent and its higher/ lower alternatives. This "anti-specificity" of SMs will arise as long as the constraint semantic vacuity is not violated. Crucially, under the current perspective, the anti-specificity of SMs arises from the collaboration between the SupP and focus presuppositions, rather than hard-wired into their semantics (cf. Nouwen 2015). Therefore, despite the parallel, SMs are fundamentally different from epistemic indefinites with respect to the nature of their anti-specificity. Second, let us consider the similarity between SMs and disjunction. Given that the domain of SMs is always anti-specific (a non-singleton set), once the existential operator E-OP kicks in, the parallel between SMs and disjunction immediately arises. More specifically, an existential claim over a (non-singleton) set amounts to a disjunctive statement of the elements in the set. Crucially, SMs do not hard-wire a disjunction in their semantics, either. This explains the long-standing puzzle for a disjunction approach on which linguistic level the disjunctive meaning comes about (see Coppock and Brochhagen 2013 for a detailed discussion). Under the current perspective, the parallel between SMs and disjunction is derived from the combination of an existential operator and a non-singleton domain.

Putting together the two parallels, the current analysis provides a deeper understanding of SMs and a shifted perspective unifying Büring (2008)'s disjunction view and Nouwen (2015)'s anti-specificity view: the common core of the two parallels actually lies in the combination of an existential operator and a non-singleton domain underlying the semantics of SMs. The latter comes from the SupP collaborating with focus presuppositions. Crucially, neither an anti-specificity condition nor a disjunction needs to be hard-wired into the semantics of SMs. Therefore, the current analysis reconciles the conflict between the two different views on the semantic nature of SMs.

5.8 Comparison with Coppock (2016)

This section presents a brief comparison between my decompositional analysis of SMs with Coppock (2016). Building on Penka (2010)'s and Solt (2011)'s insights on the compositional derivation of SMs, Coppock (2016) presents a decompositional analysis of English *at least* couched in Coppock and Brochhagen (2013)'s discourse-based analysis. There are three core ingredients in Coppock (2016)'s decompositional analysis. First, she assumes some sort of alternative semantics where natural language expressions are translated into expressions of a formal logic that denote a set of intensions. (130) illustrates how the adjective *tall* is translated under her semantic framework.

(130) $[\![tall]\!]^w = \{\lambda w \lambda d \lambda x.\ tall_w(d)(x)\}$

Next, she assumes the Containment Hypothesis, where a superlative construction structurally embeds a comparative construction. She assigns the comparative morphemes *–er* and *less* under phrasal comparatives the semantics in (131). Specifically, *-er* denotes a singleton set containing a function that expects, besides the world argument, a gradable predicate G, a comparative standard s, and a comparative target t, and returns true if the comparative target is G to a greater extent than the comparative standard. The entry of *less* encodes the opposite comparison relation.

(131) a. $[\![\text{-}er]\!]^w = \{\lambda w.\ \lambda G_{<d,\ \tau t>}\lambda s_{<\tau>}\lambda t_{<\tau>}.\ \max(\lambda d.G_w(d)(t)) > \max(\lambda d.G_w(d)(s))\}$

b. $[\![less]\!]^w = \{\lambda w.\ \lambda G_{<d,\ \tau t>}\lambda s_{<\tau>}\lambda t_{<\tau>}.\ \max(\lambda d.G_w(d)(t)) < \max(\lambda d.G_w(d)(s))\}$

Second, for the superlative morpheme -*t*, she assumes the following entry.

(132) $[\![\text{-}t]\!]^w = \{\lambda w.\ \lambda R_{<\tau,\ \tau t>}\lambda C_{<\tau>}\lambda x_{<\tau>}.\forall x'_\tau \in C.\ x \neq x' \rightarrow R_w(x, x')\}$

In order to compose meanings, she assumes a point-wise, intension-friendly version of functional application, as illustrated below. With the compositional rule, the meaning of *least* is offered in (134), where **m** is a contextually-specified gradable predicate.

(133) **Functional Application**

Let α and β be the only sub-trees of the tree γ. If

(a) $[\![\alpha]\!]^w = \alpha'$, where α' is of type $<s, <\sigma, \tau>>$

(b) $[\![\beta]\!]^w = \beta'$, where β' is of type $<s, \sigma>$

Then: $[\![\gamma]\!]^w = \{\lambda w.f(w)(a(w))\ |\ f \in \alpha' \wedge a \in \beta'\}$

(134) $[\![least]\!]^w = \{\lambda w.\ \lambda C_{<\tau>}\ \lambda x_{<\tau>}.\forall x' \in C \wedge x' \neq x \rightarrow \max(\mathbf{m}_w(x)) < \max(\mathbf{m}_w(x'))\}$

Finally, the most important insight from Coppock's analysis (I believe) is that the famous *at*-operator is actually meaningful: it introduces the alternatives and scales. The entry of *at* is provided in (135), with a sketchy compositional derivation in (136).

(135) $[\![at]\!]^w = \{\lambda w.\lambda S_{<\tau,<\tau,\ t>}\ \lambda x_{<\tau>}\ \lambda y_{<\tau>}\ |\ y \in C \wedge S_w(C)(x)\}$

‘the set of things *y* in a comparison class *C* such that *x* is *S* [least/ most] in *C*’

(136) The case of *at least three*

a. The LF: [**at** [-**t** [**less** [**m**]]]] three]

b. $[\![at\ least]\!]^w = \{\lambda w.\lambda x_{<\tau>}\ \lambda y_{<\tau>}\ |\ y \in C \wedge\ \forall x' \in C \wedge x' \neq x$

$\rightarrow \max(\mathbf{m}_w(x)) < \max(\mathbf{m}_w(x'))\}$

c. $[\![\textit{at least three}]\!]^w = \{\lambda w.\, \lambda y_{<\tau>} \mid y \in C \wedge \forall x' \in C \wedge x' \neq 3$

$\rightarrow \max(\mathbf{m}_w(3)) < \max(\mathbf{m}_w(x'))\}$

At this point, it is worth noting that under Coppock's implementation, the job of *at*-operator not only introduces the alternatives and scales, but also *encodes* a non-strict comparison relation (see the meaning of *at* above). Specifically, the meaning derived for *at least three* under Coppock's analysis is a set consisting of the prejacent (the comparative standard) and the relevant higher-ranked elements: {3, 4, 5,…}, ignoring intensionality.

Although Coppock's analysis elegantly explains the morpho-semantic puzzle of SMs, several important questions are raised by her treatment of *at*: **(a)** Why doesn't focus introduce the alternatives in the first place, as in Rooth (1985, 1992)? **(b)** What is role of focus in the analysis after all? How to incorporate the contribution of focus in the analysis? **(c)** Why do SMs have a morpheme specialized in introducing alternatives and scales? **(d)** Shall we expect such alternative-introducing or scale-introducing morpheme to occur somewhere else in natural language? Maybe also in other focus particles like *only* and *even*?

Given these considerations, the decompositional analysis proposed in this chapter differs from Coppock (2016) in several crucial respects. First, unlike Coppock's analysis, the current decompositional analysis is deeply connected with Rooth (1985, 1992)'s focus semantics. Second, assuming focus is anaphoric to some QUDs in a discourse (Rooth 1992, Roberts 1996/ 2012, among others; see also chapter 2 and chapter 4), just like the discourse function of other focus particles (e.g., *only*), SMs imposes further restrictions on the set of (possible) answers, despite their internal complexity and an "anti-specific" domain. Third, many discourse-based analyses assign a non-strict comparison relation to the semantics of SMs (e.g., Krifka 1999, Coppock and Brochhagen 2013, among others). An example lexical entry of *at least* following that approach is presented below.

(137) $[\![\textbf{\textit{at least (C)}}]\!]^g = \lambda p_{<s,\ t>}.\lambda w_{<s>}.\exists q\ [q \in C \wedge q(w) \wedge q \succeq_i p]$

Some alternative q in C and as strong as the prejacent p is true in w

However, an entry such as (137) raises a non-trivial question pertaining to the morpho-semantic puzzle of SMs: While *at least* involves an **existential** quantifier and a **non-strict comparison**; superlatives typically involve a **universal** quantifier and a **strict comparison**, as shown in (138).

(138) a. Adam ate the **least** apples. Heim (1985, 1999), Hackl (2000, 2009)

b. $[\![(137a)]\!] = 1$ iff $\forall y[y \neq \text{Adam} \wedge y \in C \rightarrow \max(\lambda d.\ \text{Adam ate } d\text{-many apples}) < \max(\lambda d.\ y \text{ ate } d\text{-many apples})]$

In Coppock's analysis, the non-strict comparison is obtained through the defining semantics of the *at*-operator. In contrast, under the current decompositional analysis, the non-strict comparison of SMs is only an illusion: it is derived from the SupP collaborating with focus presuppositions; again, there is no need to hard-wire a non-strict comparison relation into the semantics of SMs.

Finally, before leaving this section, let me briefly point out that despite the several differences discussed above, the current analysis, in fact, is largely inspired by Coppock (2016)'s analysis. In particular, the two analyses share two significant assumptions: **(a)** both analyses adopt Bobaljik's Containment Hypothesis in their decomposition of SMs; **(b)** both analyses adopt the view that the internal structure of SMs is actually an instantiation of phrasal comparatives. As mentioned in section 5.7, cross-linguistic facts indicate that there is more than one morpho-semantic mapping towards the meaning of SMs in natural language. Therefore, I am hoping that the ideas discussed in this chapter can spark some valuable discussion and shed new light on the possible morpho-semantic mappings of SMs in other languages in the future.

5.9 Conclusion

In this chapter, I have taken Chinese SMs *zuiduo*/ *zuishao* as a case study and presented a decompositional analysis that incorporates a superlative construction SupP in their syntax and semantics. This explains the morph-semantic puzzle on the morphological makeup of these focus particles. Furthermore, the SupP is further embedded under a covert existential operator E-OP. This explains why SMs parallel with disjunction and epistemic indefinites in leading to ignorance inferences.

The core pieces for the compositional mapping of Chinese SMs between their forms and meanings are listed below, where α ranges over the elements induced by focus and η represents the semantic type of those elements.

(139) $[\![duo]\!]^c = \lambda\alpha.\mu_c(\alpha)$ $\qquad\qquad$ $<\eta, d>$

(140) $[\![\mathrm{Comp}^{+}\mathrm{P}]\!]^c = \lambda\alpha\lambda\beta.\mu_c(\alpha) > \mu_c(\beta)$ $\qquad\qquad$ $<\eta, <\eta, t>>$

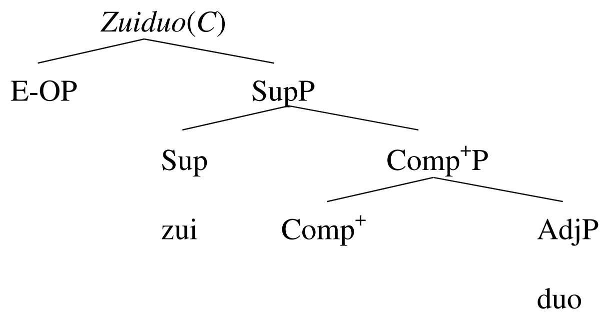
(141) $[\![\mathrm{Comp}^{-}\mathrm{P}]\!]^c = \lambda\alpha\lambda\beta.\mu_c(\alpha) < \mu_c(\beta)$ $\qquad\qquad$ $<\eta, <\eta, t>>$

(142) $[\![zui]\!]^c = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

(143) $[\![\text{E-OP}]\!]^{w, c} = \lambda SUP_{<<st, t>, <st, t>>}\lambda C_{<st, t>}\ \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge SUP\ (C, \alpha)]$

The following summarizes my decompositional proposal for Chinese SM *zuiduo*.

(144) The internal structure of Chinese SM *zuiduo* at LF



(145) A propositional version

$$[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$$

(146) A non-propositional version (by the Geach rule)

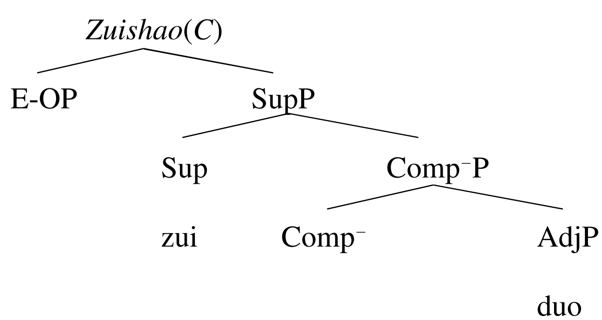
$$[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

(147) A non-propositional version (by the backward Geach rule)

$$[\![zuiduo(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

The following summarizes my decompositional proposal for Chinese SM *zuishao*.

(148) The internal structure of Chinese SM *zuishao* at LF



(149) A propositional version

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$$

(150) A non-propositional version (by the Geach rule)

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge$$

$$\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

(151) A non-propositional version (by the backward Geach rule)

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge$$

$$\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) < \mu_c(P_w(\beta))]]$$

Chapter 6 Concluding remarks

This dissertation was mainly dedicated to two intriguing puzzles posed by superlative modifiers (SMs): the ambiguity puzzle and the morpho-semantic puzzle. The former puzzle concerns the question why cross-linguistically SMs in general demonstrate an ambiguity between an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. The latter puzzle concerns the question why SMs in general involve quantity adjectives and degree morphology in their morphological makeup across languages and the question how theses morphological pieces are connected with their semantics. In addition to these two puzzles, however, there are many other puzzles posed by SMs that remain untouched in this dissertation. In this chapter, I will first discuss several puzzling facts about SMs that have attracted the attention of many recent studies, and then shift the attention to some (cross-linguistic) implications of the current analysis.

This chapter proceeds as follows. Section 6.1 discusses some interesting interaction between SMs and modals/ universal quantifiers. A particularly puzzling fact is that when SMs are under the scope of existential modals, *at most* leads to a mysterious upper bound inference (which is not predicted by the truth-conditions), while *at least* does not lead to a corresponding lower bound inference. Similar contrast holds for Chinese as well. Section 6.2 presents a puzzling fact that SMs are ill-formed under the scope of negation, while they can be well-formed in other downward-entailing contexts. Section 6.3 compares SMs with a variety of expressions that can result in ignorance inferences (such as free relatives, epistemic indefinites and disjunction) and discusses how the current analysis sheds light on the semantics and pragmatics of ignorance inferences. Section 6.4 discusses some (cross-linguistic) implications of the current analysis. In particular, four issues are addressed: **(a)** the issue of lexicalization; **(b)** the quantificational domain hypothesis for the distributional restriction on CON; **(c)** the variety of SMs and their restrictions; **(d)** the definite determiner inside English SMs. Section 6.5 summarizes this dissertation and repeats the crucial pieces of the current analysis of SMs.

6.1 The interaction with modals/ universal quantifiers

To start with, one intriguing puzzle posed by SMs is that they demonstrate a non-trivial interaction with modals (e.g., Geurts and Nouwen 2007, Büring 2008, Schwarz 2011, Coppock and Brochhagen 2013, Penka 2014, Kennedy 2015, among others). For example, English sentences like (1) containing *at least* and a necessity modal are ambiguous, depending on the relative scope relation between *at least* and the modal. When *at least* scopes above the modal, the **speaker insecurity** reading (or ignorance reading) arises: the speaker is uncertain about what the maximum number of courses that Adam is required to take is, but certain that the minimum is not less than three. When *at least* scopes below the modal, the **authoritative** reading (or variation reading) arises: it is required that Adam takes no less than three courses.

(1) Adam is required to register for **at least** three courses. *at least* > $\Box$, $\Box$ > *at least*

In the same vein, (2) is similarly ambiguous, depending on the relative scope relation between *at most* and the modal. Under the **speaker insecurity** reading (or ignorance reading), (2) conveys that the speaker is uncertain about what the minimum number of courses that Adam is required to take is, but certain that the minimum is not more than three. In contrast, under the **authoritative** reading (or variation reading), (2) conveys that it is required that Adam takes no less than three courses.

(2) Adam is required to register for **at most** three courses. *at most* > $\Box$, $\Box$ > *at most*

When it comes to the possibility modal, things become complicated. First of all, when SMs scope above the modal, the ignorance reading in (3) conveys that the minimum number of allowable courses (to register for) is three or more than three, and the speaker is uncertain about the actual cap on the number of courses; the ignorance reading in (4) conveys that the maximum number of allowable courses (to register for) is three or less than three, and the speaker is uncertain about the actual cap on the number of courses.

(3) Adam is allowed to register for **at least** three courses. *at least* > $\Diamond$, $\Diamond$ > *at least*

(4) Adam is allowed to register for **at most** three courses. *at most* > $\Diamond$, $\Diamond$ > *at most*

In contrast, some puzzling contrasts arise when SMs scope below the modal. The authoritative reading of (3) conveys that a registration in three or more courses is allowed, **without** ruling out registration in fewer courses. More specifically, the authoritative reading of (3) does NOT induce a lower-bound inference where it is not allowed that Adam registers for fewer than three courses. By contrast, the authoritative reading of (4) conveys that a registration in three or fewer courses is allowed and **a registration in more courses is not allowed**. More specifically, the authoritative reading of (4) DOES induce an upper-bound inference where it is not allowed that Adam registers for more than three courses. Crucially, the presence of the upper-bound inference is not predicted by the truth-conditions encoded in the scope relation. For purposes of illustration, assuming that $\Diamond$ represents the possibility modal and *at most* encodes a maximality operator MAX and a non-strict comparison $\leq$, the two readings of (4) can be schematically shown in (5).

(5) a. The ignorance reading of (4)

MAX { n | $\Diamond$[Adam registers for n courses]} $\leq$ 3

b. The authoritative reading of (4)

$\Diamond$ [MAX { n | Adam registers for n courses} $\leq$ 3]

As presented in (5b), for the authoritative reading, the truth-conditions generated by the scope relation only predicts a free choice inference that it is allowed that Adam registers for three or less than three courses, and crucially do NOT rule out the possibility that Adam registers for more than three courses. Many recent studies have been dedicated to the mysterious upper-bound inference arising from the combination of *at most* and a possibility modal (e.g., Blok 2015, Buccola and Haida 2016, 2017). In particular, Blok (2015) argues for a surface-scope analysis and against a scope-ambiguity analysis. In contrast, Buccola and Haida (2016, 2017) argue for a scope-ambiguity analysis while attempting to derive the upper bound inference from certain pragmatic processes. At this point, it is worth noting that **(a)** when SMs scopes above a modal, only the ignorance reading is obtained; **(b)** the same scope ambiguity is observed when SMs interact with individual quantifiers like *every*, as shown in (6).

(6) a. Every student found **at least** three typos. *every* > *at least*, *at least* > *every*

b. Every student found **at most** three typos. *every* > *at most*, *at most* > *every*

Briefly, when SMs scope above the quantifier *every*, (6) conveys the ignorance reading that the speaker is ignorant about the exact number of typos found by the students. In contrast, when SMs scope below the qualifier *every*, (6) conveys the authoritative reading that the students vary in the number of typos they found; (6a) and (6b) differ in whether the number three is the lower bound or the upper bound. In

(6a), some of the students found three typos, some have more but none have less. In (6b), some of the students found three typos, some have less but none have more.

In what follows, I show that Chinese provides a crucial piece of cross-linguistic evidence supporting a scope-based approach. More specifically, because Chinese is a scope-rigid language (Huang 1982), it is shown that the two readings (the ignorance reading and the authoritative one) are represented by the relative word order between SMs and modals. Furthermore, as in English, the combination of a possibility modal and *zuiduo* 'at most' in Chinese similarly leads to the mysterious upper bound inference. These Chinese facts indicate that a scope-based analysis is needed and the mysterious upper bound inference should be due to certain pragmatic processes. This line of thought echoes with Buccola and Haida (2016, 2017)'s view.

Now, let's first consider the interaction between Chinese SMs and the necessity modal *bixu* 'must'. Consider (7) and (8) below.

(7) a. Ignorance Reading: ***zuishao* > $\Box$**

Liubei **zuishao** **bixu** xuan [san]$_F$-men-ke.

Liubei at least must select three-CL-course

'The minimum number of courses required for registration is at least three.'

b. Authoritative reading: **$\Box$ > *zuishao***

Liubei **bixu** **zuishao** xuan [san]$_F$-men-ke.

Liubei must at least select three-CL-course

'It is required that Liubei take at least three courses.'

(8) a. Ignorance Reading: ***zuiduo* > $\Box$**

Liubei **zuiduo** **bixu** xuan [san]$_F$-men-ke.

Liubei at most must select three-CL-course

'The minimum number of courses required for registration is at most three.'

b. Authoritative reading: $\Box$ **>** ***zuiduo***

Liubei	**bixu**	**zuiduo**	xuan	[san]$_F$-men-ke.
Liubei	must	at most	select	three-CL-course

'It is required that Liubei take at most three courses.'

As demonstrated above, the relative scope relation between SMs and the modal in Chinese is preserved and transparent in terms of word order. More specifically, the ignorance reading arises only when SMs precede the modal (i.e., SMs scope above the modal; see (7a) and (8a)). In contrast, the authoritative reading arises only when the modal precedes SMs (i.e., SMs scope below the modal; see (7b) and (8b)).

Next, let's consider the interaction between Chinese SMs and the possibility modal *keyi* 'can'. Consider (9) and (10).

(9) a. Ignorance Reading: ***zuishao*** **>** $\Diamond$

Liubei	**zuishao**	**keyi**	xuan	[san]$_F$-men-ke.
Liubei	at least	can	select	three-CL-course

'The maximum number of courses allowed for registration is at least three.'

b. Authoritative Reading: $\Diamond$ **>** ***zuishao***

Liubei	**keyi**	**zuishao**	xuan	[san]$_F$-men-ke.
Liubei	can	at least	select	three-CL-course

'It is allowed that Liubei take at least three courses.'

(10) a. Ignorance Reading: ***zuiduo*** **>** $\Diamond$

Liubei	**zuiduo**	**keyi**	xuan	[san]$_F$-men-ke.
Liubei	at most	can	select	three-CL-course

'The maximum number of courses allowed for registration is at most three.'

b. Authoritative Reading: ◇ **>** ***zuiduo***

Liubei **keyi** **zuiduo** xuan [san]$_F$-men-ke.

Liubei can at most select three-CL-course

'It is allowed that Liubei take at most three courses.'

Three remarks are in order. First, as in the case of the necessity modal, the relative scope relation between SMs and the modal in Chinese is preserved and transparent in terms of word order. Second, like the situation in English, the authoritative reading in (9b) does NOT induce a lower bound inference where it is not allowed that *Liubei* registers for less than three courses. Third, resembling the situation in English again, the authoritative reading in (10b) induces an upper bound inference where it is not allowed that *Liubei* registers for more than three courses, which is not part of the meaning predicted by the scope relation (as indicated by the English translation).

Finally, like their English counterparts, Chinese SMs also show a scope interaction with the individual quantifier *mei* 'every'. In Chinese, quantificational phrases in a preverbal position generally require the particle *dou*. According to Lin (1998), the *mei*-phrase denotes a plurality while the particle *dou* is a generalized distributive operator (in the sense of Schwarzschild 1996) encoding a universal quantification. With this brief background, (11) and (12) illustrates the case of *zuishao* 'at least', while (13) and (14) present the case of *zuiduo* 'at most'.

(11) Ignorance Reading: *zuishao* takes wide scope

Zheli mei-tai-diannao **zui-shao** **dou** you liang-GB de jiyiti.

Here every-CL-computer SUP-little DOU have two-GB DE memory

'Every computer here has at least 2-GB of memory.'

(12) Authoritative Reading: *zuishao* takes narrow scope

Zheli mei-tai-diannao **dou** you **zui-shao** liang-GB de jiyiti.

Here every-CL-computer DOU have SUP-little two-GB DE memory

'Every computer here has at least 2-GB of memory.'

(13) Ignorance Reading: *zuiduo* takes wide scope

Zheli mei-zhang-xinyong-ka mei-ji **zui-duo** **dou** you

Here every-CL-credit-card every season SUP-much DOU have

wu-bai-yuan xianjin huikui.

five-hundred-dollar cash feedback

'Every credit card here has at most five hundred cash back every season.'

(14) Authoritative Reading: *zuiduo* takes narrow scope

Zheli mei-zhang-xinyong-ka mei-ji **dou** you **zui-duo**

Here every-CL-credit-card every season DOU have SUP-much

wu-bai-yuan xianjin huikui.

five-hundred-dollar cash feedback

'Every credit card here has at most five hundred cash back every season.'

In (11), where *zuishao* scopes above the particle *dou*, the sentence implies that the speaker is ignorant about the exact amount of memory that each computer has, while the amount is no less than 2GB. In contrast, in (12), where *zuishao* scopes below the particle *dou*, the sentence conveys that some computers have 2GB, some have more, but none have less. More specifically, the computers vary in the amount of memory and that amount has a lower bound 2GB.

The same scope interaction is also observed for *zuiduo*. In (13), where *zuiduo* scopes above the particle *dou*, the sentence implies that the speaker is ignorant about the exact amount of cash back that each credit card has, while she is certain that the amount is no more than five hundred. In contrast, in (14), where *zuiduo* scopes below the particle *dou*, the sentence conveys that some credit cards have 500 cash back, some have less, but none have more. That is, the credit cards vary in the amount of cash back and that amount has an upper bound 500 dollars.

Before closing this section, I'd like to point out that it is indeed possible to construct a context where the mysterious upper bound inference does not arise in the combination of a possibility modal and *at most*/ *zuiduo*. My observation is that the upper bound inference (for cases like (10b)) seems to systematically ***disappear*** in contexts where the precise course regulation is ***not*** at issue, e.g., contexts where ***time management*** on how *Liubei* can balance his priorities is under discussion: *Liubei* has multiple jobs and needs to support his family. In this case, (10b) is uttered as a ***suggestion***, not about the precise course regulations. Crucially, the speaker can continue felicitously to say: "so that *Liubei* can have more time making money to support his family. But if *Liubei* would like, of course, he can take more courses". The context and the conversation below illustrate the point.

(15) **Context**: Chris is facing an issue of how to balance his priorities: making money while taking some courses without being stressed out. Chris's friends Adam and Bill are discussing his situation and to see if there is a way out. <u>It is known that no more than five courses (15 credits) are allowed per semester</u>.

(16) a. Adam: How many courses do you think Chris can take while well supporting his family?

b. Bill: Well, Chris **can** take **at most** three courses. That should leave him enough time to study while making some money. Of course, if he just wants to graduate earlier or doesn't need that much time to study, he can take more (courses).

It is worth emphasizing that even though the mysterious upper bound inference does not arise in (15) and (16); a pragmatic mechanism is still needed to explain **(a)** why and how the upper bound inference arises; **(b)** why native speakers find it extremely difficult to cancel the upper bound inference.

To sum up, in this section, we have seen that SMs demonstrate a non-trivial interaction with modals and universal quantifiers like *every*. The Chinese facts above strongly indicate that a scope-based analysis is needed. Furthermore, we have also seen that when SMs are under the scope of a possibility modal, *at least* and *zuishao* do not induce a lower bound inference while *at most* and *zuiduo* induce an upper bound inference. Neither this contrast between pairs of SMs nor the mysterious upper bound inference is predicted by the current analysis. Given the current perspective, future studies should be devoted to the exact pragmatic mechanism explaining why and how the upper bound inference arises.

6.2 Negation and downward-entailing contexts

Another intriguing puzzle posed by SMs is that they are generally ill-formed under the scope of negation (e.g., Spector 2011), as illustrated below.

(17) ??Adam didn't solve **at least/ at most** three problems.

Crucially, however, SMs are not ill-formed across the board in downward-entailing contexts. For example, they can be well-formed in the restrictor of universal quantifier and the antecedent of conditionals (for some speakers at least), as demonstrated below.

(18) The restrictor of universal quantifier

a. Everyone who (had) solved **at least** three problems passed.

b. Everyone who (had) solved **at most** three problems failed.

(19) The antecedent of conditionals

a. If you solve **at least** three problems, you will pass.

b. If you solve **at most** three problems, you will fail.

The same empirical generalization apparently holds of Chinese SMs as well. More specifically, Chinese SMs are infelicitous under the scope of negation, regardless of whether the scale is based on semantic strength or pragmatic strength. The following examples illustrate the point.

(20) Numerical Scales (a contextual ranking: 4 ≻3 ≻2)

??Liubei **meiyou** **zui-duo/** **zui-shao** xie $[\text{san}]_F$-ben-xiaoshuo.

Linbei not SUP-many SUP-little write three-CL-novel

'Liubei did not at most/ at least write three novels.'

(21) Plurality Scales (a contextual ranking: adam &bill&chris ≻adam&bill ≻adam)

??Liubei **meiyou** **zui-duo/** **zui-shao** guyong $[\text{Adam he Bill}]_F$.

Liubei not SUP-many SUP-little hire Adam and Bill

'Liubei did not at most/ at least hired Adam and Bill.'

(22) Lexical Scales (a contextual ranking: gold medal≻silver medal≻bronze medal)

??Liubei **meiyou** **zui-duo/** **zui-shao** na $[\text{yin}]_F$-pai.

Liubei not SUP-many SUP-little take silver-medal

'Liubei did not at most/ at least got a silver medal.'

(23) Pragmatic Scales (a contextual ranking: cherries $\succ$apples $\succ$bananas)

??Liubei **meiyou** **zui-duo/** **zui-shao** mai [pingguo]$_F$.

Liubei not SUP-many SUP-little buy apple

'Liubei did not at most/ at least bought apples.'

In contrast, resembling their English counterparts, Chinese SMs are well-formed in the restrictor of the universal quantifier (see (24) and (25)) and the antecedent of conditionals (see (26) and (27)).

(24) Mei-yi-wei dadui **zui-shao** san-ge-wenti de ren dou

Every-one-CL answer SUP-little three-CL-question Mod person DOU

neng shenqing buzhu.

can apply funding

'Everyone who answered at least three questions can apply for the funding.'

(25) Mei-yi-wei dadui **zui-duo** san-ge-wenti de ren dou

Every-one-CL answer SUP-much three-CL-question Mod person DOU

bu neng shenqing buzhu.

not can apply funding

'Everyone who answered at most three questions cannot apply for the funding.'

(26) Ruguo Liubei **zui-shao** dadui san-ge-wenti,

If Liubei SUP-little answer three-CL-question

Ni jiu shu-le.

You JIU lose-ASP

'If Liubei answers at least three questions, then you lose.'

(27) Ruguo Liubei **zui-duo** dadui san-ge-wenti,

If Liubei SUP-much answer three-CL-question

Ni jiu ying-le.

You JIU win-ASP

'If Liubei answers at most three questions, then you win.'

It is worth pointing out that nothing in the proposed semantics of SMs explains why they are ill-formed under plain negation while well-formed in the restrictor of universal quantifiers and the antecedent of conditionals. It is not clear to me what factors underlying the distinction between negation and the restrictor (of universal quantifiers and conditionals) should be responsible for the contrast above, though see Mihoc (2018) for a proposal. Needless to say, more studies are needed in the future.

6.3 The landscape of ignorance inferences

In natural language, many expressions can give rise to ignorance inferences. For instance, (28) illustrates the case of free relatives (FRs; e.g., Dayal 1997, von Fintel 2000, Condoravdi 2008, Heller and Wolter 2011), (29) the case of epistemic indefinites (EIs; Alonso-Ovalle and Menédez-Benito 2010), (30) the case of disjunction and (31) the case of SMs (e.g., Geurts and Nouwen 2007, Büring 2008).

(28) Free Relatives

John will eat **whatever** Mary is cooking.

(29) Epistemic Indefinites

María se caso´ con **algún** estudiante del departamento

María SE married with ALGÚN student of the department

de lingüística.

of linguistics

'María married a linguistics student.'

(30) Disjunction

A: Who did John invite for his party?

B: John invited Adam **or** Bill.

(31) Superlative Modifiers

A: Exactly how many students came to the party?

B: **At least** $[\text{ten}]_F$ students came to the party.

In (28), the sentence conveys that the speaker does not know the thing that John is cooking. In (29), the sentence conveys that the speaker does not know which student María married. In (30), the sentence implicates that the speaker does not know exactly who John has invited for the party. In (31), the sentence implicates that the speaker does not know the exact quantity of students who came to the party.

Despite the variety of expressions, there seem to be a common core of these expressions: all of them come with a non-singleton domain (i.e., a base for the variation) and the speaker fails to identify a particular element in the domain (i.e., the indeterminacy). To see this semantic core, (32) presents von Fintel (2000)'s reformulation of Dayal (1997)'s insights on the semantics of English *whatever*. In (32), F is the modal base for *whatever*, P is the property denoted by the descriptive content of the relative clause (e.g. Mary is cooking x), and Q is the predicate expressed by the rest of the sentence (e.g. John will eat x). Next, (33) shows Alonso-Ovalle and Menédez-Benito (2010)'s semantic proposal for Spanish *algún*. Finally, (34) illustrates a propositional version of my current proposal for English SMs.

(32) whatever $(w)(F)(P)(Q)$[1]

a. Presupposition: $\exists w', w'' \in F$: $\iota x.P(w')(x) \neq \iota x.P(w'')(x)$

b. Assertion: $\forall w' \in F$: $Q(w')(\iota x.\ P(w')(x))$ von Fintel (2000: (5))

(33) $[\![algún\,]\!] = \lambda f_{<<e,t>,<e,t>>}$: anti-singleton (f). $\lambda P_{<e,t>}.\lambda Q_{<e,t>}.\exists x_{<e>}[f(P)(r) \wedge Q(r)]$

(34) a. $[\![at\ least(C)]\!]^{w,c} = \lambda \alpha_{<st>}.\exists \gamma[\gamma \in C \wedge \gamma_w \wedge \forall \beta[\beta \in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

b. $[\![at\ most(C)]\!]^{w,c} = \lambda \alpha_{<st>}.\exists \gamma[\gamma \in C \wedge \gamma_w \wedge \forall \beta[\beta \in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$

As shown in (32) and (33), for FRs, the variation base is a set of epistemic alternatives (see (32a); i-alternatives in the sense of Dayal 1997); for EIs, it is the non-singleton domain of individuals. Crucially, the indeterminacy of both types of expressions is located at the level of presuppositions. This may explain why ignorance inferences given by these expressions are difficult to cancel, if not completely impossible.[2] In contrast, it is well-known that ignorance inferences given by SMs have the hallmarks of conversational implicatures and thus are pragmatic (e.g., Coppock and Brochhagen 2013, Kennedy 2015, Schwarz 2016; see also chapters 2-3). As shown in (34), the domain of SMs will always be non-singleton, consisting of the prejacent (obtained by focus presuppositions) and the lower/ higher alternatives (obtained by the superlative component). Crucially, forming a contrast to EIs where the anti-singleton requirement

[1] The semantics of FRs illustrated here is based on von Fintel (2000)'s reformulation of Dayal (1997)'s analysis. Focusing on the nature of indeterminacy demonstrated by FRs, Heller and Wolter (2011) proposes a finer-grained notion of identification that is relativized to sorts. Readers are referred to their paper for more details about the variation requirement of FRs.

[2] According to Luis Alonso-Ovalle (p.c.), ignorance inferences associated with Spanish *algún* do not seem to be cancellable. Recall that as discussed in chapter 2, ignorance inferences of SMs do not arise in the example of TV show, where the maxim of quantity is deactivated. In that context where no ignorance inferences are intended, Spanish *algún* cannot be used; instead, a canonical indefinite *unos/ un* has to be used. As pointed out by Luis Alonso-Ovalle, this fact may be attributed to the pragmatic principle of Maximize Presuppositions (Heim 1991), given that in Spanish, *algún* imposes a non-singleton presupposition while canonical indefinites *unos/ un* do not. Although it remains to be seens whether the same contrast holds for other EIs like German *irgendein* or Italian *uno qualsiasi*, at this point, it seems clear that ignorance inferences associated with SMs may cease to arise in certain contexts (when the maxim of quantity is deactivated), while those associated with Spanish *algún* cannot and thus cannot be used in those contexts where no ignorance inference is intended.

is a presupposition, the non-singleton domain of SMs is part of their truth-conditions. This semantics of SMs together with the pragmatic nature of focus result in the sensitivity of their ignorance inferences to different QUDs.

At this point, it is worth noting that the ignorance reading is not the only possible interpretation of FRs. For example, it has been observed in von Fintel (2000) that FRs can lead to an indifference reading, as shown below.

(35) I grabbed whatever tool was handy. von Fintel (2000: (17))

Because it is highly unlikely that the speaker of (35) does not know what tool she grabbed, the most salient reading of (35) is an indifference reading. In other words, on a par with the EPI-CON ambiguity demonstrated by SMs, an ignorance-indifference ambiguity appears with FRs. This is an intriguing parallel and such parallel raises many questions: Why are SMs and FRs parallel in showing an ambiguity between an ignorance reading and some other reading (the concessive reading of SMs, while the indifference reading of FRs)?[3] Why and how does the other reading arise? How is the other reading connected with the semantics of SMs/ FRs? In this dissertation, I have offered a unified account of the EPI-CON ambiguity shown by SMs. In particular, I have argued that the two readings can be seen as pragmatic variants arising from one core semantic representation interacting with different contextual factors. For FRs, a unified account of the ignorance-indifference ambiguity has been suggested in von Fintel (2000). Very briefly, the idea there is that the indifference reading of FRs arises from the fact that the variation in the denotation of FRs is relative to a set of counterfactual alternatives. Taken together, it seems to me that neither ambiguity appears as a coincidence in natural language. More importantly, for both SMs and FRs, neither reading is derived from the other. Both ambiguities can be traced back to

[3] I thank Veneeta Dayal for drawing my attention to this parallel between SMs and FRs.

one core semantic representation, and then the other reading results from either the addition of certain pragmatic factors (the concessive reading of SMs) or the modest modification of certain semantic component (the indifference reading of FRs).

Finally, I would like to discuss how the current analysis sheds light on the distinction between comparative quantifiers (CQs) and SMs with respect to ignorance inferences, to which most of the previous studies on SMs are dedicated. To begin with, it has been observed that while both CQs and SMs may lead to ignorance inferences, those given by SMs seem more robust than those given by CQs. Consider (36) and (37) below, borrowed from Kennedy (2015).

(36) This airplane has **more than** six emergency exits.

(37) This airplane has **at least** six emergency exits

Uttered out of blue, both (36) and (37) can result in ignorance inferences. However, as pointed out by Kennedy (2015), if (36) is uttered by a flight attendant to not only provide information about emergency exist but also advise how the plane compares to others of a similar type, no ignorance inference arises. In contrast, if (37) is uttered by a flight attendant during the safety demonstration, it is most likely that the attentive passenger would look up in alarm. This is so, because (37) implicates that the flight attendant does not know what the actual number of emergency exists is. (38) below makes the same point. Assuming that the speaker knows what a pentagon is, (38a) is felicitous while (38b) is not.

(38) a. A pentagon has more than three sides.

b. #A pentagon has at least three sides.

The above contrast between CQs and SMs with respect to the robustness of ignorance inferences indicates that there is a non-trivial difference between them.

To the best of my knowledge, Nouwen (2010) is the first study suggesting an insightful distinction between CQs and SMs. In his terminology, the former belongs to the group of Class A modifiers and the latter the group of Class B modifiers, as shown in (39). Nouwen (2010) further proposes that Class A modifiers are degree quantifiers while Class B modifies are not; the latter introduce a bounding property. Although the two classes are internally heterogeneous, Kennedy (2015: (4)) suggests that the distinction between the two groups of modifiers can be understood as in (40):

(39) a. Class A modifiers: *more/ fewer/ less than n*, *over n*, *between n and m*, etc…

b. Class B modifiers: *at least*, *at most*, *up to*, *maximally*, *minimally*, etc…

(40) a. Class A modifiers express exclusive (strict) orderings relative to the modified numeral.

b. Class B modifiers express inclusive (non-strict) orderings relative to the modified numeral.

Seen in this light, the underlying distinction between CQs and SMs can be understood as pointing to what kind of ordering relation is employed in the semantics. Crucially, the current analysis is not only compatible with the suggested distinction but also decompose the non-strict ordering relation further. More specifically, in the case of SMs, the non-strict ordering is not a semantic primitive, but derived from focus presuppositions together with the semantic contribution of a superlative component. However, it remains to be seen exactly how the different ordering relation (strict vs. non-strict) is connected with the contrast between CQs and SMs in the robustness of ignorance inferences. Given the discussion in this section, a possible line of thought would be that the inclusion of the prejacent (due to focus presuppositions) makes the domain of SMs always non-singleton and thus their resulting ignorance inferences would be more robust than those given by CQs, where their domain may not be always non-singleton (cf. Schwarzschild 2002's idea on singleton indefinites).[4]

6.4 Cross-linguistic implications

This section briefly discusses four issues concerning the (cross-linguistic) implications of the current analysis of SMs: **(a)** the issue of lexicalization; **(b)** the quantificational domain hypothesis for the distributional restriction on CON; **(c)** the variety of SMs and their restrictions; **(d)** the definite determiner inside English SMs.

First, let's start with the issue of lexicalization. It has been observed in Grosz (2011) that there are languages employing different items exclusively lexicalizing the epistemic reading or the concessive reading. For example, German is a language that makes the point. In German, *mindestens* exclusively conveys the epistemic reading while *wenigstens* the concessive reading. According to Gorsz (2011: 577-578), (41) shows that *wenigstens* 'at least' does not have an epistemic reading, while *mindestens* 'at least' does; in addition, the use of *wenigstens* in (42) is infelicitous because the concessive meaning conveys that more causalities is better than less causalities.

(41)	Maria	hat	kein	Gold	gewonnen,	aber	**wenigstens /**	**#mindestens**	Silber.
	Maria	has	no	gold	won	but	at.least	at.least	silver

'Maria didn't win gold, but at least she won silver.'

[4] Cremers et al. (2017) presents an experimental study and suggests an interesting but slightly different perspective on the contrast between CQs and SMs with respect to the robustness of ignorance inferences. Roughly, the idea is that ignorance inferences associated with SMs are generated through the maxim of quality and the maxim of quantity, while those associated with CQs are generated only through the maxim of quantity. To my understanding, the proposed pragmatic difference between CQs and SMs with respect to the maxim of quality actually results from their semantic distinction between a strict ordering and a non-strict ordering. Seen in this light, the current semantic proposal of SMs is compatible with their approach, as long as the unified account of the ambiguity can be maintained.

(42) Bei dem Unfall gab es **mindestens / #wenigstens** fünf Tote.

at the accident gave it at.least at.least five casualties

'There were at least five casualties in the accident.'

German is not the only language with dedicated items lexicalizing one of the two readings. Similar issue of lexicalization arises in Romanian and Polish (Grosz 2011). These cross-linguistic facts suggest that the EPI-CON ambiguity may not be universal. In other words, the grammar should leave room for lexicalization.

However, the grammar should also explain why the EPI-CON ambiguity is cross-linguistically pervasive, namely, why the two meanings are conveyed by one single lexical item across languages. Crucially, the unified analysis proposed in this dissertation does NOT rule out the possibility of distinct lexicalization. More specifically, the semantics of English *at least* may not be exactly the same as that of German *mindestens* and *wenigstens*. That said; it is worth emphasizing that the parameter of cross-linguistic variation does NOT lie in the level of languages, but in the level of lexicons. This is evidenced by English. Although *at least* demonstrates the EPI-CON ambiguity, its related expression *at the very least* conveys only the epistemic reading (see also Alrenga 2018), as illustrated below.

(43) a. Numeral Scales (a contextual ranking: $4 \succ 3 \succ 2$)

John **at least** wrote [three]$_F$ novels. √**EPI**, √**CON**

b. Plurality Scales (a contextual ranking: adam⊕bill⊕chris ≻adam⊕bill ≻adam)

John **at least** hired [Adam and Bill]$_F$. √**EPI**, √**CON**

c. Lexical Scales (a contextual ranking: gold medal≻silver medal≻bronze medal)

John **at least** got a [silver]$_F$ medal. √**EPI**, √**CON**

d. Pragmatic Scales (a contextual ranking: cherries $\succ$apples $\succ$bananas)

John **at least** bought [apples]$_F$. √**EPI**, √**CON**

(44) a. Numeral Scales (a contextual ranking: 4 $\succ$3 $\succ$2)

At the very least, John wrote [three]$_F$ novels. **EPI**

b. Plurality Scales (a contextual ranking: adam⊕bill⊕chris $\succ$adam⊕bill $\succ$adam)

At the very least, John hired [Adam and Bill]$_F$. **EPI**

c. Lexical Scales (a contextual ranking: gold medal$\succ$silver medal$\succ$bronze medal)

At the very least, John got a [silver]$_F$ medal. **EPI**

d. Pragmatic Scales (a contextual ranking: cherries $\succ$apples $\succ$bananas)

At the very least, John bought [apples]$_F$. **EPI**

What (43) and (44) demonstrate is that in one single language like English, having an expression showing the EPI-CON ambiguity does not rule out the possibility of having another expression lexicalizing one of the two readings.[5] This means that the parametric distribution of the two meanings vary from one lexicon to another, not (necessarily) between languages.[6] Seen in this light, a unified analysis of the two meanings is needed ultimately, in order to explain how the EPI-CON ambiguity arises (recall that Chinese makes a case where multiple lexical items in one single language all demonstrate the ambiguity) and why it is so pervasive across languages.

Second, based on empirical facts about English and Chinese SMs, I have pursued the quantificational domain hypothesis as an explanation for the distributional

[5] Gorsz (2011: 578) mentions that Russian is another language with lexical differentiation of the two readings. However, Vera Gor (p.c.) suggests that Russian does have some lexical item showing the EPI-CON ambiguity. Given the contrast between *at least* and *at the very least* in English that we have just seen, I fully expect that Russian may be a language with some expressions showing the EPI-CON ambiguity while some exclusively conveying one of the two readings.

[6] Under this perspective, future studies should take a closer look at Romanian and Polish to see whether they have expressions showing the EPI-CON ambiguity, because these two languages have been claimed to have dedicated items exclusively for one of the two meanings. To my knowledge and based on my informants, German is the only language without any expression showing the ambiguity.

restriction on the concessive reading. Recall that the idea is that the concessive meaning of SMs requires a propositional domain because what's evaluated is actually a set of contextually relevant but different circumstances (i.e., propositions). A syntactic reflex of this semantic-pragmatic requirement then is that the concessive reading of SMs is only available when SMs adjoin to a syntactic constituent with propositional content. This seems to correctly capture the facts concerning English and Chinese SMs. It is worth noting that the quantificational domain hypothesis pursed here crucially relies on the assumption that focus adverbs are interpreted at the place where they are merged in the structure. Put differently, it is not the case that focus adverbs always undergo quantifier-raising (QR) and thus take a clausal scope (contra Rooth1985). Along this line of thought, because the hypothesis ties the syntax of SMs closely with their semantics-pragmatics, there are at least two factors potentially leading to the apparent absence of the distributional restriction as observed in English/ Chinese: **(a)** the syntax of focus adverbs in a given language; **(b)** the QR of focus adverbs (or other types of syntactic movement). For example, Japanese makes the case at hand, as demonstrated in (45). Descriptively speaking, Japanese *sukunaku-temo* 'at least' seems to be only available at two syntactic positions (see 45a and 45b) and the two readings are both available at the two syntactic positions.

(45) Japanese

a. **Sukunaku-temo** John-wa [san]$_F$-ko ringo-o kat-ta √**EPI**, √**CON**
few-even.if John-Top three-CL apple-Acc buy-Past
'At least John bought three apples.'

b. John-wa **sukunaku-temo** [san]$_F$ -ko ringo-o kat-ta √**EPI**, √**CON**
John-Top few-even.if three-CL apple-Acc buy-Past

c.? John-wa [san]$_F$-ko **sukunaku-temo** ringo-o kat-ta
John-Top three-CL few-even.if apple-Acc buy-Past

d.? John-wa [san]$_F$-ko ringo-o **sukunaku-temo** kat-ta
John-Top three-CL apple-Acc few-even.if buy-Past

e.*? John-wa [san]$_F$ -ko ringo-o kat-ta **sukunaku-temo**
John-Top three-CL apple-Acc buy-Past few-even.if

Obviously, the pattern of Japanese *sukunaku-temo* seems very different from those of English *at least* and Chinese *zuishao/ zhishao/ qima* 'at least'. However, Japanese is a well-known language allowing for syntactic scrambling. It may well be that some syntactic operations are involved in (45a) or (45b), thus rendering the concessive reading available in both cases. Of course, without a thorough examination of Japanese data, this is only a speculation. However, the point here is that in extending the quantificational domain hypothesis to other languages, future studies should be careful about how the syntax of SMs works in a given language and whether some syntactic operations are involved, thus changing the scope of SMs in a given example.

At this point, in addition to the two syntactic factors, still another possible factor potentially leading to the absence of the distributional restriction on CON concerns the distinction between constituent focus particles versus sentential focus particles, as discussed in chapter 1 (section 1.3). Though unrelated to the quantificational domain hypothesis pursued here, the apparent distributional restriction of CON observed in English/ Chinese is attributed to the fact that only sentential focus particles conveying CON are available and constituent focus particles conveying CON are *accidentally* missing in these two languages. On this view, the apparent distributional restriction on CON is simply because of the accidental absence of certain lexical items in the inventory. Although I am skeptical about this explanation for English/ Chinese because it runs foul of any attempt to unify the two meanings (such as the analysis in this dissertation), I believe that it may well be the case for other languages where one of the two meanings is lexicalized in a dedicated item, as in German.

Third, for purposes of this dissertation, the attention has been restricted to two particular items of English SMs: *at least* and *at most*. As observed in Coppock (2016), the pattern of English SMs is quite productive: *at (the) best*, *at (the) worst*, *at the earliest*, *at (the) latest*, *at the longest*, *at the highest*, *at the lowest*, and *at the fastest*. The following examples, borrowed from Coppock (2016), illustrate the point.

(46) The officers have arguments which I feel are fatuous, fallacious, erroneous and **at best** equivocal.

(47) I hope to be back in action for the second race or, **at the worst**, the third.

(48) I must be away by eleven **at the latest**, though.

(49) A final decision is not expected until the end of the month **at the earliest**.

(50) Capital would return 15 percent **at the lowest**.

(51) These groups move much more slowly than trains—only several miles per hour **at the fastest**.

Despite the productivity, the pattern of English SMs is not without restrictions. There are at least two restrictions. First, Coppock (2016) suggests that SMs only modify expressions that occupy a position on the scale named by the gradable predicate. This is evidenced by the ill-formedness of (52) and (53), where Sarah is not an age and no table is a length.

(52) *You may invite Sarah **at the oldest**.

Intended: You may invite (someone as old as) Sarah or someone younger.

(53) *The room will fit this table **at the longest**.

Intended: The room will fit this table or something shorter

Second, not every gradable adjective participates in the pattern of English SMs. It seems difficult to construct examples with SMs in (54). Moreover, the pattern of English SMs seems restricted to those (monosyllabic or bisyllabic) adjectives that are compatible with the bound superlative morpheme *–est*, as shown in (55).

(54) (?)at the smartest, (?)at the happiest, (?)at the wettest, (?)at the easiest, etc…

(55) #at the most expensive, #at the most beautiful, #at the most confortable, etc...

None of these restrictions are addressed in my current analysis of SMs. An observation here is that most adjectives participating in the pattern of SMs seem compatible with measure phrases. In particular, many of them are dimensional adjectives (in the sense of Bierwisch 1989, e.g., *long*, *wide* and *high*). This observation may bring together the two restrictions as to whether a conventionalized measure unit is utilized in the measurement. Of course, a deeper question is why. Moreover, pairs of SMs like *at best/ at worst* and *at least/ at most* would constitute some notorious exceptions. In the future, more studies are needed in order to better understand why those restrictions on the construction of SMs should exist and how they are connected with the morpho-semantics of SMs.

Finally, as illustrated above, some acute readers may have noticed that most SMs in English require the presence of the definite determiner *the* inside them, except for *at best/ at worst* and *at least/ at most*. The obligatory presence of *the* in English SMs is intriguing and is not captured under the current analysis. In addition to *the*, Coppock (2016: footnote 4) observes that possessives may be involved in English SMs. Below, (56) illustrates her point.

(56) at his/ her heaviest, at his/ her earliest, at his/ her fastest, at his/ her worst, etc…

The current analysis proposed in this dissertation captures the parallel between SMs and disjunction, and that between SMs and epistemic indefinites by analyzing SMs as an existential quantifier whose domain is always non-singleton, obtained by focus presuppositions together with the superlative component. For future studies, it would be very interesting to see how to factor the contribution of the definite article *the* and possessives into the semantics of SMs, while maintaining the two types of parallel.

6.5 Conclusion

This dissertation has studied an ambiguity generally demonstrated by SMs across languages: an epistemic reading (EPI) conveying speaker ignorance and a concessive reading (CON) conveying speaker concession. In this work, I have argued for a uniform semantic representation of those SMs (e.g., English *at least* and Chinese *zuishao*, *zhishao, qima*) showing the EPI-CON ambiguity in natural language. In particular, I have suggested that the semantic core of the two meanings is scalarity and the notion of scalarity can be understood as (57). Furthermore, I have proposed that a uniform semantic representation of the two meanings should encode scalarity.

(57) **Scalarity** (the semantic core of EPI and CON)

The set of focus alternatives (the set of answers addressing the CQ) is ordered along a contextually given scale.

In chapter 2, building on Nakanishi and Rullmann (2009)'s observations and Biezma (2013)'s insights, I took English *at least* as a case study and proposed that the two meanings can be pragmatic variants in natural language and the EPI-CON ambiguity arises from one single semantic entry combining with different pragmatic factors such as informativity and evaluativity. Furthermore, three novel observations on the parallel between the two meanings were presented: both meanings are focus-sensitive, compatible with various scales and demonstrate two scalar effects.

The core ingredients for the unified analysis of the EPI-CON ambiguity shown by English *at least* and Chinese *zuishao*/ *zhishao*/ *qima* are repeated below.

(58) **A propositional version of *at least***

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$$

(59) **A non-propositional version (by the Geach rule)**

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\ st>}\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

(60) **A non-propositional version (by the backward Geach rule)**

$$[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\lambda P_{<\eta,\ st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) < \mu_c(P_w(\beta))]]$$

(61) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

(62) **CON and the issue of evaluativity**

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is true; while it is not the best situation, it is not the worst situation either.

In chapter 3, I took English as a case study and discussed in detail the discourse properties of concessive *at least* and epistemic *at least*. In the spirit of the unified account proposed in chapter 2, I presented a formal analysis and illustrated how concessive *at least* and epistemic *at least* are pragmatic variants. The central idea there was that although concessive *at least* and epistemic *at least* share one single semantic representation (as proposed in chapter 2), they have different discourse profiles. Adopting the discourse model (i.e., the conversational scoreboard, incorporating the insights from Lewis 1979) presented by Farkas and Bruce (2010) and further developed in Malamud and Stephenson (2015) and Beltrama (2018), it was argued the discrepancy in their discourse profiles crucially comes from where propositions are added in the conversational scoreboard. More specifically, assertions with concessive *at least* add associated propositions to the speaker's present commitments, while assertions with epistemic *at least* add associated propositions to the speaker's projected commitments. Furthermore, in this chapter, a number of issues were discussed in detail: What is put forward in the discourse under assertions with concessive/ epistemic *at least*? What is placed on the Table under assertions with concessive/ epistemic *at least*? What are the participants' discourse commitments under assertions with concessive/ epistemic *at least*? What is updated/ added to the discourse under assertions with concessive/ epistemic *at least*? How are assertions with concessive/ epistemic *at least* different from (factual) assertions? Finally, the connection between the unified semantic representation of *at least* (proposed in chapter 2) and their different discourse profiles under the two readings (i.e., ignorance vs. concession) was also discussed in this chapter.

The core of my proposal for the discourse profile of concessive *at least* and epistemic *at least* are repeated below.

(63) (CG_i represents the input CG;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts $\neg q$ and then asserts p with concessive *at least*

	Before the move	After the move
DC_A	{ }	$\{\{\neg q\}, \{p\}\}$
DC_A*	{ }	{ }
DC_B	{ }	{ }
Table	< >	$\langle\{\neg q\}, \{p\}\rangle$
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$(CG_i \cap \{q \succ_c p\}) \cap \{\neg q\} \cap \{p\}$

(64) (CG_i represents the input CG and $\{\neg q\}$ is part of CG_i;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts p with concessive *at least*

	Before the move	After the move
DC_A	{ }	$\{p\}$
DC_A*	{ }	{ }
DC_B	{ }	{ }
Table	< >	$\langle\{p\}\rangle$
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$(CG_i \cap \{q \succ_c p\}) \cap \{p\}$

(65) (CG_i represents the input CG;

$q \succ_c p$ represents that q is ranked higher than p in context c)

A asserts p with epistemic *at least*

	Before the move	After the move
DC_A	{ }	{ }
DC_A*	{ }	$\{\{q\}, \{p\}\}$
DC_B	{ }	{ }
Table	< >	$<\{q\}, \{p\}>$
CG	CG_i	$CG_i \cap \{q \succ_c p\}$
CG*	CG_i	$\{(CG_i \cap \{q \succ_c p\}) \cap \{q\}, (CG_i \cap \{q \succ_c p\}) \cap \{p\}\}$

In chapter 4, I extended the unified analysis in chapter 2 by taking English *at most* as a case study and proposed a uniform semantic representation of *at most*. In particular, it has been shown that *at most*, like *at least*, delivers the EPI-CON ambiguity and the two meanings are pragmatic variants resulting from one unified semantic entry interacting with different pragmatic factors such as informativity and evaluativity. Furthermore, three contrasts between *at most* and *at least* were discussed: First, *at most* shows a mirror image of *at least* with respect to the discrepancy between the two scalar effects TSE and BSE. Second, the concessive reading of *at most* looks like an epiphenomenon, while that of *at least* does not. Third, concessive *at most* gives rise to a "settle-for-less" flavor slightly different than that given by concessive *at least*. Crucially, these three contrasts are not arbitrary, but systematic: all the three contrasts result from different semantic bounding properties of *at most* and *at least*.

The core ingredients for the unified analysis of the EPI-CON ambiguity shown by English *at most* and Chinese *zuiduo* are repeated below.

(66) **A propositional version of *at most***

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$$

(67) **A non-propositional version (by the Geach rule)**

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta,\ st>}\ \lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$$

(68) **A non-propositional version (by the backward Geach rule)**

$$[\![at\ most(C)]\!]^{w,c} = \lambda\alpha_{<\eta>}\ \lambda P_{<\eta,\ st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$$

(69) **EPI and the issue of informativity**

Ignorance inferences arise pragmatically to justify the failure of providing the maximally informative unique answer in a given discourse.

(70) **CON and the issue of evaluativity**

Given the set of alternatives evaluated and ranked in the discourse, the prejacent is the best situation.

In chapter 5, I shifteded the attention to a longstanding and intriguing morpho-semantic puzzle posed by SMs: Why do SMs morphologically involve a quantity adjective and the superlative morpheme? What is the role of quantity adjective and superlative morpheme inside SMs? How are these morphological pieces connected with the semantics of SMs? In this respect, Chinese makes the situation even more puzzling: the same expressions (morphologically consisting of a quantity adjective and superlative morpheme) *zui-duo* ‘at most’ and *zui-shao* ‘at least’ are used as superlative modifiers and quantity superlatives. I took Chinese as a case study and presented a decompositional analysis of the proposed semantics in chapters 2 and 4.

The core ingredients for the compositional mapping of Chinese SMs between form and meaning are repeated below, where α ranges over the elements induced by focus and η represents the semantic type of those elements.

(71) $[\![duo]\!]^{c} = \lambda\alpha.\mu_c(\alpha)$ $\quad <\eta, d>$

(72) $[\![\text{Comp}^{+}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_c(\alpha) > \mu_c(\beta)$ $\quad <\eta, <\eta, t>>$

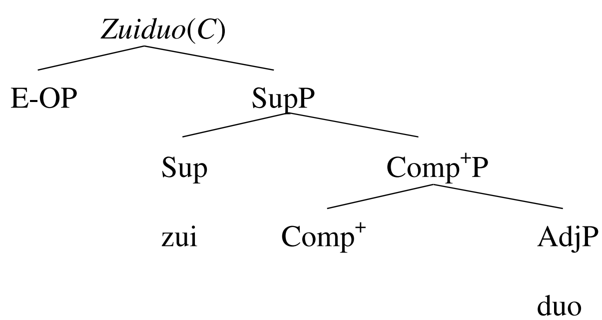
(73) $[\![\text{Comp}^{-}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_c(\alpha) < \mu_c(\beta)$ $\quad <\eta, <\eta, t>>$

(74) $[\![zui]\!]^{c} = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

(75) $[\![\text{E-OP}]\!]^{w, c} = \lambda SUP_{<<st, t>, <st, t>>}\lambda C_{<st, t>}\ \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge SUP\ (C, \alpha)]$

The following summarizes my decompositional proposal for Chinese SM *zuiduo*.

(76) The internal structure of Chinese SM *zuiduo* at LF



(77) A propositional version

$[\![zuiduo(C)]\!]^{w, c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$

(78) A non-propositional version (by the Geach rule)

$[\![zuiduo(C)]\!]^{w, c} = \lambda\alpha_{<\eta, st>}\ \lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge$

$\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) > \mu_c(\beta(P))]]$

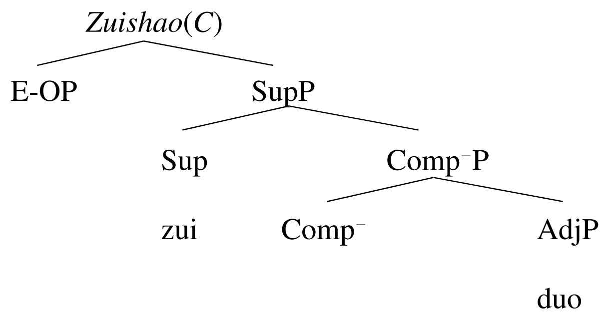
(79) A non-propositional version (by the backward Geach rule)

$[\![zuiduo(C)]\!]^{w, c} = \lambda\alpha_{<\eta>}\ \lambda P_{<\eta, st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge$

$\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) > \mu_c(P_w(\beta))]]$

The following summarizes my decompositional proposal for Chinese SM *zuishao*.

(80) The internal structure of Chinese SM *zuishao* at LF



(81) A propositional version

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$$

(82) A non-propositional version (by the Geach rule)

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta,\,st>}\,\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$$

(83) A non-propositional version (by the backward Geach rule)

$$[\![zuishao(C)]\!]^{w,\,c} = \lambda\alpha_{<\eta>}\,\lambda P_{<\eta,\,st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P_w(\alpha)) < \mu_c(P_w(\beta))]]$$

Bibliography

Al Kahtib, Sam. 2013. Only and association with negative antonyms. Doctoral Dissertation, Massachusetts Institute of Technology.

Alrenga, Peter. 2018. *Epistemic vs. Concessive at least: the least at least can do*. Handout of talk given at the Workshop on Evaluative meanings: Theoretical and Computaitonal perspectives, University of Stuttgart.

Alonso-Ovalle, Louis, and Paula Menendez-Benito. 2010. Modal indefinites. *Natural Language Semantics* 18: 1−31.

Alonso-Ovalle, Louis, and Paula Menendez-Benito. 2013a. Two views of epistemic indefinites. *Language and Linguistic Compass*: 105-122.

Alonso-Ovalle, Louis, and Paula Menendez-Benito. 2013b. Modal determiners and alternatives: quantity and ignorance effects. In *Proceedings of Semantics and Linguistic Theory 23*.

AnderBois, Scott, Adrian Brasoveanu and Robert Henderson. 2010. Crossing the appositive / at-issue meaning boundary. In *Proceedings of Semantics and Linguistic Theory 20*, N. Li & D. Lutz (eds.), 328−346.

AnderBois, Scott, Adrian Brasoveanu and Robert Henderson. 2015. At-issue proposal and appositive impositions in discourse. *Journal of Semantics* 32: 93−138.

Barker, Chris. 2007. Parasitic scope. *Linguistics and Philosophy* 30(4): 407−444.

Beaver, David I., and Brady Z. Clark. 2008. *Sense and sensitivity: how focus determines meaning*. Malden, MA: Wiley-Blackwell.

Beltrama, Andrea. 2018. Totally between subjectivity and discourse. Exploring the pragmatic side of intensification. *Journal of Semantics* 35: 219−261.

Bierwisch, Manfred. 1989. The semantics of gradation. In Manfred Bierwisch and Ewald Lang (eds.), *Dimensional adjectives: Grammatical structure and conceptual interpretation*, 71－261. Springer-Verlag.

Biezma, Maria. 2013. Only one *at least*. In *Proceedings of Penn Linguistics Colloquium*, (ed.) Kobey Shwayder, volume 36, 12–19.

Biezma, Maria, and Kyle Rawlins. 2012. Responding to alternative and polar questions. *Linguistics and Philosophy* 35(5): 361－406.

Biezma, Maria, and Kyle Rawlins. 2017a. Rhetorical questions: Severing questioning from asking. In *Proceedings of Semantics and linguistic theory 27*, Dan Burgdorf & Jacob Collard (eds.), LSA and CLC Publications.

Biezma, Maria, and Kyle Rawlins. 2017b. Or what?. *Semantics and Pragmatics*.

Blok, Dominique. 2015. Scope interactions between modals and modified numerals. Amsterdam Colloquium, ILLC University of Amsterdam

Bobaljik, Jonathan. 2012. *Universals in comparative morphology*. Cambridge, MA: MIT Press.

Bumford, Dylan. 2017. Split-scope definites: Relative superlatives and Haddock descriptions. *Linguistics and Philosophy* 40(6): 549－593.

Büring, Daniel. 2003. On D-trees, beans, and B-accents. *Linguistics and Philosophy* 26: 511－545.

Büring, Daniel. 2007. Cross-polar nomalies. In *Proceedings of Semantics and Linguistic Theory 17*, Tova Friedman and Masayuki Gibson (eds.), 37–52. Ithaca, NY: Cornell University.

Büring, Daniel. 2008. The least *at least* can do. In *Proceedings of West Coast Conference on Formal Linguistics (WCCFL) 26*, 114–120.

Büring, Daniel, and Katharina Hartmann. 2001. The syntax and semantics of focus-sensitive particles in German. *Natural Language and Linguistic Theory* 19: 229–281.

Bresnan, Joan. 1973. Syntax of the comparative clause construction in English. *Linguistic Inquiry, 4*(3), 275–343.

Buccola, Brian, and Benjamin Spector. 2016. Modified numerals and maximality. *Linguistics and Philosophy* 39: 151－199.

Buccola, Brian, and Andreas Haida. 2016. A surface-scope analysis of authoritative readings of modified numerals. In *Proceedings of Sinn und Bedeutung 21*.

Buccola, Brian, and Andreas Haida. 2017. Obligatory irrelevance and the computation of ignorance inferences. Manuscript.

Bylinina, Lisa, and Rick Nouwen. 2018. On zero and semantic plurality. To appear in *Glossa: A Journal of General Linguistics*.

Charlow, Simon. 2014. *On the semantics of exceptional scope*. Doctoral Dissertation, New York University.

Chen, Liping. 2008. *DOU: distributivity and beyond*. Doctoral Dissertation, Rutgers University, New Brunswick.

Chierchia, Gennaro. 1998. Reference to kinds across language. *Natural language semantics* 6:339–405.

Chierchia, Gennaro, and Sally McConnell-Ginet. 2000. *Meaning and grammar*. Cambridge, MA: MIT Press.

Chierchia, Gennaro. 2006. Broaden your views: Implicatures of domain widening and the logicality of language. *Linguistic Inquiry* 37: 535−590.

Chierchia, Gennaro. 2013. *Logic in grammar: Polarity, free choice, and intervention*. Oxford University Press, Oxford, UK.

Chierchia, Gennaro, Danny Fox, and Benjamin Spector. 2012. Scalar implicature as a grammatical phenomenon. In *Semantics: An international handbook of natural language meaning*, volume 3, pages 2297−2331. Berlin & Boston: de Gruyter.

Ciardelli, Ivano, Floris Roelofsen, and Nadine Theiler. 2017. Composing alternatives. *Linguistics and Philosophy* 40(1): 1−36.

Cohen, Ariel. 2001. Relative readings of many, often and generics. *Natural Language Semantics* 9: 41−67.

Cohen, Ariel, and Manfred Krifka. 2014. Superlative quantifiers and modifiers of meta-speech acts. *Linguistics and Philosophy* 37: 41−90.

Condoravdi, Cleo. 2008. *Free choice and uncertainty*. Handout of talk given at the Workshop on Inferential Mechanisms and their Linguistic Manifestations, University of Kyoto.

Coppock, Elizabeth. 2016. Superlative modifiers as modified superlatives. In *Proceedings of Semantics and Linguistic Theory 26*, University of Texas at Austin.

Coppock, Elizabeth, and David Beaver. 2013. Principles of the exclusive muddle. *Journal of Semantics* 31: 371 – 432.

Coppock, Elizabeth, and David Beaver. 2014. A superlative argument for a minimal theory of definiteness. In *Proceedings of Semantics and Linguistic Theory 24*, New York University.

Coppock, Elizabeth, and Thomas Brochhagen. 2013. Raising and resolving issues with scalar modifiers. *Semantics and Pragmatics* 6: 1–57.

Coppock, Elizabeth, Ivano Ciardelli, Floris Roelofsen. 2016. *Implicatures of modifed numerals: quality or quantity?* Handout of *Sub21*. University of Edinburgh, UK.

Cremers, Alexandre, and Elizabeth Coppock, and Floris Roelofsen. 2017. Modified numerals: two routes to ignorance. Manuscript.

Cummins, Chris, and Napoleon Katsos. 2010. Comparative and superlative quantifiers: Pragmatic effects of comparison type. *Journal of Semantics* 27(3): 271–305.

Davis, Christopher, Christopher Potts, and Margaret Speas. 2007. The pragmatic values of evidential sentences. In *Proceedings of Semantics and Linguistic Theory 17,* T. Friedman and M. Gibson (eds), 71-88, Ithaca, NY: Cornell University

Dayal, Veneeta. 1996. *Locality in Wh Quantification: Questions and Relative Clauses in Hindi*. Dordrecht: Kluwer.

Dayal, Veneeta. 1997. Free relatives and ever: identity and free choice readings. In *Proceedings of Semantics and Linguistic Theory 7*, A. Lawson (ed.), 99–116. Ithaca, NY: CLC Publications.

Dayal, Veneeta. 2016. *Questions*. Oxford: Oxford University Press.

Dubar, Ewan and Alexis Wellwood. 2016. Addressing the "two interface" problem: comparatives and superlatives. *Glossa: A Journal of General Linguistics* 1(1): 1–29.

Erlewine, Michael Yoshitaka. 2014. *Movement out of focus*. Doctoral Dissertation, Massachusetts Institute of Technology.

Erlewine, Michael Yoshitaka. 2017. Vietnamese focus particles and derivation by phase. *Journal of East Asian Linguistics* 26(4): 325–349.

Farkas, Donka, and Katalin É. Kiss. 2000. Comparative and absolute readings of superlatives. *Natural Language and Linguistic Theory* 18: 417−455.

Farkas, Donka, and Kim B. Bruce. 2010. On reacting to assertations and polar questions. *Journal of Semantics* 27: 81–118.

von Fintel, K. 1994. *Restrictions on Quantifier Domains*. Doctoral Dissertation, University of Massachusetts, at Amherst..

von Fintel. 2004. Would You Believe It? The King of France is Back! (Presuppositions and Truth-Value Intuitions). In *Descriptions and Beyond*, 315−341. Oxford University Press, Oxford

von Fintel 2000. Whatever. In *Proceedings of Semantics and Linguistic Theory 10*, B. Jackson and T. Matthews (eds.), 27–40. Ithaca, NY: CLC Publications.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In *Presupposition and implicature in compositional semantics*, Sauerland, U. and Stateva, P. (eds), pages 71−120. Palgrave Macmillan.

Fox, Danny. 2012. The semantics of questions. Class notes, MIT.

Fox, Danny. 2014. Cancelling the maxim of quantity: another challenge to Gricean theory of scalar implicatures. *Semantics and Pragmatics* 7: 1−20.

Fox, Danny, and Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19: 87–107.

Geurts, Bart. 2010. *Quantity implicatures*. Cambridge University Press

Geurts, Bart, and Rick Nouwen. 2007. *At least* et al.: The semantics of scalar modifiers. *Language* 83(3): 533–559.

Greenberg, Yael. 2016. A novel problem for the likelihood-based semantics of *even*. *Semantics and Pragmatics* 9: 1–28.

Greenberg, Yael. 2018. A revised, gradability-based semantics for *even*. *Natural Language*

Semantics 26(1): 51–83.

Grimshaw, Jane. 2005. Extended Projections. In *Words and Structure*. CSLI Lecture Notes Number 151, Stanford, CA: CSLI Publications.

Groenendijk, Jeroen. 1999. The logic of interrogation. In *Proceedings of Semantics and Linguistic Theory* 9, T. Matthews & D. Strolovitch (eds), pp.109-126, Ithaca, NY: CLC Publications.

Groenendijk, Jeroen, and Martin Stokhof. 1984. *Studies on the Semantics of Questions and the Pragmatics of Answers*. Doctoral Dissertation, Universiteit van Amsterdam.

Grosz, Patrick. 2011. A Uniform Analysis for Concessive *At Least* and Optative *At Least*. In *Proceedings of Semantics and Linguistic Theory 21*, Rutgers University.

Hackl, Martin. 2000. *Comparative quantifiers*. Doctoral Dissertation, Massachusetts Institute of Technology.

Hackl, Martin. 2009. On the grammar and processing of proportional quantifiers: most versus more than half. *Natural Language Semantics* 17: 63−98.

Hacquard, Valentine. 2006. *Aspects of modality*. Doctoral Dissertation, Massachusetts Institute of Technology.

Hamblin, Charles L. 1973. Questions in Montague English. *Foundations of Language* 10(1): 41−53.

Heim, Irene. 1985. Notes on comparatives and related matters. Manuscript, University of Texas, Austin.

Heim, Irene. 1991. Artikel und Definitheit. In *Semantics: An International Handbook of Contemporary Research*. 487–535: Berlin: de Gruyter.

Heim, Irene. 1999. Notes on superlatives. Manuscript, MIT, Cambridge.

Heim, Irene. 2001. Degree operators and scope. In *Proceedings of Semantics and Linguistic Theory* 10, Brendan Jackson & Tanya Mathews (eds.), 40−64.

Heim, Irene. 2006a. Little. In *Proceedings of Semantics and Linguistic Theory 16*, Masayuki Gibson and Johnathan Howell (eds.), 35−58.

Heim, Irene. 2006b. Remarks on comparative clause as generalized quantifiers. Ms., MIT, Cambridge.

Heller, Daphna, and Lynsey Wolter. 2011. On identification and transworld identity in natural language: the case of *–ever* free relatives. *Linguistics and Philosophy* 34: 169−199.

Herberger, Elena. 1997. Focus and weak noun phrases. *Natural Language Semantics* 5:53–78.

Huang, C.-T. James. 1982. *Logical relations in Chinese and the theory of grammar*. Doctoral Dissertation, Massachusetts Institute of Technology.

Huang, C.-T. James, Y.-H. Audrey Li, and Yafei Li. 2009. *The Syntax of Chinese*. Cambridge: Cambridge University Press.

Jackendoff, Ray. 1972. *Semantic interpretation in generative grammar*. Cambridge: MIT Press.

Jacobson, Pauline. 1999. Toward a variable-free semantics. *Linguistics and Philosophy* 22: 117−185.

Jacobson, Pauline. 2016. The short answer: implications for direct compositionality (and vice versa). *Language* 92: 331−375.

Kay, Paul. 1992. At least. In A. Lehrer & E. F. Kittay (eds.), *Frames, Fields, and Contrasts: New Essays in Semantic and Lexical Organization*. Hillsdayle, NJ: Lawrence Erlbaum Associates, 309-331.

Kennedy, Christopher. 1999. *Projecting adjectives: the syntax and semantics of gradability and comparison*. New York: Garland.

Kennedy, Christopher. 2007. Vagueness and grammar: The semantics of relative and absolute gradable predicates. *Linguistics and Philosophy* 30: 1-45.

Kennedy, Christopher. 2015. A "de-fregean" semantics (and neo-gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8: 1–44.

Kennedy, Christopher, and Louise McNally. 2005. Scale structure, degree modification, and the semantic typology of gradable predicates. *Language* 81: 345-381.

Klein, Ewan. 1980. A semantics for positive and comparative adjectives. *Linguistics and Philosophy* 4: 1−45.

Klein, Ewan. 1982. The interpretation of adjectival comparatives. *Journal of Linguistics*

18: 113–36.

Koev, Todor. 2012. On the information status of appositive relative clauses. In *Proceedings of Amsterdam Colloquium 18*, 401–410, Berlin: Springer.

Koev, Todor. 2013. *Apposition and the Structure of Discourse*. Doctoral Dissertation, Rutgers University, New Brunswick.

Kratzer, Angelika. 1996. Severing the external argument from its verb. In J. Rooryck, & L. Zaring (eds.), *Phrase structure and the lexicon* (pp. 109–137). Dordrecht: Kluwer Academic Publishers.

Kratzer, Angelika. and Junko, Shimoyama. 2002. Indeterminate Pronouns: The View from Japanese. In Yukio Otsu (ed.), *The Proceedings of the Third Tokyo Conference on Psycholinguistics*, pp. 1–25. Hituzi Syobo, Tokyo.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics'. In Renate Bartsch, Johan van Benthem & Peter von Emde Boas (eds.), *Semantics and Contextual Expressions*. Foris. Dordrecht. 75–115.

Krifka, Manfred. 1995. Common Nouns: A Contrastive Analysis of Chinese and English. In G. Carlson and J. Pelletier (eds), *The Generic Book*, University of Chicago Press, Chicago.

Krifka, Manfred. 1999. *At least* some determiners aren't determiners. In *The semantics/ pragmatics interface from different points of view*, ed. K. Turner, volume 1. Elsevier Science B.V.

Krifka, Manfred. 2014. Embedding illocutionary acts. In *Recursion, Complexity in Cognition (Studies in Theoretical Psycholinguistics 43)*, Tom Roeper & Margaret Speas (eds.), 125–155. Berlin: Springer.

Krifka, Manfred, Francis Jeffry Pelletier, Gregory N. Carlson, Alice ter Meulen, Godehard Link and Gennaro Chierchia. 1995. Genericity: an introduction. In G. Carlson and J. Pelletier (eds.), *The Generic Book*. University of Chicago Press. Chicago. 1–124.

Lassiter, Daniel. 2017. *Measurement and modality: The scalar basis of modal semantics*. Oxford University Press.

Lechner, Winfried, and Nobert Corver. 2017. Comparative deletion and comparative subdeletion. In *The Blackwell Companion to Syntax*, Martin Everaert & Henk van Riemsdijk (eds.), Malden: Blackwell.

Lewis, David. 1979. Scorekeeping in a language game [Reprinted 2002]. In P. Portner & B. H. Partee (eds.), *Formal Semantics: The Essential Readings*. Blackwell. Oxford. 162–177.

Lin, Jowang. 1998. Distributivity in Chinese and its implications. *Natural Language Semantics* 6: 201-243.

Lin, Jonah Tzong-Hong. 2001. *Light Verb Syntax and The Theory of Phrase Structure*. Doctoral Dissertation, University of California, Irvine.

Link, Godehard. 1983. The logical analysis of plurals and mass terms. In R. Bauerle, C. Schwarze and A. von Stechow (eds.), *Meaning, use and interpretation of language*, 302–323. De Gruyter.

Liu, Luther Chen-Sheng. 2018. Projecting adjectives in Chinese. *Journal of East Asian Linguistics* 27: 67−109.

Liu, Mingming. 2016. *Varieties of alternatives*. Doctoral Dissertation, Rutgers University, New Brunswick.

Mayr, Clemens. 2013. Implicatures of modified numerals. In *From grammar to meaning:the spontaneous logicality of language*, (ed.), Carlo Caponigro, Ivan and Cecchetto, 139– 171. Cambridge University Press

Malamud, Sophia, and Tamina Stephenson. 2015. Three ways to avoid commitments: declarative force modifiers in the conversational scoreboard. *Journal of Semantics* 32: 275−331.

McCready, Eric. 2008. What man does. *Linguistics and Philosophy* 31(6): 671–724.

McCready, Eric. 2010. Varieties of conventional implicature. *Semantics and Pragmatics* 3: 1–57.

Mendia, Jon Ander. 2016a. Focusing on Scales. In *Proceedings of North East Linguistic Society (NELS) 46*, Concordia University, Montreal.

Mendia, Jon Ander. 2016b. Reasoning with partial orders: Restrictions on Ignorance Inferences of Superlative Modifiers. In *Proceedings of Semantics and Linguistic Theory 26*, Univesity of Texas Austin.

Mendia, Jon Ander. 2016c. *Known unknowns*. Manuscript, University of Massachusetts at Amherst.

Merchant, Jason. 2004. Fragments and ellipsis. *Linguistics and Philosophy* 22: 661−738.

Meyer, Marie-Christine, and Uli Sauerland. 2009. A pragmatic constraint on ambiguity detection. *Natural Language and Linguistic Theory* 27: 139–150.

Mihoc, Teodora. 2018. Numerals and their modifiers: how morphology constrains alternatives. Manuscript. Harvard University.

Murray, Sarah E. 2010. *Evidentiality and the structure of speech acts*. Doctoral Dissertation, Rutgers University, New Brunswick.

Murray, Sarah E. 2014. Varieties of update. *Semantics and Pragmatics* 7: 1–53.

Nakanishi, Kimiko. 2007. Measurement in the nominal and verbal domains. *Linguistics and Philosophy* 30: 235–276.

Nakanishi, Kimiko, and Hotze Rullmann. 2009. Epistemic and Concessive Interpretation of at least. In CLA 2009.

Nouwen, Rick. 2010. Two kinds of modified numerals. *Semantics and Pragmatics* 3, 1–41.

Nouwen, Rick. 2015. Modified numerals: The epistemic effect. In Luis Alonso-Ovalle and Paula Menéndez-Benito (eds.), *Epistemic Indefinites: Exploring modality beyond the verbal domain*, Oxford: Oxford University Press.

Novel, Marc, and Maribel Romero. 2010. Movement, Variables and Hamblin Alternatives. In *Proceedings of Sinn und Bedeutung 14*, Martin Prinzhorn, Viola Schmitt & Sarah Zobel (eds.), 322–338.

Pearson, Hazel. 2010. A modification of the‘‘Hey, Wait a Minute’’ test. Snippets 22: 7−8.

Potts, Christopher. 2005. *The Logic of Conventional Implicatures*. Oxford: Oxford University Press.

Potts, Christopher. 2007. The expressive dimension. *Theoretical Linguistics* 33(2): 165–198.

Reis, Marga. 2005. On the syntax of so-called focus particles in German: A reply to B..uring and Hartmann 2001. *Natural Language and Linguistic Theory* 23: 459–483.

Rett, Jessica. 2008. *Degree modification in natural language*. Doctoral Dissertation, Rutgers University, New Brunswick.

Rett, Jessica. 2014. The polysemy of measurement. *Lingua* 143: 242–266.

Rett, Jessica. 2015. The semantics of evaluativity. Oxford: Oxford University Press.

Rizzi, Luigi. 1997. The fine structure of the left periphery. In *Elements of grammar*, Liliane Haegeman (ed), Dordrecht: Kluwer, 281-337

Romero, Maribel. 2015. The conservativity of many. In *Proceedings of the 20th Amsterdam Colloquium*, T. Brochhagen, F. Roelofsen & N. Theiler (eds.), 20−29.

Rooth, Mats. 1985. *Association with focus*. Doctoral Dissertation, University of Massachusetts, at Amherst.

Rooth, Mats. 1992. A theory of focus interpretation. *Natural Language Semantics* 1(1): 75–116.

Rooth, Mats. 1996. Focus. In Lappin, S., editor, *The Handbook of Contemporary Semantic Theory*, pages 271–297. Blackwell Publishers, Oxford.

Roberts, Craige. 1996/ 2012. Information structure in discourse: Towards an integrated formal theory of pragmatics. *Semantics and Pragmatics* 5, 1–69.

Schwarz, Bernhard. 2011. *Remarks on class B numeral modifiers*. Handout of talk given at Workshop on Indefinites and beyond, University Gottingen.

Schwarz, Bernhard. 2016a. Consistency preservation in Quantity implicature: the case of *at least*. *Semantics and Pragmatics* 9: 1–47.

Schwarz, Bernhard. 2016b. At least and ignorance: a reply to Coppock and Brochhagen 2013. *Semantics and Pragmatics* 9: 1–17.

Schwarzschild, Roger. 1996. *Pluralities*. Dordrecht: Kluwer.

Schwarzschild, Roger. 2002. Singleton indefinites. *Journal of Semantics* 19: 289–314.

Schwarzschild, Roger. 2005. Measure phrases as modifiers of adjectives. *Recherches Linguistiques de Vincennes 34*: 207–228.

Schwarzschild, Roger. 2006. The role of dimensions in the syntax of noun phrases. *Syntax* 9: 67–110.

Schwarzschild, Roger. 2008. The semantics of comparatives and other degree constructions. *Language and Linguistics Compass* 2(2): 308–331.

Shan, Chung-chieh. 2004. Binding alongside Hamblin alternatives calls for variable-free semantics. In *Proceedings of Semantics and Linguistic Theory14*, Kazuha Watanabe & Robert B. Young (eds.), 289–304. Ithaca, NY: Cornell University.

Shannon, Benny. 1976. On the two kinds of presuppositions in natural language. *Foundations of Language* 14:247–249.

Sharvit, Yael, and Penka Stateva. 2002. Superlative expressions, context, and focus, *Linguistics and Philosophy* 25: 453−504.

Simons, Mandy, Judith Tonhauser, David Beaver and Craige Roberts. 2010. What projects and why. In *Proceedings of Semantics and Linguistic Theory 20*, N. Li & D. Lutz (eds.), 309–327.

Smeets, Liz, and Michael Wagner. 2018. Reconstructing the syntax of focus operators. *Semantics and Pragmatics*.

Solt, Stephanie. 2009. *The semantics of adjectives of quantity*. Doctoral Dissertation, City University of New York.

Solt, Stephanie. 2015. Q-adjectives and the semantics of quantity. *Journal of Semantics* 32: 221–273.

Sportiche, Dominique. 1988. A theory of floating quantifiers and its corollaries for constituent structure. *Linguistic Inquiry* 19: 425−449.

Spychalska, Maria. 2018. At least not false, at most possible: between truth and assertibility of superlative quantifiers. *Synthese* 195(2): 571–602.

Stalnaker, Robert. 1978. Assertion. In Peter Cole (ed.), *Syntax and Semantics*, New York: Academic Press, volume 9: Pragmatics, 315–332.

Stalnaker, Robert. 1998. On the representation of context. *Journal of Logic, Language and Information*, 7: 3–19.

Stalnaker, Robert. 2002. Common ground. *Linguistics and Philosophy* 25: 701–721.

von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of*

Semantics 3(1): 1–77.

Syrett, Kristen, and Todor Koev. 2015. Experimental evidence for the truth conditional contribution and shifting information status of appositives. *Journal of Semantics* 32: 525−577.

Tomaszewicz, Barbara. 2015. Superlative ambiguities: a comparative perspective. Doctoral Dissertation, University of Southern California.

Tonhauser, Judith. 2012. Diagnosing (not-)at-issue content. In *Proceedings of Semantics of Under-Represented Languages of the Americas 6*, E. Bogal-Allbritten (ed.), 239–254.

Tonhauser, Judith, David Beaver, Craige Roberts, and Mandy Simons. 2013. Toward a taxonomy of projective content. *Language* 89: 66–109.

Tsai, Chengyu. 2016. *Toward a theory of Mandarin quantification*. Doctoral Dissertation, Harvard University.

Weir, Andrew. 2014. Fragments and clausal ellipsis. Doctoral Dissertation, University of Massachusetts, at Amherst.

Wellwood, Alexis. 2014. *Measuring predicates*. Doctoral Dissertation, University of Maryland, College Park.

Wellwood, Alexis. 2015. On the semantics of comparison across categories. *Linguistics and Philosophy* 38: 67–101.

Wellwood, Alexis, Hacquard Valentine and Roumyana Pancheva. 2012. Measuring and comparing individuals and events. *Journal of Semantics* 29(2): 207–228.

Westera, Matthijs, and Adrian Brasoveanu. 2014. Ignorance in context. In *Proceedings of Seamntics and Linguistic Theory 24*, (ed.), Todd Snider, Sarah D'Antonio, and Mia Weigand, volume 24, 414–431.

Westerstahl, 1985. Logical constants in quantifier languages. *Linguistics and Philosophy* 8: 387−413.

Xiang, Yimei. 2016. *Interpreting questions with non-exhaustive answers*. Doctoral Dissertation, Harvard University.

Yalcin, Seth. 2007. Epistemic modals. *Mind* 116(464): 983 –1026.

Yalcin, Seth. 2010. Probability operators. *Philosophy Compass* 5(11): 916–937.

Yang, Rong. 2001. *Common nouns, classifiers and quantification in Chinese*. Doctoral Dissertation, Rutgers University, New Brunswick.